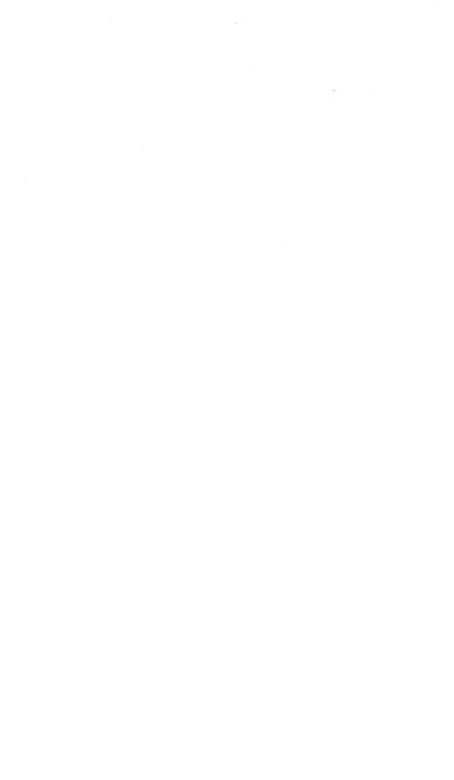




M Amackenzie
Tricily College Toronto







# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—Bacon.

## VOL. XXXIV.

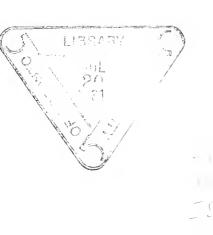
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# INSTITUTE OF ACTUARIES.

On the Treatment of Endowment Assurance Policies in Periodical Valuations. By H. Archer Thomson, B.A. Cantab., F.I.A., Assistant Actuary of the Hand-in-Hand Fire and Life Insurance Society.

[Read before the Institute, 20 December 1897.]

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ORIGIN AND GROWTH OF ENDOWMENT ASSURANCE BUSINESS.

ENDOWMENT ASSURANCE can boast of no great antiquity, though its constituent elements, the temporary assurance and the pure endowment, both date from early times. Traces of the latter arc found as long ago as the 16th century; thus, in a pamphlet by one Thomas Wilson, published in 1572, and entitled "A Discourse upon Usurie, by waie of Dialogue and oracions, for "the better varietie and more delight of all those that shall read "this treatise", it is stated that, "Whosoever lendeth such a summe "of money (£100), and hath a childe of one yeere, shall have for "his childe, if the same childe doe live till he be full fifteene years "of age, 500li (£) of money: but if the childe die before that "time, the father to lose his principal for ever." It is evident that the rate of mortality in those days must have been alarmingly high, either among children or among the institutions granting these benefits. A form of assurance, similar in some respects to the above, called "Apprenticeship Assurance", was introduced towards the beginning of the 18th century. In a prospectus dated 1710 it is stated that, "Anyone who now is, or that is to be an "apprentice, may by paying 1s. entrance and 2s. 6d. per quarter, "gain probably from £40 to £50 up to £500 or £600." reading this astonishing statement, one cannot but admit that the tontine and other estimates of American Companies, about which we hear so much in the present day, pale their uneffectual fire before the similar extravagant statements made by our predecessors in the reign of Queen Anne.

The temporary assurance is also undoubtedly one of the earliest forms of life assurance. But when, and with whom, the happy idea originated of wedding the term assurance to the pure endowment, is wrapped in mystery. A generation ago comparatively few offices granted endowment assurances, but at the present day there is not, I believe, a single life assurance company in this country that has not some of these policies on its books.

The growing popularity of this class of assurance is evidenced by the following table:

TABLE I.

| (1)                            | (2)  | (3)  | (4)                           |
|--------------------------------|--|--|-------------------------------|
| Blue Book<br>issued<br>in year | Total Assurances<br>in force, excluding<br>Re-assurances | Endowment Assurances in force, excluding Re-assurances | Ratio Col. (3)<br>to Col. (2) |
|                                | £  | €  |                               |
| 1888                           | 421,061,768  | 25,980,743   | .062                          |
| 1889                           | 437,510,512  | 31,368,522   | .072                          |
| 1890                           | 442,436,266  | 34,307,580   | .078                          |
| 1891                           | 443,362,228  | 35,866,867   | ·0S1                          |
| 1892                           | 455,184,135  | 42,483,492   | .093                          |
| 1893                           | 480,196,330  | 57,336,223   | .119                          |
| 1894                           | 490,856,082  | 64,769,652   | 132                           |
| 1895                           | 500,006,066  | 67,955,919   | 136                           |
| 1896                           | 529,184,344  | 85,144,665   | .161                          |
| 1897                           | 551,645,412  | 98,799,984   | 179                           |
| Total increase                 | 130,583,644  | 72,819,241   | .588                          |

As all companies do not collect their statistics on the same date, these figures are, of course, only approximate, and do not refer to particular epochs of time. For instance, the figures for the year 1897 are derived for the most part from the valuation returns of companies as on 31 December 1891, 1892, 1893, 1894, and 1895. It follows then that the figures, when they appear, represent the state of affairs rather more than three years previously. Hence an increase of £130,583,644 in the total sums assured and bonuses at risk in this country under all classes of policies, and of £72,819,241 in the amount assured under endowment assurances took place, roughly speaking, between the end of 1884 and 1893, the increase in the endowment assurances being considerably greater than that in all other kinds of assurance together. It does not follow that as much endowment assurance business has been written during the period as other business, because the total waste from deaths, &c., is much heavier under the latter than the former. The ratio of new endowment assurances to total new business varies in different companies from less than 1 per-cent to over 85 per-cent. In the most recent reports of individual companies there is every indication that the relative increase in this class of business has been well maintained, and it is probable that not less than 25 per-cent of the total assurances in force at the present time belongs to this category.

We are thus confronted with the fact that endowment assurance is steadily supplanting the whole-term policy as the staple commodity of life assurance. It seems probable that this rapid increase in the number of endowment assurance policies will lead most companies at no distant date to employ some method of grouping them in their periodical valuations, not for the mere purpose of placing a check upon a detailed valuation, as is often done at present, but in order to obtain an estimate of the requisite reserves sufficiently accurate for all purposes, the individual valuation and its accompanying labour being discarded unnecessary.

### DIFFERENT KINDS OF ENDOWMENT ASSURANCES AND APPROPRIATE VALUATION FACTORS.

Endowment assurances assume three forms.

Form 1. Where the sum assured is payable at a given age exactly, i.e., on a specified birthday, and premiums payable periodically until that age is reached. If m be the difference between the age next birthday at entry and the exact age at maturity, the number of years' premiums under the contract will be m+1 if payable yearly, m+1 or  $m+\frac{1}{2}$  if payable half-yearly, and m+1,  $m+\frac{3}{4}$ ,  $m+\frac{1}{2}$  or  $m+\frac{1}{4}$  if payable quarterly, the number of half-yearly and quarterly premiums payable depending on the interval elapsing between the date of the policy and the next birthday of the life assured. For such policies the following are the valuation factors:

VALUATION FACTORS FOR POLICIES OF FORM 1 Age nearest birthday at Number of Reversion Annuity date of Valuation Future Premiums  $\frac{1}{2}\left(1+a_{x:\overline{n-1}}+a_{x:\overline{n}}\right)$  $A_{x:n}$ r $n + \frac{1}{4}$  $\frac{1}{8}(4+3a_{x},\overline{n-1}+4a_{x},\overline{n}+a_{x},\overline{n+1})$  $A_{x:n+\frac{1}{4}}$  $\mathcal{T}$  $n + \frac{1}{2}$ , or  $n + \frac{2}{4}$  $\frac{1}{4}(2 + a_{x:\overline{n-1}} + 2a_{x:\overline{n}} + a_{x:\overline{n+1}})$  $A_x : \overline{n+1}$  $n + \frac{3}{4}$  $\frac{1}{5}(4+a_{x:\overline{n-1}}+4a_{x:\overline{n}}+3a_{x:\overline{n+1}})$  $A_{r:n+4}$ x

TABLE 2.

Form 2. Where the sum assured is payable at a given age exactly, i.e., on a specified birthday, as in Form 1, but the number of years' premiums is limited to the difference between the age at maturity and the age next birthday at entry. In policies of this form a period of from one to two years elapses after payment of the last annual premium before the endowment matures.

TABLE 3.

| Age nearest<br>birthday at<br>date of<br>Valuation | Number of<br>Future Premiums | Reversion  | Annuity  |
|--|------------------------------|--|--|
| x  | n (if annual)                | $\Lambda_{x:n+1}$                                    | $\frac{1}{2}\left(1+a_{x;\overline{n-1}]}+a_{x;\overline{n}}\right)$                                     |
| x  | n (if half-yearly)           | $\Lambda_{x:n+1}$                                    | Do.  |
| $\cdot v$  | n (if quarterly)             | $A_{\mathbb{Z}:n^{\frac{-}{4}\frac{-}{8}}}$          | Do.  |
| $\boldsymbol{x}$                                   | $n + \frac{1}{2}$            | $\mathbf{A}_{\mathbb{Z}:n\overline{+1}\overline{4}}$ | $\frac{1}{4}\left(2+\sigma_{x:\overline{n-1}}+2\sigma_{x:\overline{n}}+\alpha_{x:\overline{n+1}}\right)$ |
| x  | $n + \frac{1}{4}$            | $A_{x:\overline{n+	ilde{\hat{s}}}}$                  | $\frac{1}{\kappa} \left( 4 + 3a_{\kappa:n-1} + 4a_{\kappa:n} + a_{\kappa:n+1} \right)$                   |
| x  | $n + \frac{2}{4}$            | $\Lambda_{x;\overline{n+1\frac{1}{3}}}$              | $\frac{1}{4}(2 + \sigma_{x:\overline{n-1}} + 2\sigma_{x:\overline{n}} + \sigma_{x:\overline{n+1}})$      |
| x  | $n + \frac{3}{4}$            | $A_{x:\overline{n+1\frac{2}{8}}}$                    | $\frac{1}{8}(4+a_{x:\overline{n-1}}+4a_{x:\overline{n}}+3a_{x:\overline{n+1}})$                          |

Form 3. Where the sum assured is payable on an anniversary of the policy and not on a birthday, and the number of years' premiums is equal to the number of years the policy has to run, so that the last annual premium is payable exactly one year before the endowment matures.

TABLE 4.

| Age nearest<br>birthday at<br>date of<br>Valuation | Number of<br>Future Premiums | Reversion                                    | Annuity  |
|--|------------------------------|--|--|
| x  | n (if annual)                | $\Lambda_{x:\widehat{n}+1}$                  | $\frac{1}{2}\left(1+a_{\mathcal{L},\overline{n-1}}+a_{\mathcal{L},\overline{n}}\right)$                    |
| $\boldsymbol{x}$                                   | u (if half-yearly)           | $\Lambda_{x:ar{n}+rac{1}{4})}$              | Do.  |
| x  | n (if quarterly)             | $\Lambda_{x}, \overline{n+rac{1}{\lambda}}$ | Do.  |
| x  | $n + \frac{1}{2}$            | $\Lambda_{r:n+z}$                            | $\frac{1}{4} \left( 2 + a_{x:\overline{n-1}} + 2a_{x:\overline{n}} + a_{x:\overline{n+1}} \right)$         |
| x  | $n + \frac{1}{4}$            | $\Lambda_{x:\overline{n+3}}$                 | $\frac{1}{2} \left( 1 + 3a_{x;n-1 } + 4a_{x;\overline{n}} + a_{x;\overline{n+1}} \right)$                  |
| x  | $n + \frac{2}{4}$            | $\Lambda_{x:\overline{n+rac{\xi}{2}}}$      | $\frac{1}{4}\left(2+a_{x;n+1}+2a_{x;\overline{n+}}+a_{x;\overline{n+1}}\right)$                            |
| $\cdot$  | $n + \frac{n}{4}$            | $\Lambda_{x:\widehat{n}}$                    | $\frac{1}{5} \left( 1 + a_{x, \overline{n} - 1} + 1 a_{x, \overline{n}} + 3 a_{x, \overline{n+1}} \right)$ |

In the above expressions the usual assumptions are made as to the policies being effected on an average midway between two birthdays, and as to the premiums being evenly distributed over the calendar year. Each of these assumptions involves a small error. The first slightly under-estimates the value of the reversion and therefore the liability in Forms 1 and 2, but does not affect Form 3. The second error will be removed if the valuation factors be determined by the "Nearest Renewal Date" method, referred to later on. At the present day endowment assurances are usually issued in Form 3, which seems to have found favour with the public, and to be supplanting the two older forms, several companies having ceased to issue policies in Forms 1 and 2, and having adopted the more popular Form 3.

The first form of policy is open to the obvious objection that a premium may be payable almost immediately before the sum assured becomes due, and the policyholder feels aggrieved at what he considers a deduction of one premium from the sum assured, and perhaps offers, as a compromise, to pay a proportionate part of the premium for the short period intervening. Sometimes, when such a case arises, the company, finding perhaps that its present rate for a similar policy under Form 3, with only m premiums to pay, is actually less than that which has been charged under Form 1 with m+1 premiums to pay, will waive the payment of the last premium altogether, at the same time taking the opportunity of calling attention to its marked generosity in its dealings with policyholders. Presumably, companies in the habit of adopting this course make reserves beforehand for their prospective generosity by ignoring in their valuations the last premiums payable under policies of this form.

THE "VALUATION DATE", "NEAREST RENEWAL DATE", AND "NEXT ANNIVERSARY" METHODS OF OBTAINING THE VALUATION FACTORS.

Instead of taking the valuation factors on the date of valuation, as in the expressions given above, we may take the valuation age as before, but otherwise value the policies as on the nearest renewal date. If, for instance, there be n annual premiums remaining to be paid at the date of valuation, then the annuity of n payments will be used if the nearest renewal date falls in the preceding six months (since the premium then due has been paid), and the annuity-due of n payments will be

used if the nearest renewal date falls in the following six months (since the premium then due has not been paid), (Text-Book, Part II, chap. xviii, § 104). This method of obtaining the valuation factors is superior to the previous one, because it automatically makes a correction for the premiums not being evenly distributed over the year, which is otherwise somewhat troublesome to estimate.

Some companies calculate the valuation factors as on the next anniversary of the policy, assuming, of course, the premium then due to be unpaid. This method is scarcely so accurate as the two preceding, but has the advantage of simplicity. For instance, if the policy be of *Form* 3 and the premiums be payable annually, the reversion to be used is  $A_{x+t:\overline{n}}$ , x being the age at entry, t the number of premiums paid, and n the number of future premiums. Similarly, if the premiums be payable annually, the annuity is, of course,  $1 + a_{x+t:\overline{n-1}}$ .

In the case of policies renewable more often than annually, we should, in order to be strictly accurate, use  $P^m$  and  $a^m$ , but it is customary to treat them as annual and to employ the premium and annuity payable annually, it being assumed that

$$\left(\frac{1}{m} + a_{x+t}^{(n)}, \overline{1 - \frac{1}{n}}\right) P_{x, \overline{n+t}}^{(n)} = (1 + a_{x+t}, \overline{1 - 1}) P_{x, \overline{n+t}}$$

a relation which, of course, is only strictly true when t=0, i.e., at the inception of the policy.

For convenience, we may call these the "Valuation Date", the "Nearest Renewal Date", and the "Next Anniversary" methods of obtaining the valuation factors.

Comparison of the Reserves brought out by the "Valuation Date", "Nearest Renewal Date", and "Next Anniversary" Methods of obtaining the Valuation Factors.

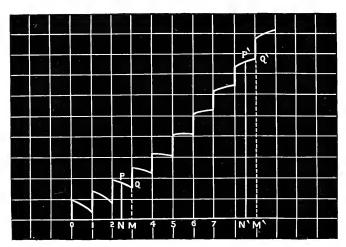
The first two methods have, practically, the same effect, if the premiums be evenly spread over the calendar year. The "Next Anniversary" method does not necessarily bring out larger reserves than the other two methods. This will depend on what proportion of the endowment period has elapsed. In the earlier years of assurance the reserve will be smaller under the "Next Anniversary" method, and in the later years larger.

In the following diagram (I) the curve represents roughly the reserve at various stages under an endowment assurance policy, assumed to mature at a reasonable age.

#### Diagram I.

Abscissa represents duration of policy.

Ordinate represents reserve on an Endowment Assurance policy at annual premiums.



At the beginning of each policy-year there is a constant increment equal to the net premium, and during the year the reserve steadily falls, but by a less amount each year, until, when a certain stage is reached, the reserve thereafter rises during each successive year.

The valuation date will fall between two anniversaries of the policy, say, at N and at N', and the true reserves are P N and P' N', while the reserves brought out by the "Next Anniversary" method of valuation are Q M and Q' M' respectively, the former (Q M) being too small and the latter (Q' M') too large. In the case of new business, the "Next Anniversary" method may very well bring out reserves too small by 2 or 3 per-eent.

On an H<sup>M</sup>  $2\frac{1}{2}$  per-cent basis, for policies maturing at age 55, the point at which the reserve ceases to decrease during the year is reached when the policy has run about one-third of its course, no matter what the age at entry may have been; and the same applies to policies maturing at age 60, if the rate of interest be 3 per-cent, and at age 65 if  $3\frac{1}{2}$  per-cent. For older ages at maturity, in each case this stage will not be reached until rather more than one-third of the premiums have been paid, and *vice versa*.

This point may be treated analytically as follows, either for whole-term policies or endowment assurances:

$$\begin{split} t \mathbf{V}_x + \mathbf{P}_x > &= <_{t+1} \mathbf{V}_x \\ \text{according as } v(q_{x+t} + p_{x+t} \times_{t+1} \mathbf{V}_x) > &= <_{t+1} \mathbf{V}_x \\ ,, \qquad q_{x+t} > &= <(1+i-p_{x+t})_{t+1} \mathbf{V}_x \\ ,, \qquad q_{x+t} > &= <(i+q_{x+t})_{t+1} \mathbf{V}_x \\ \text{Hence} \quad t \mathbf{V}_x + \mathbf{P}_x =_{t+1} \mathbf{V}_x \\ \text{if} \qquad q_{x+t} = (i+q_{x+t})_{t+1} \mathbf{V}_x \\ i.e., \text{ if} \qquad q_{x+t} = i(_t \mathbf{V}_x + \mathbf{P}_x) + q_{x+t} \times_{t+1} \mathbf{V}_x \\ i.e., \text{ if } i(_t \mathbf{V}_x + \mathbf{P}_x) = (1-_{t+1} \mathbf{V}_x) q_{x+t} \end{split}$$

i.e., if the interest on the reserve with the premium at the beginning of the year is equal to the expected death-strain during the year, as might have been anticipated.

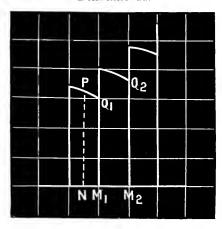
With whole-life policies valued by the  $H^M$  Table this stage is never reached, the interest being always less than the expected death-strain, unless the policy be effected at a comparatively young age, and even in that case the yearly increase in the reserve does not continue, for as the life grows old and q becomes large, the interest becomes again less than the expected death-strain during the year.

In half-yearly eases, when the anniversary of the policy falls in the first half of the year, the reserve brought out by the "Next Anniversary" method will be less than the reserve by the "Valuation Date" method in the early years of assurance for similiar reasons to those above, but will generally be greater if the anniversary fall in the second half of the year, because one more premium is assumed to have been paid than is actually the ease.

In the accompanying diagram (II), N represents the valuation date, P N the true reserve at that date, and  $Q_1$   $M_1$ ,  $Q_2$   $M_2$  represent the reserves by the "Next Anniversary" method, according as the anniversary falls in the first or second half of the year,  $M_1$   $M_2$  representing a period of six months.

Similar remarks apply to quarterly eases.

#### DIAGRAM II.



VALUATION OF ENDOWMENT ASSURANCES IN GROUPS. THREE CLASSES OF METHODS.

In an interesting paper read before this Institute on 30 December 1895 by Mr. Frederick Schooling, "On some methods of grouping Policies for the purpose of Valuation", a paragraph was devoted to endowment assurances, to which a prominent place was assigned in the discussion that followed. that the subject is not yet exhausted, especially as regards the difference that may be expected between the results of valuation individually and in groups, or, in other words, the errors in the latter, I have ventured to bring the subject forward again for discussion.

It has been suggested that the variety in the forms of endowment assurances in existence, which has already been referred to, presents a barrier to the successful employment of any method of valuation in groups. But, if a company has policies of Forms 1 and 3, say, on its books, the former can easily be separated from the latter, and the method applied to each batch separately. In most cases there will have been some definite epoch at which the one form of contract was discarded and the other adopted, so that the separation, if considered necessary, would involve no trouble whatever.

The principal methods of valuing endowment assurances in groups may be divided into the following classes:

Class 1.—Where the groups depend both on the valuation age and the age at maturity.

Class 2.—Where the groups depend on fixed values of the reversion or of the annuity for valuing the premiums, but the valuation age and age at maturity are otherwise ignored.

Class 3.—Where the groups depend on the calendar year of maturity or on the number of future premiums at the date of valuation, an average valuation age being employed in each group.

# Class 1.—Where the Groups depend on the Valuation Age and the Age at Maturity.

Method I.—The method most often adopted is to divide the policies into batches depending on the age at maturity, and to sub-divide each batch into groups of policies having the same valuation age. The reversion and annuity employed in valuing a group are identical with those that would be used for each policy of the group in a detailed valuation, so that no error is involved by the method. If all the policies mature at quinquennial ages, the number of groups is considerable, while if the policies mature at intermediate ages also, as is the case with many companies, the number of groups is practically multiplied by five, and the labour of valuation is enormous.

Method II.—A modification of the above method is sometimes employed, whereby the number of groups is rendered less unwieldy. All policies maturing at other than quinquennial ages are treated as maturing at the nearest quinquennial age, and a corresponding alteration is made in the age at entry (and therefore in the net premium) and in the valuation age, so that the original term of the policy and the term unexpired both remain unchanged. Thus a policy effected at age 33 and maturing at 63, would be treated throughout as if effected at 35 and maturing at 65. The greatest difference between the true ages and the assumed ages is two years, and the consequent error in the value of any individual policy is very small.

According to the last valuation returns of a company which values its endowment assurances by this method, the number of resulting groups was 300. This, however, was rather an extreme case, as both the valuation ages and the ages at maturity extended over unusually wide ranges, namely, from 8 to 68 and from 35 to 75 respectively. Nevertheless, even in a normal case the number of groups involved by the method is considerable.

Method III.—Select ages at maturity differing by, say, 10 or 11 years, the number of such selected ages depending on the range

and incidence of the true ages at maturity of the policies to be valued. Assume each policy to mature at the selected age nearest to its true age at maturity, and make a corresponding alteration in the age at entry and the valuation age, the necessary increase or decrease in the ages will be 0, 1, 2, 3, 4, or 5 years, and the original term and the term unexpired will remain unchanged. The net premium is at the outset calculated from the assumed age instead of the true age, and remains fixed throughout the duration of the policy. In most cases three selected ages would be sufficient; if there be no policies maturing at ages beyond 65, two selected ages would generally suffice; if, further, policies maturing at the younger ages be excluded, so that the ages at maturity extend over about 15 years only, reliable results would generally be obtained by using one central age only, provided the number of policies is sufficiently large. We thus obtain one, two, or three batches of policies, as the case may be, and in each batch the policies with the same number of future premiums will have the same valuation age and can be valued together in a group by means of one reversion and one annuity.

This method is merely an extension of *Method* II, but it removes its principal defect, viz., the largeness of the number of groups. For instance, if the ages at entry be greater than 20, and the policies mature between 40 and 65, and two maturityages be selected, e.g., 45 and 60, or 47 and 58, the maximum number of groups would be only 77.

In altering the ages in the manner described above, there is a balance of error, because, on an average, the ages at entry, valuation and maturity are as often increased as decreased; and there is, as it were, a further balance of error, since an increase in the ages will sometimes cause an increase and sometimes a decrease in the reserve. An addition to the valuation age will increase the reversion and diminish the annuity, and therefore increase the liability, while an addition to the age at entry will increase the premium valued and therefore diminish the liability, and the effect of the latter is often more than sufficient to counterbalance the effect of the former. It is difficult to formulate a general rule showing in which cases the reserve will be increased, and in which cases diminished by an addition to the ages, as there are three variables involved, viz., the age at entry, the original term, and the duration of the policy. Generally speaking, however, if the original term be short, the reserve will be decreased

by an addition of a few years to the ages, and if the original term be long, say over 25 years, the reserve will (for ages at entry over 20) be increased, except when the policy is approaching maturity, when, whatever the original term, the reserve will be decreased.

The following eight diagrams, numbered III, IV, . . . X, relate to endowment assurance policies of 10, 15, . . . 45 years' term. In any diagram each curve represents the reserves (less a constant) after a specified number of years on policies of the same original term (denoted in the heading of the diagram), the abscissa representing the age at entry. The constants have been deducted from the reserves in order to keep the diagrams within reasonable limits. The relative heights of the ordinates cannot be compared, because the curves have different imaginary base lines lying below the axis of x, the object being merely to show the shape of the curves and the differences of the ordinates. Beneath each diagram is given a table showing the corresponding reserves.

It will be noticed that the curves are, for the most part, fairly level, and that the differences in the reserves caused by an alteration of five years in the ages at entry, valuation and maturity, are generally very small and never considerable. In the specimen reserves on policies for £100 represented in the diagrams, this difference is greater than unity in 16 cases out of 169. The corresponding portions of the curves are marked thus (\*). If policies maturing above age 70 be omitted, this difference is greater than unity in only 4 cases out of the remaining 138, all of which occur when the age at maturity is altered from 70 to 65.

It is interesting to compare these differences in endowment assurance reserves with the errors involved in an ordinary valuation of whole-life policies.

If x be the age at entry, and x+t the age nearest birthday at date of valuation,

$$\begin{split} \text{Reserve } &= \mathbf{A}_{x+t} - (\tfrac{1}{2} + a_{x+t}) \mathbf{P}_x \\ &= \tfrac{a_x - a_{x+t}}{1 + a_x} + \tfrac{1}{2} \mathbf{P}_x \end{split}$$

So that the error in the reserve on a whole-life policy caused by the largest possible error in the valuation age, viz., six months, is

$$\frac{\frac{1}{2}\Delta a_{x+t}}{1+a_x}.$$

The differences in the endowment assurance reserves depicted in these diagrams, caused by an alteration of five years in the ages, are, in most cases, considerably less than the largest possible error in the reserve on a corresponding whole-life policy, by which is meant a policy effected at the same age, and carrying a reserve of about the same amount. In only 19 cases out of the 169 are the differences greater than the possible errors for the corresponding whole-life policies. Such portions of the curves are marked thus (‡). If policies maturing above age 70 be omitted, the number of such cases is reduced to 9 out of the remaining 138, and if policies effected under age 20 be also excluded, the number is further reduced to 4 out of the remaining 115 cases.

It appears then, that if we assume all policies to mature at certain selected ages, differing by 10 or 11 years (the greatest alteration in the ages for any policy being then five years), and if we value them by the present method, the results will be sufficiently accurate.

As stated above, maturity-ages still further apart might safely be selected in many cases.

#### DIAGRAM III.

Endowment Assurance Policies of 10 years term.

HM 3 per-cent Reserves.

Abscissa represents age at entry.

Ordinates represent  $_3V_{x:\overline{10}}$  -23.5,  $_5V_{x:\overline{10}}$  -41,  $_7V_{x:\overline{10}}$  -61.

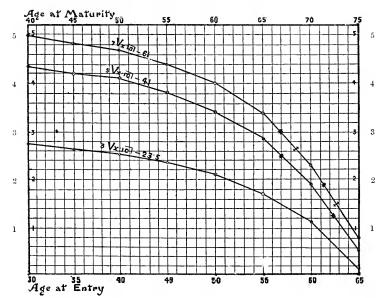


Table 5.—HM 3 per-cent.

| Age at<br>Entry | $_3V_{z:1\overline{0}}$ | $_{5}\mathrm{V}_{z\cdot\overline{10}}$ | , V <sub>z:10</sub> | Age at<br>Maturity |
|-----------------|-------------------------|--|---------------------|--------------------|
| 30              | 26:238                  | 45.380                                 | 65.983              | 40                 |
| 35              | 26.135                  | 45.217                                 | 65·\$29             | 45                 |
| 40              | 26.029                  | 45.102                                 | 65.704              | 50                 |
| 45              | 25.884                  | 44.829                                 | 65.413              | 55                 |
| 50              | 25 603                  | 44.448                                 | 65.015              | 60                 |
| 55              | 25.253                  | 43.891                                 | 64.390              | 65                 |
| 60              | 24.635                  | 42.883                                 | 63.291              | 70                 |
| 65              | 23.656                  | 41.525                                 | 61.827              | 75                 |

#### DIAGRAM IV.

Endowment Assurance Policies of 15 years term.

H<sup>M</sup> 3 per•cent Reserves.

Abscissa represents age at entry.

Ordinates represent  $_{5}\overline{V}_{x:\overline{15}} - 25.5$ ,  $_{10}V_{x:\overline{15}} - 56$ .

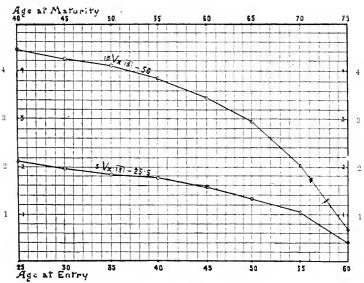


Table 6.—HM 3 per-cent.

| Age at<br>Entry | <sub>5</sub> V <sub>z:15</sub> | $_{10}V_{z:\overline{15}}$ | Age at<br>Maturity |
|-----------------|--------------------------------|----------------------------|--------------------|
| 25              | 27:624                         | 60.467                     | 40                 |
| 30              | 27.468                         | 60.266                     | 45                 |
| 35              | 27.311                         | 60.095                     | 50                 |
| 40              | 27.286                         | 59.883                     | 55                 |
| 45              | 27:061                         | 59.481                     | 60                 |
| 50              | 26.854                         | 58.953                     | 65                 |
| 55              | 26.574                         | 58.062                     | 70                 |
| 60              | 25.896                         | 56.668                     | 75                 |

#### DIAGRAM V.

Endowment Assurance Policies of 20 years term.

 ${\rm H}^{\rm M}$  3 per-cent Reserves.

Abscissa represents age at entry.

Ordinates represent  ${}_5V_{\chi,\overline{20|}}-18.5,\ {}_{10}V_{\chi,\overline{20|}}-39,\ {}_{15}V_{\chi,\overline{20|}}-63.$ 

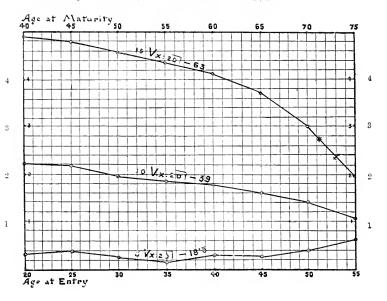


Table 7.—HM 3 per-cent.

| Age at<br>Entry | $_{5}\mathrm{V}_{x}$ : $_{20}$ | $_{10}\mathrm{V}_{x},_{\overline{20} }$ | $_{15}\mathbf{V}_{x},\overline{_{20}},$ | Age at<br>Maturity |
|-----------------|--------------------------------|---|---|--------------------|
| 20              | 18.806                         | 41.234                                  | 67.902                                  | 40                 |
| 25              | 18.913                         | 41.185                                  | 67.780                                  | 45                 |
| 30              | 18.776                         | 40.959                                  | 67.587                                  | 50                 |
| 35              | 18.686                         | 40.873                                  | 67:379                                  | 55                 |
| 40              | 18.801                         | 40.774                                  | 67.099                                  | 60                 |
| 45              | 18.773                         | 40.578                                  | 66.659                                  | 65                 |
| 50              | 18.913                         | 40.461                                  | 65.993                                  | 70                 |
| 55              | 19.149                         | 40.087                                  | 64.967                                  | 75                 |

## DIAGRAM VI.

#### Endowment Assurance Policies of 25 years term.

H<sup>M</sup> 3 per-cent Reserves.

Abscissa represents age at entry.

 $\text{Ordinates represent } {}_5V_{x:2\overline{5}} - 13 \cdot 5, \ {}_{10}V_{x:2\overline{5}} - 29 \cdot 5, \ {}_{15}V_{x:2\overline{5}} - 47, \ {}_{20}V_{x:2\overline{5}} - 68.$ 

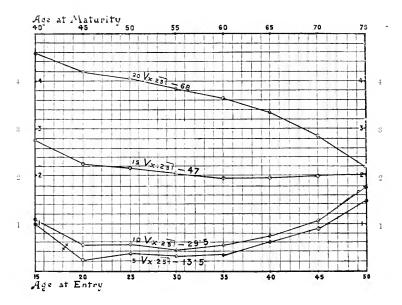


Table 8.—HM 3 per-cent.

| Age at<br>Entry | $_{5}\mathrm{V}_{x}_{:\overline{25}}$ | $_{10}\mathrm{V}_{x:\overline{25}}$ | 15V <sub>x.25</sub> | 29 V <sub>21:25</sub> | Age at<br>Maturity |
|-----------------|---------------------------------------|-------------------------------------|---------------------|-----------------------|--------------------|
| 15              | 14.503                                | 30.582                              | 49.757              | 72.557                | 40                 |
| 20              | 13.714                                | 30.033                              | 49.252              | 72.199                | 45                 |
| 25              | 13.869                                | 30.041                              | 49.147              | 72.083                | 50                 |
| 30              | 13.797                                | 29.905                              | 49.031              | 71.880                | 55                 |
| 35              | 13.815                                | 30.019                              | 48.956              | 71.644                | 60                 |
| 40              | 14.125                                | 30.247                              | 48.972              | 71:369                | 65                 |
| 45              | 14.355                                | 30.579                              | 49.025              | 70.886                | 70                 |
| 50              | 14.971                                | 31.253                              | 49.056              | 70.210                | 75                 |

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#### DIAGRAM VII.

### Endowment Assurance Policies of 30 years term.

HM 3 per-cent Reserves.

Abscissa represents age at entry.

Ordinates represent  ${}_{5}V_{x:\overline{30}} - 10^{\circ}4$ ,  ${}_{10}V_{x:\overline{30}} - 22^{\circ}4$ ,  ${}_{15}V_{x:\overline{30}} - 36^{\circ}5$ ,  ${}_{20}V_{x:\overline{30}} - 52$ ,  $_{25}V_{x:\overline{30}}-70.$ 

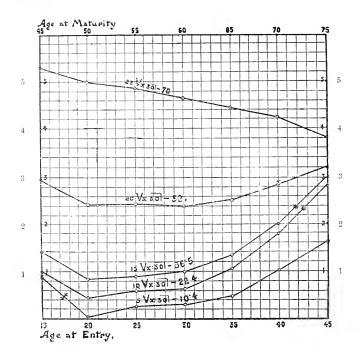


Table 9.—HM 3 per-cent.

| Age at<br>Entry | $_{5}\mathrm{V}_{x:\overline{30}]}$ | 10Vx:30 | 15V <sub>x:30</sub> | 20V <sub>x:30</sub> | 25V <sub>x:30</sub> | Age at<br>Maturity |
|-----------------|-------------------------------------|---------|---------------------|---------------------|---------------------|--------------------|
| 15              | 11.262                              | 23.432  | 37:913              | 54.967              | 75.330              | 45                 |
| 20              | 10.466                              | 22.883  | 37.363              | 54.469              | 75.004              | 50                 |
| $^{25}$         | 10.689                              | 23.011  | 37:397              | 54.479              | 74.885              | 55                 |
| 30              | 10.705                              | 23.042  | 37.512              | 54.421              | 74.680              | 60                 |
| 35              | 10.874                              | 23.464  | 37.832              | 54.521              | 74.482              | 65                 |
| 40              | 11.433                              | 24.176  | 38.516              | 54.854              | 74.215              | 70                 |
| 45              | 12.023                              | 25.194  | 39.518              | 55.181              | 73.792              | 75                 |

#### DIAGRAM VIII.

Endowment Assurance Policies of 35 years term.

HM 3 per-cent Reserves.

Abscissa represents age at entry.

 $\text{Ordinates represent } _5V_{\chi;\,3\overline{5}|} - 8\cdot 2, \ _{10}V_{\chi;\,\overline{35}|} - 17\cdot 5, \ _{20}V_{\chi;\,3\overline{5}|} - 41\cdot 5, \ _{30}V_{\chi;\,\overline{35}|} - 72.$ 

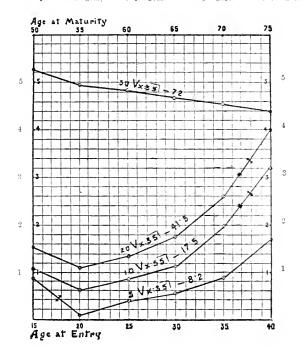


Table 10.—HM 3 per-cent.

| Age at<br>Entry | 5 V <sub>x:35</sub> | 10 V <sub>x:35</sub> | 20 V <sub>x:35</sub> | $_{30}$ V $_{x\cdot \overline{35} }$ | Age at<br>Maturity |
|-----------------|---------------------|----------------------|----------------------|--------------------------------------|--------------------|
| 15              | 9.062               | 18.579               | 43.038               | 77:270                               | 50                 |
| 20              | 8.291               | 18.093               | 42.587               | 76.968                               | 55                 |
| 25              | 8.594               | 18.379               | 42.881               | 76.856                               | 60                 |
| 30              | 8.728               | 18.653               | 43.258               | 76.709                               | 65                 |
| 35              | 9.084               | 19.478               | 44.101               | 76.558                               | 70                 |
| 40              | 9.906               | 20.738               | 45.509               | 76.387                               | 75                 |

#### DIAGRAM IX.

#### Endowment Assurance Policies of 40 years term.

HM 3 per-cent Reserves.

Abscissa represents age at entry.

Ordinates represent  ${}_{5}V_{x;\overline{40}} - 6.8$ ,  ${}_{10}V_{x;\overline{40}} - 14$ ,  ${}_{20}V_{x;\overline{40}} - 33$ ,  ${}_{30}V_{x;\overline{40}} - 57.5$ ,  $_{35}V_{x:\overline{40}} - 72.5$ .

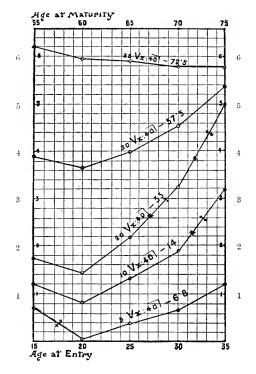


Table 11.—HM 3 per-cent.

| Age at<br>Entry | $_{5}\mathrm{V}_{x:\overline{40}}$ | 10Vx:40          | $_{20}\mathrm{V}_{x:\overline{40}}$ | $_{30}\mathrm{V}_{x:\overline{40} }$ | 35 V <sub>x:40</sub> | Age at<br>Maturity |
|-----------------|------------------------------------|------------------|-------------------------------------|--------------------------------------|----------------------|--------------------|
| 15              | 7.528                              | 15.196           | 34.711                              | 61:395                               | 78.705               | 55                 |
| 20<br>25        | 6·799<br>7·203                     | 14·808<br>15·302 | 34·438<br>35·176                    | 61·170<br>61·480                     | 78·429<br>78·387     | 60<br>65           |
| 30<br>35        | 7·483<br>8·038                     | 15.887 $17.148$  | $36.222 \\ 38.022$                  | 62·027<br>62·867                     | 78.312 $78.287$      | 70<br>75           |

#### DIAGRAM X.

## Endowment Assurance Policies of 45 years term.

H<sup>M</sup> 3 per-cent Reserves.

Abscissa represents age at entry.

Ordinates represent  ${}_5V_{x:\overline{45}}=5.5$ ,  ${}_{15}V_{x:\overline{45}}=19$ ,  ${}_{25}V_{x:\overline{45}}=37$ ,  ${}_{35}V_{x:\overline{45}}=60$ ,  ${}_{40}V_{x:\overline{45}}=74$ .

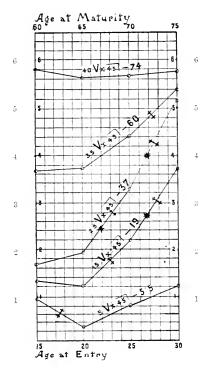


Table 12.—H<sup>M</sup> 3 per-cent.

| Age at<br>Entry | 5V <sub>x:45</sub> | 15V <sub>x:45</sub> | $_{25}V_{x:45}$ | 35V <sub>x:45</sub> | $_{40}\mathbf{V}_{x:\overline{45}}$ | Age at<br>Maturity |
|-----------------|--------------------|---------------------|-----------------|---------------------|-------------------------------------|--------------------|
| 15              | 6.444              | 20:298              | 38.661          | 63.672              | 79:820                              | 60                 |
| 20              | 5.780              | 20.198              | 35.922          | 63.707              | 79.636                              | 65                 |
| 25              | 6.300              | 21.186              | 40.239          | 64.418              | 79.678                              | 70                 |
| 30              | 6.737              | 22.731              | 42.197          | 65:369              | 79.750                              | 75                 |

CLASS 2.—WHERE THE GROUPS DEPEND ON FIXED VALUES OF THE REVERSION OR ANNUITY.

Method IV.—In 1889 the late Mr. Woolhouse read a paper before this Institute on "An easy method of getting out a rough estimate valuation of a whole-life assurance business." (J.I.A., xxvii, 433.)

His method is briefly as follows: Separate the groups by the hypothetical series of values A=15, 25, . . . 85, 95; then, assuming an even distribution, the average values of A for the successive groups are 2, 3, 4, 5, 6, 7, 8, 9. Let S, P, denote the amount of sums assured and net premiums respectively in a group, and let  $\Sigma$  denote summation for all the groups, then the total value of the sums assured  $\Sigma$ 

Also since 
$$1 + a = (1 - A)\left(1 + \frac{1}{i}\right)$$

$$\therefore (1 + a)P = (P - AP)\left(1 + \frac{1}{i}\right)$$

$$\therefore \Sigma\{(1 + a)P\} = \{\Sigma P - \Sigma(AP)\}\left(1 + \frac{1}{i}\right)$$

Giving to A the values 2, 3, ... &c., as before, the last expression is readily evaluated, and the total value of the net premiums is obtained by deducting therefrom  $\frac{1}{2}\Sigma P$ , or some similar expression depending on the incidence of the premium income over the year.

Mr. Woolhouse pointed out that "Endowment assurances are in all respects analogous to ordinary whole-life assurances if we only assume the purely imaginary hypothesis that, on reaching the stipulated age, all the then existing lives become immediately extinct, as they do at the end of an ordinary mortality table", and that, using  $a_{x:\overline{n-1}}$  for  $a_x$  and  $A_{x:\overline{n}}$  for  $A_x$ , and forming a skeleton table of the values of  $A_{x:\overline{n}}$ , the above method would apply equally well to endowment assurances.

If the "Next Anniversary" method of obtaining the valuation factors be employed, no deduction from the above expression for  $\Sigma\{(1+a)P\}$  is necessary. If, however, the "Valuation Date" method be used, we must use the reversion  $A_{x:\overline{n+\frac{1}{2}}}$  and the corresponding annuity  $\frac{1}{2}(a_x:\overline{n-1}]+a_x:\overline{n}$ ), and we must deduct  $\frac{1}{2}\Sigma P$  from the expression for  $\Sigma\{(1+a)P\}$ ; if the premium income is

not evenly spread over the year, a further adjustment must be made.

Mr. Woolhouse stated that his method was not put forward as a substitute for ordinary methods of valuation, but merely as an auxiliary check upon them.

Method V.—As we have tables of  $a_{x:\overline{n-1}}$  for all values of x and n, but  $A_{x:\overline{n}}$  on the contrary is only tabulated for quinquennial values of x+n, and it is necessary for other ages at maturity to enter conversion tables with the corresponding annuity, it will obviously save some trouble if the groups be segregated according to selected values of the annuity rather than of the reversion. We may, for instance, separate the groups by the hypothetical series of values a=5,  $1\cdot5$ ,  $2\cdot5$ ,  $3\cdot5$ , &c., and assume the average values of a in the groups to be  $1, 2, 3, \ldots$ ; then the value of the net premiums is  $\Sigma(aP) + \frac{1}{2}\Sigma P$  for whole-term policies, and  $\Sigma(aP) + \Sigma P$  for Endowment Assurances valued by the "Next Anniversary" method.

Also since A = 1 - d(1 + a) = v - da,

 $\therefore$  value of sums assured  $= \Sigma(AS) = v\Sigma S - d\Sigma(aS)$ .

We thus obtain an expression for the value of the sums assured analogous to Mr. Woolhouse's expression for the value of the net premiums; it would, however, take but little more time to obtain A for each group by entering conversion tables with the values, a=1,2,3, &c., and to multiply out in the ordinary way. The values of A will, of course, be v-d, v-2d, v-3d. . . . . The number of groups in this method will be  $\frac{1}{d}$  times the number in the last method, and the working will be proportionately heavier, but as the largest possible individual errors in the values of the sum assured and net premiums will be reduced in the ratio of  $\frac{d}{1}$  (or say  $\frac{1}{4}$ , if the rate of interest be  $2\frac{1}{2}$  per-cent), the results will be more accurate.

Method VI.—If instead of a mere check valuation we require to bring out results on which reliance can be placed, it will be necessary to separate the series of hypothetical annuity-values by smaller intervals than unity. It will be quite sufficient to take intervals of 25, the greatest difference between the true and assumed values of the annuity will then be 125, and the largest possible individual errors per unit assured will be \$\frac{1}{2}d\$ in the

value of the sum assured and  $\frac{1}{8}P$  in the value of the net premiums. The error in the value of each is equally likely to be positive or negative, and the results brought out by the method will be very near the truth.

Method VII.-A method of valuing endowment assurance policies by means of whole-life reversions and annuities has been in use for a long time in a company having a considerable number of such policies on its books. Instead of the temporary annuity for valuing the premiums, the whole-life annuity nearest to it in value is taken, and the corresponding reversion employed. For instance, if the valuation age be 40 and the number of future premiums 10, then using the HM 21 per-cent table and the "Valuation Date" method, the annuity for valuing the premiums =  $\frac{1 + a_{x:\overline{0}} + a_{x:\overline{10}}}{9} = 8.397 = \frac{1}{2} + a_{67}$  nearly, and the policy is treated as a whole-term policy at age 67, the premium valued being of course the true endowment assurance premium.

The method is substantially the same in principle as the Suppose for instance that in the valuation of an endowment assurance business the temporary annuities range in value from 0 up to 20; then in Method VI we employ, instead of the true temporary annuity, the nearest of the series of 81 hypothetical annuity-values 20, 19.75, 19.5, 19.25, .........5, 25, 0, and in Method VII we take the nearest of the series of 63 hypothetical annuity-values 20:006, 19:687, 19:365, . . . . 418, 179, 0, these being the ordinary whole-life annuities; the only difference between the two methods is that in the former the selected values are equi-different, and in the latter they are not, the intervals ranging from 173 to 404. At the ordinary valuation rates of interest the average decrement in  $a_x$  is > 25, so that there is a larger margin of possible error in the present method than in the last, but the number of groups involved will generally be slightly less.

CLASS 3 .- WHERE THE GROUPS DEPEND ON THE CALENDAR YEAR OF MATURITY OR ON THE NUMBER OF FUTURE Premiums.

Whether we group the policies according to calendar year of maturity or according to the number of future premiums is

immaterial if the premiums be payable yearly. For instance, if the policies be of Form 3, described above, the sum assured being payable exactly one year after the last renewal date, and if the "Valuation Date" method of obtaining the valuation factors be employed, x being the age nearest birthday at the date of valuation, say 31 December 1897, then the reversion and annuity for a policy maturing in 1910, or, having 12 premiums remaining to be paid, are

 $A_{x:\overline{12}\frac{1}{2}}$  and  $\frac{1+a_{x:\overline{11}}+a_{x:\overline{12}}}{2}$  by both systems of grouping. If,

however, the premiums are payable half-yearly, there may be either 12 or  $12\frac{1}{2}$  future premiums on a policy maturing in 1910. The former system takes no account of this, and therefore slightly overstates the liability on such policies; similar remarks apply to quarterly cases, so that practically the calendar-year-of-maturity system treats cases where there are  $n+\frac{1}{4}$ ,  $n+\frac{1}{2}$ , and  $n+\frac{3}{4}$ , future premiums, just as if there were only n premiums. Of course if we treat such cases as forming separate groups in employing the alternative system, the number of groups, and therefore the labour involved, is considerably increased, but this disadvantage may be met, without much loss of accuracy, by taking  $n+\frac{1}{4}$  and  $n+\frac{3}{4}$  as n and n+1, respectively, and  $n+\frac{1}{2}$  alternately as n and n+1.

After grouping the policies according to the number of future premiums, there are various ways of proceeding to obtain an average valuation age, X, for each group. In all the methods, after calculating X (which is usually fractional, and is taken to the nearest integer or to the nearest first decimal place), the corresponding reversion and annuity are employed for the valuation of the total sums assured and the total net premiums of the group.

Let S, P and x represent the sum assured, net premium, and valuation age respectively of a policy, and X the average valuation age for a group (obtained by one of the following methods), and let  $\Sigma$  denote summation for all the policies in a group.

Method VIII.— $X = \frac{\Sigma(x)}{m}$ , where m = number of policies in a group.

The results of a valuation on this basis cannot be depended on, the error may be considerable. Moreover, we cannot tell beforehand whether the liability brought out will be too large or too small, the balance of probability, however, is in

favour of its being understated. The margin of possible error will be greatly reduced by the employment of one or other of the following methods, the first two of which involve very little additional labour.

Method IX.—
$$X = \frac{\Sigma(xS)}{\Sigma(S)}$$
.

This method often produces a greater error in the value of the premiums than in the value of the sum assured. partly due to valuing the premiums by the annuity corresponding to an average age which is found by weighting the individual valuation ages with the sums assured instead of with the premiums, while the latter are not proportional to the former. For instance, take the group of policies having 9 future premiums, the original terms of two such policies may have been, say, 10 years and 40 years respectively, in which case, if the policies be of equal amounts, the premium on one would be roughly four times that on the other: so the weights to be applied to the valuation ages in obtaining an average age for valuing the premiums should be in the ratio of 4 to 1, whereas if we employ for this purpose the average age found as above, the weights applied are equal. Hence, if there is a great disparity between the two valuation ages in question, say, 20 or 30 years, as may well be the case, an appreciable error is introduced in the value of the premiums.

Method X.-Errors of the above description may be avoided by finding two average ages, and using one for the reversion and the other for the annuity.

$$\mathbf{X}_1 = \frac{\Sigma(x\mathbf{S})}{\Sigma(\mathbf{S})}, \ \mathbf{X}_2 = \frac{\Sigma(x\mathbf{P})}{\Sigma(\mathbf{P})}.$$

The second weighted age can be run in very quickly by Crelle's Tables, it being quite sufficient to take the net premiums to the nearest pound. In many cases the two ages X1, X2, will be very close, but in others, especially if the groups contain only a few policies, there will be a difference of a year or two, as in the following illustration.

Table 13.

A Group of Policies, Method X.

| Policy<br>No. | Age<br>at<br>Entry | Sum<br>Assured | Rever-<br>sionary<br>Bonus | Ye  | ffice<br>early<br>mium | ì              | Net<br>Yearly<br>Premium<br>H <sup>M</sup> 2½°/° | Valua-<br>tion<br>Age | Prems. to Pay | Valuation Age × Sum Assured | Valua-<br>tion<br>Age<br>× Net<br>Premium |
|---------------|--------------------|----------------|----------------------------|-----|------------------------|----------------|--|-----------------------|---------------|-----------------------------|---|
|               |                    | £              | £                          | £   | s. (                   | 1.             | £  |                       |               |                             |   |
|               | 27                 | 1,000          | 92.7                       | 41  | 1.4                    | $\overline{2}$ | 36.830   | 39                    | 10            | 390                         | 1,443                                     |
|               | 40                 | 100            | 6.2                        | 5   | 12                     | 5              | 4.798  | 49                    | 10            | 49                          | 245                                       |
|               | 37                 | 500            | 23.2                       | 30  | 2                      | 6              | 24.900   | 45                    | 10            | 225                         | 1,125                                     |
|               | 33                 | 250            | 7.7                        | 15  | 0 1                    | 0              | 12.998   | 40                    | 10            | 100                         | 520                                       |
|               | 33                 | 50             | 1.2                        | 3   | 0                      | $^2$           | 2.600  | 39                    | 10            | 19                          | 117                                       |
|               | 23                 | 200            | 3.1                        |     | 10                     | 2              | 10.834   | 29                    | 10            | 58                          | 319                                       |
|               | 53                 | 1,000          | 15.2                       | 76  | 8                      | 4              | 64.520   | 59                    | 10            | 590                         | 3,835                                     |
| 7             |                    | 3,100          | 149.9                      | 184 | 8                      | 7              | 157.480  | 46·2, 7               |               | 1,431                       | 7,604                                     |

Value of sums assured =  $3,100 \times \Lambda_{46/2; \overline{10_{\frac{1}{2}}}}$ .

Value of net premiums =  $157.480 \times \frac{1}{2} (1 + a_{48.4:9} + a_{48.4:10})$ .

Method XI.—If a mere check on a more accurate valuation be required, a considerable saving of labour may be effected in Methods VIII, IX, and X by pairing the groups and so halving their number. The successive groups would then contain policies on which 0 or 1, 2 or 3, 4 or 5, . . . . premiums remain to be paid, the average age for each group being obtained as before. If the "Valuation Date" method of obtaining the valuation factors be employed, the form of the reversion and annuity is simplified by pairing the groups; for if there be n future premiums, the valuation factors are

$$A_{x:n+\frac{1}{2}}$$
 and  $\frac{1+a_{x:n-1}+a_{x:n}}{2}$ ,

and if n+1 future premiums,

$$\Lambda_{r:n+1\frac{1}{2}}$$
 and  $\frac{1+a_{r:\overline{n}}+a_{r:\overline{n+1}}}{2}$ ,

so that, if all policies having n or n+1 future premiums be combined in one group, the valuation factors become

$$\begin{array}{l} \frac{1}{2}(\mathbf{A}_{x:\,n+\frac{1}{2}|}+\mathbf{A}_{x:\,\overline{n+1}\frac{1}{2}}) \ \ \text{and} \ \ \frac{1}{2}\binom{1+a_{x:\,\overline{n-1}}+a_{x:\,\overline{n}|}}{2}+\frac{1+a_{x:\,\overline{n}|}+a_{x:\,\overline{n+1}}}{2}) \\ \text{or} \qquad \qquad \mathbf{A}_{x:\,\overline{n+1}} \ \ \text{and} \ \ \frac{1}{2}+a_{x:\,\overline{n}|} \ \ \text{approximately}. \end{array}$$

If there be no great disparity in the amount of assurances in the groups which are paired (which is generally the case when the total number of policies dealt with is considerable), a satisfactory check will be furnished by the method, but if the contrary is the case, the results may be somewhat wide of the mark.

Method XII.—First separate the policies into sections depending on the age at maturity, say four sections comprising policies maturing between 40 and 50, 50 and 60, 60 and 70, 70 and 75. Then group the policies in each section according to the number of future premiums, and proceed exactly as in Method IX for each section.

This method, of course, involves considerably more labour than either of the four preceding methods, as we get three or four times as many groups, but the results must be close to the truth, and can be relied on. (J.I.A., xxxii, 312.)

Method XIII.—If the number of policies in the groups of the last method is small, it might be advisable to use two average ages, as in Method X, instead of one. Since, however, the valuation ages of any two policies in a group cannot differ by more than 10 years, there is not the same liability to error in the value of the net premiums through the use of an average age weighted with the sum assured, as there is when all policies having n premiums to pay are gathered into one group. If the number of policies in each group is considerable, it would be exceptional to find any appreciable difference between the two average ages, and the extra labour involved in obtaining the second weighted age would be thrown away.

If the valuation factors are to be obtained by the "Nearest Renewal Date" method instead of the "Valuation Date" method, the methods of valuation in groups just described can be equally well applied. The number of groups will be the same as before, but the arrangement will be slightly different. Taking Method IX for instance, each group will consist of two sections:

- (a) Policies with n future premiums where the nearest renewal date precedes the date of valuation (marked  $\overline{n}$ ) in the numerical example).
- (b) Policies with n+1 future premiums, where the nearest renewal date *succeeds* the date of valuation (marked, n+1, in the numerical example).

Having obtained the average valuation age X, the total sum assured of the group is multiplied by  $A_{x:\overline{n+1}}$  and the total net premiums by  $a_{x:\overline{n}}$ , and to the latter must be added the total net premiums in the *second* section of the group.

Table 14.

Two Groups of Policies, Method IX, when the Valuation Factors are obtained by the "Nearest Renewal Date" Method.

| Policy<br>No. | Age at<br>Entry | Original<br>Term | Sum<br>Assured | Offic<br>Pr | e Ye<br>emit | arly<br>un   | Net<br>Yearly<br>Premium<br>H* 4 °/5 | Valuation<br>Age | Prems.<br>to Pay | Valuation<br>Age ×<br>Sum<br>Assured |
|---------------|-----------------|------------------|----------------|-------------|--------------|--------------|--------------------------------------|------------------|------------------|--------------------------------------|
|               |                 |                  | £              | £           | 8.           | <i>d</i> .   | £                                    |                  |                  |                                      |
|               | 30              | 30               | 300            | 9           | 10           | 6            | 7:191                                | 30               | $\overline{29}$  | 90                                   |
|               | 15              | 30               | 300            | 8           | 9            | 6            | 6.348                                | 15               | 29               | 45                                   |
|               | 19              | 30               | 250            | 7           | 13           | $\mathbf{s}$ | 5.202                                | 19               | 29               | 48                                   |
|               | 20              | 30               | 300            | 9           | 4            | 4            | 6.654                                | 20               | 29               | 60                                   |
|               |                 |                  |                | 34          | 18           | 0            | 25:700                               |                  |                  |                                      |
|               | 25              | 35               | 1,000          | 26          | 5            | 0            | 19:390                               | 29               | 30               | 290                                  |
|               | 19              | 31               | 250            |             | 11           | 0            | 5.297                                | 20               | 30               | <b>5</b> 0                           |
|               | 29              | 31               | 300            | 9           | 3            | 3            | 6.879                                | 29               | 30               | 87                                   |
|               |                 |                  |                | 42          | 19           | 3            | 31.566                               |                  |                  |                                      |
| 7             |                 |                  | 2,700          | 77          | 17           | 3            | 57.266                               | 24.8             |                  | 670                                  |
|               | 15              | 35               | 500            | 11          | 19           | 2            | 8.740                                | 19               | 30               | 95                                   |
|               | 27              | 33               | 100            | 2           | 16           | 7            | 2.105                                | 29               | 30               | 29                                   |
|               | 24              | 32               | 250            | 7           | 14           | 6            | 5.277                                | 25               | 30               | 63                                   |
|               |                 |                  |                | 22          | 10           | 3            | 16.122                               | -                |                  |                                      |
|               | 15              | 36               | 250            | 5           | 19           | 7            | 4.225                                | 20               | 31               | 59                                   |
|               | 15              | 36               | 100            | 2           |              | 10           | 1.690                                | 19               | 31               | 19                                   |
|               | 26              | 34               | 100            | 2           | 14           | 6            | 2.019                                | 28               | 31               | 28                                   |
|               |                 |                  |                | 11          | 1            | 11           | 7.934                                |                  |                  |                                      |
| 6             |                 |                  | 1,300          | 33          | 12           | 2            | 24.056                               | 21.8             | .,,              | 284                                  |

In the first group,

Value of sums assured = £2,700 ×  $A_{24.8:\overline{30}}$ 

Value of office premiums = £77. 17s.  $3d. \times a_{24\%} = £42. 19s. 3a$ .

Value of net premiums = £57·266 ×  $a_{24*8:\overline{29}}$  + £31·566.

PERSISTENT ERRORS IN METHODS OF THE THIRD CLASS.

It may be pointed out that a small error is involved in all the above methods of grouping endowment assurances according to the number of future premiums, and using an average age for each group. There is no balance of this error, but it is persistent, occurring in each group.

Suppose the curve,  $y = A_{x:n}$ , drawn, the abscissa representing the age, and n being constant throughout the curve. If n=1, we get the straight line, y=r, and for each higher value of n we get a curve falling below this straight line and above the curve,  $y = A_x$ , and merging into the latter at age  $\omega - n$ .

In diagram XI, the eight curves,  $y = A_{x:\overline{10}}$ ,  $y = A_{x:\overline{10}}$ , . . .  $y = A_{x:\overline{40}}$ , are drawn (H<sup>M</sup>  $2\frac{1}{2}$  per-cent). The five steeper curves are the following,  $y = A_{x:\overline{40-x}}$ ,  $y = A_{x:\overline{50-x}}$ ,  $y = A_{x:\overline{60-x}}$ ,  $y = A_{x:\overline{70-x}}$ , and  $y = A_{x:\overline{70-x}}$ , respectively, while the lowest curve represents the whole-life reversion. In order to avoid confusion, all the curves,  $y = A_{x:\overline{10}}$ , with the exception of  $y = A_{x:\overline{5}}$  and  $y = A_{x:\overline{15}}$ , are terminated in the diagram by the curve,  $y = A_{x:\overline{15}-x}$ , instead of being continued until they merge into the curve,  $y = A_x$ .

The curve,  $y = A_x$ , is convex with respect to the axis of x from age 10 to age 15, concave from 16 to 21, convex from 21 to 57, concave from 57 to 85 (with the exception of age 68), convex from 85 to 92, and concave from 92 onwards. (See Diagram XI).

For distinctness the portions of the curves concave to x are dotted. The curves,  $y = A_{x:\overline{n}}$ , are mostly convex to x between ages 10 and 15, and concave from 15 to 21, at which point they one and all change again to convex, and remain so through the greater part of their length, but as they approach the curve,  $y = A_x$ , they become deflected and follow its curvature. For instance, the curve,  $y = A_{x:\overline{0}}$ , is convex to x from 21 to 80, concave from 80 to 85, and convex from 85 to 92; and the curve,  $y = A_{x:\overline{10}}$ , is convex from 21 to 71, and concave from 71 to 83, where it joins the curve  $y = A_x$ . Those curves,  $y = A_{x:\overline{n}}$ , which join the curve,  $y = A_x$ , before age 57 (at which age the latter changes from convex to concave) are convex throughout at all ages above 21.

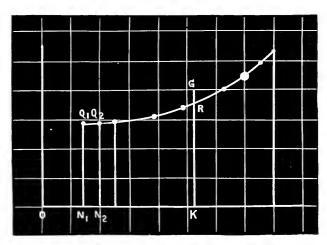
It thus appears that the portions of the curves,  $y = A_{x:\overline{n}|}$ , within the limits with which we have to deal in practice, i.e., where x > 21 and x + n < 75, are convex with respect to the axis of x. This holds for the  $H^M$  Table at different rates of



interest, and also for the  $H^{M,3}$  Table with this slight difference, namely, the age at which the curves change to convex is not 21 for all durations, but varies from 22 when n is large to 30 when n is small, also there is a slight unevenness about ages 35 to 38 in the curves for the shorter durations.

Now, if we take a group of policies having n future premiums

### DIAGRAM XII.



and represent the various sums assured,  $S_1$ ,  $S_2$ , . . . . , by proportionate weights distributed along the corresponding endowment assurance reversion curve at  $Q_1$ ,  $Q_2$ , . . . . (see Diagram XII), so that the abscisse O  $N_1$ , O  $N_2$ , . . . . represent the corresponding valuation ages, and if we take G the centre of gravity of the weights, and G K its ordinate, then

$$G~K\times \Sigma(S) = S_1\times Q_1~N_1 + S_2\times Q_2~N_2 + \ldots = \Sigma(S\times Q~N)~.~(1)$$

$$O K \times \Sigma(S) = S_1 \times O N_1 + S_2 \times O N_2 + \dots = \Sigma(S \times O N) \quad . \quad (2)$$

The first equation shows that if we multiply the total sums assured in the group by the reversion corresponding to G K, the ordinate of the centre of gravity, the result will exactly agree with a detailed valuation of the sums assured individually; and the second equation shows that the process of finding the average age in Methods IX, X, XII and XIII, namely,  $X_1 = \frac{\Sigma(xS)}{\Sigma(S)}$ , is equivalent to finding O K, the abscissa of the centre of gravity; further

to finding OK, the abscissa of the centre of gravity: further, taking the reversion corresponding to the average age so found,

is equivalent to taking K R, the ordinate up to the curve, and not up to the centre of gravity. But the centre of gravity of any number of weights distributed along a curve whose curvature is in the same direction throughout must be on the concave side of the curve, i.e., above the curve in this case, hence the reversion used is a little too small. That is to say, in a grouped valuation of endowment assurances by Methods IX and X, and to a smaller extent by Methods XII and XIII, the value of the sums assured will be somewhat understated in all the groups.

Considering now the annuities for valuing the premiums, suppose the curves,  $y = a_{x:\overline{n-1}|}$ , drawn; for each value of n we get a curve lying above the axis of x and below the curve,  $y = a_x$ , and merging into the latter at age  $\omega - n$ . In Diagram XIII the eight curves,  $y = a_{x:\overline{4}|}$ ,  $y = a_{x:\overline{9}|}$ , . . .  $y = a_{x:\overline{9}|}$ , are drawn, and also the five curves,  $y = a_{x:\overline{39-x}|}$ ,  $y = a_{x:\overline{49-x}|}$ ,  $y = a_{x:\overline{59-x}|}$ ,  $y = a_{x:\overline{69-x}|}$ ,  $y = a_{x:\overline{69-x$ 

Since 
$$A = 1 - d(1 + a)$$
  
 $\therefore \Delta A = -d \cdot \Delta a$ , and  $\Delta^2 A = -d \cdot \Delta^2 a$ .

Now a curve in the first quadrant is convex with respect to the axis of x if the second difference of its ordinate is positive, and concave if negative, hence where the curve,  $y = A_{x:\overline{n}}$ , is convex, the corresponding curve,  $y = a_{x:\overline{n-1}}$ , is concave, and vice versa. Thus the temporary annuity curves are mostly concave to x between ages 10 and 15, and convex from 15 to 21, at which point they all change to concave and remain so throughout the greater part of their length, but, just as in the case of the reversions, they become deflected as they approach the curve,  $y = a_x$ , and follow its curvature, all the points of inflexion occurring at the same ages as in the corresponding reversion curves. This is true for the  $H^M$  Table at different rates of interest, and also for the  $H^{M(5)}$  Table with the slight differences mentioned above.

Now if we take, as before, a group of policies having n future premiums, and represent the net premiums,  $P_1, P_2, \ldots$  by proportionate weights distributed along the corresponding

90  $y=a_{x:(0)-x}$ ,  $y=a_{x:(1)-x^i}$ , and the curve  $y=a_{x^i}$ The abscissa represents the age, and the ordinate the annuity,  $11^{M} 2_{2}^{1}$  °/. Portions of curves convex to the axis of x are marked thus  $\gamma$ 9 20 40 いとこのまってい 3- a 2 4 30 y=03

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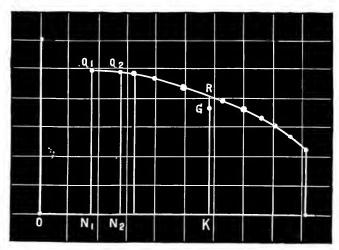
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#### DIAGRAM XIV.



temporary life annuity curve at  $Q_1$ ,  $Q_2$ , . . . ., and take G the centre of gravity of the weights and GK its ordinate, then

$$GK.\Sigma(P) = P_1 \times Q_1N_1 + P_2 \times Q_2N_2 + \dots = \Sigma(P \times QN)$$
  

$$OK.\Sigma(P) = P_1 \times O N_1 + P_2 \times O N_2 + \dots = \Sigma(P \times QN)$$

So that if we multiply the total net premiums by the annuity corresponding to GK, the result will exactly agree with a detailed valuation; also, in obtaining the second average age, as described in Methods X and XIII, by means of the premiums as weights, we are really finding OK the abscissa of the centre of gravity, and in employing the temporary annuity corresponding to this average age, we are taking KR the ordinate up to the curve instead of KG; but the centre of gravity will in this ease lie below the curve, hence the annuity used is a little too large, and the value of the future premiums payable is exaggerated in each group.

These small errors in the values of the sums assured and net premiums would be negligeable were it not for the fact that they are (1) persistent, occurring in every group, and (2) additive, both having the effect of bringing out too small a reserve.

If only one average age be used, namely, that weighted with the sum assured, it is impossible to gauge beforehand the effect on the value of the future premiums. The range of error will be widened by the use of the incorrect weights, and it is quite possible that in some cases the error in the value of the premiums

will be negative. Similarly, we cannot say beforehand what the effect of any of these methods will be on the value of the bonuses. It seems, however, that there will be a tendency for it to be under-estimated, for, in any group, the policies with the higher valuation ages will, as a rule, have been longer in force than those at the younger ages, and will therefore carry heavier bonuses in proportion to the sums assured; consequently, the average reversion for valuing the bonuses should be greater than the average reversion for valuing the sums assured. business have extended over a considerable period, so that the bonuses on the older policies are large, it might be advisable to obviate this last error by using an age weighted with the sum assured and bonus together, instead of with the sum assured alone.

## Suggested Adjustment to counteract the Persistent Errors.—Method XIV.

The adjustment which most readily presents itself to counterbalance the errors arising, as explained above, from the curvature of the reversion and annuity curves, is to make some small addition to the average age brought out; for where the curvature is least, i.e., at the younger ages and shorter terms, the errors in question are necessarily small, while a small addition to the age has but little effect on  $A_{x:\overline{n}|}$  and  $a_{x:\overline{n-1}|}$ , but where the curvature is greater, the error is greater, and a small addition to the age has a greater effect on the reversion and annuity.

It is clear that if the valuation ages in a group extend over a considerable number of years, so that the sums assured and net premiums of the group are distributed over a considerable length of the corresponding curves in Diagrams XI and XIII, the errors introduced by neglecting the curvature will be greater than if the valuation ages extend over a few years only. Better results then will be obtained if the average ages be adjusted as follows:

Method XIV.—Calculate the average ages as in Method IX or X, and increase each by an amount depending on the "range" of valuation ages in its group (that is, the difference between the highest and lowest ages in the group), and use the reversions and annuities corresponding to the adjusted average ages.

Referring to Diagrams XI and XIII, suppose the valuation ages in a group to range from 30 to 50, so that the policies are distributed over the curves QRS, it is obvious that the error will be least when most of the policies have the same valuation age, and that the error will be greatest if the policies are congregated at the two extremities of the section Q, S, in about equal amounts. We may assume, then, that an average error would arise when the policies are evenly distributed over the ages comprised in the section. In such a case the average age would be the central age of the section, and the total net premiums would be multiplied by the corresponding annuity.

Take, for example, the section having n future premiums, the valuation ages extending from x-r to x+r. In a grouped valuation the total net premiums would be multiplied by  $a_{x:\overline{n}}$ , say, this being near enough for present purposes, while a detailed valuation of the net premiums would be equivalent to multiplying the total by  $\frac{1}{nr} / \frac{1}{2} a_{x-r:\overline{n}} + a_{x-r+1:\overline{n}} + \dots$ 

$$+ \ldots + a_{x:\overline{n}} + \ldots + a_{x+r-1:\overline{n}} + \frac{1}{2}a_{x+r:\overline{n}}$$

The average adjustment (a) necessary for a "range" of 2r may be found by equating the last expression to  $a_{x+\alpha;\overline{n}|}$ .

Provided r is constant, then  $\alpha$  will be found to be fairly constant, however x and n may vary, and the suggested adjustments are as follows:

Table 15.

| "Range" of Valuation<br>Ages in a group.<br>Additive Adjustment<br>totheaverageValua- | 0-5 | 6,7 | 8,9 | 10,11 | 12  | 13  | 14  | 15  | 16 1 | 17 18 | 19  | 20  |
|---|-----|-----|-----|-------|-----|-----|-----|-----|------|-------|-----|-----|
| tion Age  | 0   | ·1  | •2  | .3    | •.1 | .2  | .6  | .7  | ·s   | 9 1:0 | 1.1 | 1.2 |
| "Range" of Valuation  |     |     |     |       |     |     |     |     |      |       |     |     |
| Ages in a group .<br>Additive Adjustment  | 21  | 22  | 23  | 24    | 25  | 26  | 27  | 28  | 29   | 30    | 31  | 32  |
| to the average Valuation Age  | 1.3 | 1.4 | 1.6 | 1.8   | 2.0 | 2.1 | 2.3 | 2.5 | 2.7  | 2.9   | 3.1 | 3.3 |
| "Range" of Valuation  |     |     |     |       |     |     |     |     | 1    |       |     | _   |
| Ages in a group Additive Adjustment to the average Valua-                             | 33  | 34  | 35  | 36    | 37  | 38  | 39  | 40  |      |       |     |     |
| tion Age  | 9.5 | 2.7 | 3.0 | 1.1   | 4.9 | 1.5 | 0.7 | z.0 |      |       |     |     |

It is assumed that we are dealing only with valuation ages over 21. In the unusual case where the number of ages below 21 is considerable compared with the number over 21, the errors due to neglecting the curvature of the reversion and annuity curves will in most cases (owing to the points of inflexion at age 21 and about age 15) be of small dimensions, and may even have the effect of over-stating the liability, so that it is of little use to attempt any adjustment to counteract them. In any case, in finding the "range" for a group, ages below 21 should be ignored, as it is at that point that the curvature changes.

It takes very little time to run the eye down the column of valuation ages for each group, and select the highest and lowest, so that the suggested adjustment practically adds no labour to the working of Methods IX and X.

The adjustment can be equally well applied to other methods of the Third Class-Method XII, for instance. Here each group having n future premiums is sub-divided, and we deal with only a small section of the reversion and annuity curves, QR for instance, instead of with the whole of the curves PQRST. The curvature of a section, extending over ten years, say, is small, and the centre of gravity is only just above the reversion curve and just below the annuity curve. Hence, the error created by taking the ordinate up to the curve instead of up to the centre of gravity is small, though not entirely eradicated. Assuming an even distribution as before, a detailed valuation of the premiums would be equivalent to multiplying the total by  $\frac{1}{10}\{\frac{1}{2}a_{x-5:\overline{n}}+a_{x-4:\overline{n}}+\dots+a_{x+4:\overline{n}}+\frac{1}{2}a_{x+5:\overline{n}}\}\}$ , while in a grouped valuation by Method XII we should multiply the total by  $a_{x,\overline{n}}$ . From Table 15 the correction for a range of 10 years is 3, so that if the proposed adjustment be applied, we should multiply the total premiums by  $a_{x+3:\overline{n}|}$ . In the following table these three expressions (H<sup>M</sup>  $2\frac{1}{2}$  per-cent) are compared, x and n being so chosen that the policies in each case would mature between ages 40 and 50, 50 and 60, or 60 and 70.

# [See p. 37.]

It will be seen that the fifth column agrees very closely with the third column. It is therefore suggested that if in the application of Method XII or XIII, where the valuation ages in a group range over 10 years, the average ages (which, of course, usually come out fractional) be increased by 3, a closer approximation to the true reserves will be obtained.

This seems a very trifling adjustment to make, but if the policies have been recently issued, its effect is appreciable.

 $\begin{array}{ccc} & \text{Table 16.} \\ & \text{H}^{\text{M}} & 2\frac{1}{2} & \textit{per-cent Annuities.} \end{array}$ 

| n     | x  | $\frac{\frac{1}{10}(\frac{1}{2}a_{z-5;n} + a_{z-4;n} + \dots + a_{z+4;n} + \frac{1}{2}a_{z-5;n})}{\dots + a_{z+4;n} + \frac{1}{2}a_{z-5;n})}$ | $a_{x:\overline{n}}$ | a <sub>2+3:n</sub> |
|-------|----|---|----------------------|--------------------|
| 39    | 25 | 20:955  | 20.999               | 20:961             |
| 34    | 30 | 19:238  | 19:269               | 19.231             |
| 29    | 25 | 18·228  | 18·254               | 15·234             |
| 29    | 35 | 17·324  | 17·359               | 17·321             |
| 24    | 30 | 16:040  | 16:052               | 16:035             |
| 24    | 40 | 15:187  | 15:231               | 15:194             |
| 19    | 25 | 13:925  | 13:93\$              | 13:929             |
| 19    | 35 | 13:555  | 13:569               | 13:554             |
| 19    | 45 | 12:785  | 12:818               | 12:783             |
| 14    | 30 | 10°994  | 10:996               | 10:990             |
| 14    | 40 | 10°717  | 10:734               | 10:722             |
| 14    | 50 | 10°090  | 10:122               | 10:095             |
| 9     | 35 | 7°609   | 7:611                | 7:607              |
| 9     | 45 | 7°439   | 7:146                | 7:438              |
| 9     | 55 | 7°035   | 7:057                | 7:038              |
| 1 1 1 | 40 | 3:664   | 3:665                | 3:664              |
|       | 50 | 3:606   | 3:609                | 3:607              |
|       | 60 | 3:166   | 3:473                | 3:466              |
|       |    | 211:857   | 212-202              | 211:569            |

For instance, the  $H^M$   $2\frac{1}{2}$  per-cent reserves after two years and five years on a policy for £1,000, effected at age 25 and payable at 60, are as follows:

|  | After<br>2 Years | After<br>5 Years |
|--|------------------|------------------|
| Value of sum assured<br>Value of future premiums | 510°80<br>174°75 | 539·71<br>446·71 |
| Reserve  | <br>36:05        | 53.00            |

If the valuation ages used be 27.3 and 30.3 instead of 27 and 30, the unexpired terms being 33 and 30 years as before, we get the following figures:

|  |      | After<br>2 Years | After<br>5 Years |
|--|------|------------------|------------------|
| Value of sum assured<br>Value of future premiums | <br> | 511·46<br>474·08 | 540.34 $446.09$  |
| Rese   |      | 37:38            | 94.25            |

the difference being £1:33, or 3.7 per-cent of the reserve after two years, and £1:25, or 1:3 per-cent of the reserve after five years. After 10 years the corresponding reserves on the same policy would be £196:76 and £197:96, the difference being £1:2, or only 6 per-cent of the reserve. As the duration of the policy increases, the actual difference between the two corresponding reserves will slowly decrease, and the difference reckoned as a percentage of the reserve will rapidly diminish and become negligeable.

In order to admit of *Method* XII or XIII being employed, the number of policies to be valued must be large. If, for instance, there be only a few hundreds of policies altogether, we might get an average of about three policies only in each group. Of course, two-thirds of the multiplications would be thereby avoided, but this would scarcely compensate for the more careful scheduling necessary, and the trouble of weighting the ages, &c.

It is suggested, however, that if the adjustment specified in Table 15 be applied, a single grouping of the policies according to the number of future premiums, as in *Methods* IX and X, will be sufficient, and that it will be unnecessary to sub-divide the groups as in *Methods* XII and XIII.

# NUMERICAL RESULTS OF METHODS IX AND X.

In a valuation on which I was working a few years ago there were 797 Endowment Assurance policies, which were separated for convenience into three distinct batches, namely:

- (1) 243 with-profit policies which had been in force more than 5 years.
- (2) 393 with-profit policies which had been in force less than 5 years.
- (3) 161 policies, under a special scheme, which had been in force less than 5 years.

In each batch the policies were first valued individually, and then, in order to obtain a check, they were thrown into groups depending on the number of future premiums, and the first batch were valued by *Method* X, and the second and third batches by *Method* IX, with the following slight modifications.

Firstly, in the grouped valuation, but not in the detailed valuation,  $n+\frac{1}{4}$  and  $n+\frac{3}{4}$  future premiums were treated as n and n+1 respectively, and  $n+\frac{1}{2}$  was taken alternately as n and n+1.

Secondly, the average ages were taken to the next higher integer instead of to the nearest integer.

The results of the detailed and grouped valuations are shown in the following Table.

Table 17.

Numerical Results of Methods IX and X (modified).

| (1)   | (-2)                                 | (3)  | (4)  | (5)         | (6)      |
|---|--------------------------------------|--|--|-------------|----------|
| (1)   | Detailed<br>Valuation                | Grouped  |  | (3) -: (2)  |          |
| First Batch. 243 Police   |                                      | *  | _  | oer in Grou | p = 8.   |
|   | МЕТНОВ                               | X (modif   | ied).  |             |          |
| Нм(5) 2½ per-cent   |                                      |  |  |             |          |
| Value of Sums Assured .   | $51397 \cdot 2$                      | 51373.4  | - 23.8   |             |          |
| Value of Bonuses  | 1887.5                               | 1888-5   | + 1.0  | ***         |          |
| Value of Net Premiums.  | 31439.2                              | 31520.7 $21741.2$                                      | + 81·5<br>-104·3                                     | ·99523      | .477     |
|   |                                      |  |  |             |          |
| Net Liability   |                                      |  | erage num  |             |          |
|   | icies, 35 G                          | roups. Av  | erage num  |             |          |
| Second Batch. 393 Poli  | icies, 35 G                          | roups. Av  | erage num  |             |          |
| Second Batch. 393 Poli<br>11 × 2½ per-cent<br>Value of Sums Assured .<br>Value of Net Premiums.   | 084524<br>822825                     | roups. Av<br>IX (modif<br>98408-0<br>82466-6           | erage num<br>hed).<br>- 444<br>+1844                 | ber in Gro  | ap = 11. |
| Second Batch. 393 Poli<br>11 × 2½ per-tent<br>Value of Sums Assured.  | icies, 35 G<br>Метнор<br>98452:1     | roups. Av<br>IX (modif                                 | erage num<br>ied).                                   | ber in Gro  |          |
| Second Batch. 393 Poli<br>11 M 2½ Per-tent<br>Value of Sums Assured .<br>Value of Net Premiums.<br>Net Liability .                                  | 984524<br>82282-5<br>16169-6         | roups. Av<br>IX (modif<br>984080<br>824666<br>459414   | erage num<br>hed).<br>- 441<br>+1841<br>-228:2       | ber in Gro  | up = 11. |
| Second Batch. 393 Poli<br>11 M 2½ Per-cent<br>Value of Sums Assured .<br>Value of Net Premiums.<br>Net Liability .                                  | 984524<br>822825<br>161696           | roups. Av<br>IX (modif<br>984080<br>824666<br>459414   | erage num  1 + 15F1  1 - 228*2                       | ber in Gro  | np = 11. |
| Second Batch. 393 Poli<br>11 M 2½ Per-cent<br>Value of Sums Assured .<br>Value of Net Premiums.<br>Net Liability .                                  | 984524<br>822825<br>161696           | 98408;0<br>98408;0<br>82466;6<br>15941;4<br>roups. Av  | erage num  1 + 15F1  1 - 228*2                       | ber in Gro  | np = 11. |
| Second Batch. 393 Poli  11 ** 2½ per-cent  Value of Sums Assured .  Value of Net Premiums .  Net Liability .  Third Batch. 161 Poli                 | 984524<br>822825<br>161696           | 98408;0<br>52466;6<br>45941;1<br>roups. Av             | erage num  - 441 +1841 -228:2  rerage num fied)      | ber in Gro  | np = 11. |
| Second Batch. 393 Poli  11 M 2½ per-cent  Value of Sums Assured .  Value of Net Premiums.  Net Liability .  Third Batch. 161 Poli  11 M 2½ per-cent | 984524<br>822825<br>161696<br>METHOD | 98408:0<br>52466:6<br>45941:4<br>roups. Av<br>1X (modi | erage num led).  - 4F1 +18F1 -228·2 rerage num fied) | ber in Gro  | mp = 11. |

With reference to the error of over 4 per-cent in the reserve for the third batch of policies, it was noticed when the policies were being scheduled for the check valuation that in a particular group the results must necessarily be wide of the mark, and it was accordingly car-marked.

This group contained only three policies and two valuation ages, and these differed by no less than 28 years. In such a case, the errors alluded to above, arising from the curvature of the curves being disregarded, are considerable. The policy creating the disturbance matured at age 78, thirteen years later than any of the other 160 policies; the other two policies in the same group matured at age 50. The sum assured by the policy in question was £3,000, the average sum assured in the batch being only £375.

The figures relating to this group are given in the following table, and, for comparison, the results of applying *Methods* VIII, IX, X, and XIV to the group are also inserted.

Table 18.

Results of applying Methods VIII, IX, X, and XIV to an unsatisfactory group.

| Method of Valuation                           | Value of<br>Sums<br>Assured | Value of<br>Net<br>Premiums | Net<br>Liability | Amouut<br>of<br>Error | Percentage<br>of<br>Error |
|---|-----------------------------|-----------------------------|------------------|-----------------------|---------------------------|
| Detailed Valuation Grouped Valuations:        | 2998.2                      | 2565.6                      | 432.6            |                       |                           |
| Method VIII                                   | 2882.8                      | 2978.2                      | - 95.4           | -528.0                | 122.0                     |
| Method IX                                     | 2958·5<br>2958·5            | 2735·0<br>2693·8            | 223·5<br>254·7   | -209.1<br>-177.9      | 48·3<br>41·1              |
| Method X (average ages<br>being taken to next | 20000                       | 20000                       | 2011             | 1110                  |                           |
| higher integer)                               | 2958.5                      | 2649.6                      | 308.9            | -123.7                | 28.6                      |
| Method XIV (one age) .                        | 2985.2                      | 2602.6                      | 382.6            | - 50.0                | 11.6                      |
| Method XIV (two ages) .                       | 2985.2                      | 2552.7                      | 432.5            | - 0.1                 | 0                         |

When the various methods are put to such a severe test as this, their comparative defects are at once manifest.

This one refractory group is responsible for the greater part of the error of 4.389 per-cent shown above. If this group be omitted, the figures for the third batch are as follows.

Table 19.

Numerical Results of Method IX (modified).

| Third Batch (one Group omitted). 158 Policies, 32 Groups. Average number in Group = 5.<br>Method IX (modified). |                              |                              |                            |                      |                               |  |  |  |
|---|------------------------------|------------------------------|----------------------------|----------------------|-------------------------------|--|--|--|
| (1)   | (2) Detailed Valuation       | (3)<br>Grouped<br>Valuation  | (4)<br>(3) - (2)           | $(5)$ $(3) \div (2)$ | (6)<br>Percentage<br>of Error |  |  |  |
| H <sup>M</sup> 2½ per-cent<br>Value of Sum Assured .<br>Value of Net Premiums<br>Net Liability .                | 35396·7<br>29653·2<br>5743·5 | 35382·8<br>29701·3<br>5681·5 | - 13·9<br>+ 48·1<br>- 62·0 | <br>·98921           | <br>1·079                     |  |  |  |

In order to find out what the errors would be when the methods are applied without any modification or adjustment, the policies on which the number of future premiums payable is fractional have been eliminated, and the average ages re-calculated and taken to the *nearest* integer. The results appear in the following table 20.

Table 20.

Numerical Results of Methods IX and X.

| (1)  | (2)  | (3)  | (4)  | (5)          | (6)                    |
|--|--|--|--|--------------|------------------------|
|  | Detailed<br>Valuation  | Grouped<br>Valuation   | (3) - (2)  | (3) : (2)    | Percentage<br>of Error |
| First Batch. 216 Poli  | cies, 30 Gr  | oups. Ave  | erage numl   | er in Grou   | p=7.                   |
|  | ME   | тнор Х.  |  |              |                        |
| 11x(5) 2½ per-cent   |  |  |  |              |                        |
| Value of Sum Assured .<br>Value of Bonuses .<br>Value of Net Premiums .<br>Net Liability . | $\begin{array}{r} 442813 \\ -16140 \\ 268454 \\ -190829 \end{array}$ | $\begin{array}{c} 44208:7 \\ 1641:9 \\ 26987:2 \\ 18863:4 \end{array}$ | $ \begin{array}{r} -75.6 \\ -2.1 \\ +141.8 \\ -219.5 \end{array} $ | <br>:98850   | 1:15                   |
| S 1 D-4 1 979 D 1  | 9° C   |  |  | bunda (tua   | · 10                   |
| Second Batch. 353 Pol  |  | roups. Av<br>тнор IX.  | erage num  | ber 111 Grot | p = 10.                |
| II™ 2½ per-cent  |  |  |  |              |                        |
| Value of Sum Assured .   | 81302.7  | 84176.0  |  |              |                        |
| Value of Net Premiums.<br>Net Liability  | 70402·8<br>13899·9   | 70615·4<br>13530·6   | +242.6 $-369.3$  | 97314        | 2.656                  |
| Third Batch. 139 Po  |  | roups. A   |  | ber in Gro   | up=1                   |
|  | ME   | тнор ІХ.   |  |              |                        |
| 11M 2½ per-cent  |  |  |  |              |                        |
| Value of Sum Assured .   | 30942.7  | 30903.7  | - 39:0   |              |                        |
| Value of Net Premiums.<br>Net Liability  | 25913·6<br>5029·1  | 25995·5<br>4908·2  | + 81·9<br>- 120·9  | .97596       | 2:401                  |

The figures in the following table relate to precisely the same policies as in the preceding table, but the average ages have been taken to the next higher integer instead of the nearest integer. The results show that this adjustment is not sufficient in these particular eases to remove the errors in the reserves, but the errors are thereby reduced in varying proportions in the three eases, namely, from 1:15 per-cent, 2:656 per-cent, and 2:404 per-eent to '752 per-cent, 1:29 per-cent, and '31 per-cent respectively.

Table 21. Numerical Results of Methods IX and X (modified).

| (1)  | (2)                                     | (3)                                     | (4)   | (5)             | (6)                    |
|--|---|---|---|-----------------|------------------------|
|  | Detailed<br>Valuation                   | Grouped<br>Valuation                    | (3) - (2)   | $(3) \div (2)$  | Percentage<br>of Error |
| First Batch. 216 Poli  |   | oups. Av<br>X (modif                    | 0   | er in Grou      | p=7.                   |
| II M(5) 2½ per-cent  Value of Sums Assured .  Value of Bonuses .  Value of Net Premiums .  Net Liability . | 44284·3<br>1644·0<br>26845·4<br>19082·9 | 44239·7<br>1642·9<br>26943·2<br>18939·4 | - 44·6<br>- 1·1<br>+ 97·8<br>-143·5                               | <br><br>.99248  | <br><br>.752           |
| Second Batch. 353 Poli   | icies, 35 Gr<br>Метнор                  | roups. Av<br>IX (modi                   | erage numl<br>fied).  | er in Grou      | ip = 10.               |
| 11 <sup>M</sup> 2½ per-cent<br>Value of Sums Assured .<br>Value of Net Premiums .<br>Net Liability         | 84302·7<br>70402·8<br>13899·9           | 84254·5<br>70533·9<br>13720·6           | -48.2 + 131.1 - 179.3   | <br><br>.98710  | 1.29                   |
| Third Batch. 139 Pol   | (one Gr                                 | roups. Av<br>oup omitte<br>IX (modi     | ,   | ber in Gro      | up = 4                 |
| <b>Н</b> м(5) 2½ per-cent  |   |   |   |                 |                        |
| Value of Sums Assured .<br>Value of Net Premiums .<br>Net Liability  | 30942·7<br>25913·6<br>5029·1            | 30945·2<br>25931·5<br>5013·7            | $\begin{array}{rrrr} + & 2.5 \\ + & 17.9 \\ - & 15.4 \end{array}$ | ·99 <b>69</b> 4 |                        |

In a recent valuation of a Colonial company which had been six years in existence, there were 549 endowment assurances, which were divided into two batches of 167 and 382 policies, (referred to in this paper as the "Fourth Batch" and "Fifth Batch" respectively). The policies were valued and checked individually,  $H^{\mathrm{M}}$  4 per-cent reversions and annuities being employed, and the premiums valued being slightly greater than  $H^{\mathrm{M}}$  4 per-cent net premiums.

These policies have been grouped and re-valued by Method IX. The "Nearest Renewal Date" method of obtaining the valuation factors was employed, so that the grouping took the form shown in Table 14.

The results are as follows:

Table 22.

Numerical Results of Method IX.

| (1)                               | (2)<br>Detailed<br>Valuation | (3)<br>Grouped<br>Valuation | (4)<br>(3)-(2) |            | (6)<br>Percentag |
|-----------------------------------|------------------------------|-----------------------------|----------------|------------|------------------|
| Fourth Batch. 167 P               |                              |                             |                |            |                  |
| Fourth Baten. 107 1               |                              | топря. Ал<br>гнор IX.       | rerage num     | ber in ere | , ap = 0,        |
| H <sup>M</sup> 4 per-cent         |                              |                             |                |            |                  |
| Value of Sums Assured.            |                              |                             | - 75.1         |            |                  |
| Value of Premiums . Net Liability |                              |                             |                | ·95258     | 4.712            |
| Fifth Batch. 382 Pol              | licies, 35 Gı                | oups. Ave                   | rage numb      | er in Grov | np=11.           |
| Titte Daten. 952 To.              | 3.5                          | 137                         |                |            |                  |
| Titte Batta. 932 10               | ME                           | гнор ІХ.                    |                |            |                  |
| Il <sup>M</sup> 4 per-cent        | ME                           | гнор ІХ.                    |                |            |                  |
|                                   | 1                            |                             | -265·1         |            |                  |
| II <sup>M</sup> 4 per-cent        | 76767·7<br>66716·9           | 76502°6<br>67008°3          | $\pm 291.4$    | <br>•94463 | <br>5:537        |

It will be noticed that in all the above illustrations of the application of *Methods* IX and X, the grouped valuation underestimates the value of the sums assured, and over-estimates the value of the future premiums, so that the reserve brought out is

too small, so far confirming the remarks made above as to the persistent errors involved in methods of this class.

## NUMERICAL RESULTS OF METHOD XIV.

The same five batches of policies have been re-valued after adjusting the average valuation age of each group, as described in Method XIV.

The results are given in Tables 23 and 24.

Table 23.

Numerical Results of Method XIV.

| (1)  | (2)<br>Detailed<br>Valuation                                | (3)<br>Grouped<br>Valuation             | (4) $(3) - (2)$  | (5)<br>(3) : (2) | (6)<br>Percentage<br>of Error |
|--|---|---|--|------------------|-------------------------------|
|  |   | •                                       | erage numb   |                  | p = 7.                        |
| HM(5) 2½ per-cent  Value of Sums Assured .  Value of Bonuses .  Value of Net Premiums .  Net Liability . | 44284·3<br>1644·0<br>26845·4<br>19082·9                     | 41266·6<br>1644·0<br>26882·4<br>19028·2 | -17.7 $0.0$ $+37.0$ $-54.7$                              |                  |                               |
|  |   | •                                       | erage numb   |                  | p = 10.                       |
| IIM 2½ per-cent  Value of Sums Assured .  Value of Net Premiums .  Net Liability                         | 84302·7<br>70402·8<br>13899·9                               | 84332·8<br>70377·4<br>13955·4           | +30·1<br>-25·4<br>+55·5                                  | <br>1·0040       |                               |
|  | (one Gr   | oup omitte                              | verage numb<br>d).<br>nation Age).                       |                  | 1p = 4                        |
| 11 <sup>M</sup> 2½ per-cent  Value of Sums Assured .  Value of Net Premiums.  Net Liability              | $\begin{array}{c} 30942.7 \\ 25913.6 \\ 5029.1 \end{array}$ | 30941·8<br>25931·2<br>5010·6            | $ \begin{array}{r} - 9 \\ + 17.6 \\ - 18.5 \end{array} $ | <br>·99632       |                               |

Table 24.

Numerical Results of Method XIV.

| (1)                 |       | (2)     | (3)                  | (4)                   | (5)      | (6)                    |
|---------------------|-------|---------|----------------------|-----------------------|----------|------------------------|
|                     |       |         | Grouped<br>Valuation | (3) - (2)             | (3): (2) | Percent of<br>of Error |
| Fourth Batch. 16    |       |         | •                    |                       |          | oup = 6.               |
|                     | Метн  | OD XIV. | (One Valu            | iaticn Age)<br>—————  |          |                        |
| HM 4 per cent       |       |         |                      |                       |          |                        |
| Value of Sums Assur |       |         |                      |                       |          |                        |
| Value of Premiums   |       |         | 19815.6              |                       |          |                        |
| Net Liability .     |       | 3112.2  | 3100.7               | - 115                 | -99631   | .369                   |
|                     |       |         | •                    | erage numbuation Age) |          | up = 11.               |
| Value of Sums Assu  | red . | 76767:7 | 76768:5              | + .8                  |          |                        |
| Value of Premiums   |       |         | 66757-2              |                       |          |                        |
| Net Liability .     |       | 10050.8 | 10011.3              | <b>-</b> 39·5         | .99607   | .393                   |
|                     |       |         |                      |                       |          |                        |

# NUMERICAL RESULTS OF METHOD V.

The Fourth and Fifth Batches of policies have been re-grouped and valued by *Method V.*, *i.e.*, according to the nearest integral annuity-value. The results are given in Table 25.

Table 25.

Numerical Results of Method V.

| (1)   | (2)                           | (9)                           | (1)                         | (*)            | (2)                           |
|---|-------------------------------|-------------------------------|-----------------------------|----------------|-------------------------------|
| (1)   | (2) Detailed Valuation        | (3)<br>Grouped<br>Valuation   | (3) - (2)                   | $(3) \div (2)$ | (6)<br>Percentage<br>of Error |
| Fourth Batch. 167 Pol   | icies, 12 G<br>ME             | roups. Av                     | erage num                   | ber in Gro     | ap=14.                        |
| H <sup>M</sup> 4 per-cent  Value of Sums Assured .  Value of Premiums  Net Liability    | 22916·0<br>19803·8<br>3112·2  | 22974·2<br>19753·3<br>3220·9  | + 58·2<br>- 50·5<br>+ 108·7 | <br>1·03493    | <br>3·493                     |
| Fifth Batch. 382 Poli   |                               | roups. Av                     | rerage num                  | ber in Grou    | np = 25.                      |
| II <sup>M</sup> 4 per-cent  Value of Sums Assured .  Value of Premiums .  Net Liability | 76767·7<br>66716·9<br>10050·8 | 76784·7<br>66642·1<br>10142·6 | + 17·0<br>- 74·8<br>+ 91·8  | 1.00913        | <br>.913                      |
| Fourth and Fifth  | rerage num                    | mbined.<br>aber in Gro        | 549 Policie<br>oup=37.      | s, 15 Grou     | ns.                           |
| H <sup>M</sup> 4 per cent  Value of Sums Assured .  Value of Premiums .  Net Liability  | 99683·7<br>86520·7<br>13163·0 | 99758·9<br>86395·4<br>13363·5 | + 75·2<br>-125·3<br>+200·5  | <br>1.01523    | <br>1·523                     |

It is, perhaps, a mere coincidence that the errors are comparatively so small, and, considering that only 12 and 15 groups were involved in the valuations, it is somewhat surprising that the results should be more accurate than those brought out by Method IX. (See Table 22). This affords an illustration of the principle that comparatively large errors, which are equally likely to be positive or negative, can be more readily ignored than much smaller errors which are persistent.

Method V is useful for a check valuation. The values of the sums assured and the premiums should be tolerably close to the true values, and if the error in the latter is negative, showing that the annuity-values used are on the whole too small, we may expect a positive error in the value of the sums assured, through the reversions being on the whole too large. In the case of an individual policy, if the error in the annuity used be = -e, the error in the reversion will be = de, since A = v - da. So that, if  $\pi$  be the premium per unit assured,

$$\frac{\text{Error in value of sum assured}}{\text{Error in value of premiums}} = \frac{de}{-\pi e} = -\frac{d}{\pi}.$$

If, then, all the policies in a batch carried the same rate of premium, the total errors would be in the above ratio. We can obtain an average premium,  $\pi$ , by dividing the total premiums by the total sums assured, but, as the actual premiums per unit will vary considerably in practice, we cannot expect the above relation to hold for the whole batch unless the number of policies is very large.

In the fourth batch, 
$$-\frac{d}{\pi} = -\frac{.03846}{.0304} = -1.27$$
, Error in value of sums assured

Error in value of premiums

$$= \frac{+58.2}{-50.5} = -1.05$$

and in the fifth batch, 
$$-\frac{d}{\pi} = -\frac{.03846}{.0366} = -1.05$$
,   

$$\frac{\text{Error in value of sums assured}}{\text{Error in value of premiums}} = \frac{+17.0}{-74.8} = -.227.$$

### NUMERICAL RESULTS OF METHOD III.

Table 26.

Ages at Maturity.

| The Fourth and Fifth Batches have been        |
|---|
| re-grouped and valued by Method III.          |
| The annexed table shows the ages at which     |
| the policies would actually mature. In        |
| applying the method, two maturity ages        |
| were selected, viz., 50 and 61, all the       |
| policies above the dividing line in the table |
| being assumed to mature at age 50, and all    |
| below at age 61. For the Fourth Batch of      |
| 167 policies, this necessitated an alteration |
| of more than 5 years in 22 cases, and of      |
| more than 10 years in 8 cases, while in       |
| the Fifth Batch of 382 policies the ages      |
| were altered by more than 5 years in 32       |
| cases, by more than 10 years in 5 cases,      |
| and by no less than 30 years in one case.     |

In these valuations, it was found convenient to use H<sup>M</sup> 4 per-cent net premiums instead of the premiums employed above.

The results are given in Table 27. If all the policies were assumed to mature at one age, 55 say, instead of two, the results would doubtless be sufficiently accurate for a check valuation.

| $\Lambda ge$    | No. of          | Policies        |
|-----------------|-----------------|-----------------|
| at<br>Maturity  | Fourth<br>Batch | Fifth<br>Batch  |
| 20              |                 | 1               |
| 34<br>35        | 1               | <br>1           |
| 36              | , ,             | 1               |
| 37              | 1               |                 |
| $\frac{38}{39}$ | <br>5           | <br>3           |
| 40              | 9               | 14              |
| 41              | 1               | 2               |
| 42<br>43        |                 |                 |
| 44              | <br>4           | 8               |
| 45              | 10              | 25              |
| $\frac{46}{47}$ |                 | <br>1           |
| 48              |                 |                 |
| 49              | 20              | 30              |
| 50              | 35              | 89              |
| $\frac{51}{52}$ | $\frac{2}{1}$   | $\frac{3}{1}$   |
| 53              |                 |                 |
| 54<br>55        | 10<br>16        | $\frac{25}{55}$ |
|                 |                 |                 |
| 56              | 2               | 1               |
| 57<br>58        | •••             | 3<br>3          |
| 59              | 12              | 23              |
| 60              | 23              | 49              |
| 61              | 1               | 2               |
| 62<br>63        | 1               | $\frac{2}{5}$   |
| 64              | ${2}$           | 13              |
| 65              | 10              | 19              |
| 66<br>67        |                 | 1 1             |
| 68              |                 |                 |
| 69              |                 | 1               |
| 70              |                 | 1               |
| Total           | 167             | 382             |
|                 |                 |                 |

Table 27.

Numerical Results of Method III.

| (1)  | (2)                          | (3)                          | (4)                      | (5)         | (6)                    |
|--|------------------------------|------------------------------|--------------------------|-------------|------------------------|
|  | Detailed<br>Valuation        | Grouped<br>Valuation         | (3)-(2)                  | (3) ÷ (2)   | Percentage<br>of Error |
| Fourth Batch. 167 Pe   |                              | Groups. A                    | verage nur               | nber in Gro | oup = 4.               |
| Н <sup>м</sup> 4 per-cent  |                              |                              |                          |             |                        |
| Value of Sums Assured .<br>Value of Net Premiums.<br>Net Liability | 22916·0<br>18976·5<br>3939·5 | 22919·0<br>18988·2<br>3930·8 | + 3·0<br>+ 11·7<br>- 8·7 |             | <br>·221               |
| Fifth Batch. 3S2 Pol   | ,                            | roups. Av                    | erage num                | ber in Gro  | np = 7.                |
| H <sup>™</sup> 4 Per-cent  |                              |                              |                          |             | 1                      |
| Value of Sums Assured . Value of Net Premiums.                     | 76767·7<br>64022·7           | 76631·8<br>63924·5           |                          |             | •••                    |
| Net Liability  | 12745.0                      | 12707:3                      | - 37·7                   | 99704       | .296                   |
|  |                              |                              |                          | 04.61       |                        |
| Fourth and Fifth   |                              |                              | 549 Policie              | s, 61 Group | os.                    |
|  | verage nun                   |                              |                          | s, 61 Group | )S.                    |
|  | verage nun                   | aber in Gro                  |                          | s, 61 Group | )\$,                   |
| A<br>H× 4 per-cent   | verage nun                   | aber in Gro                  | oup = 9.                 | s, 61 Group |                        |
| A  | verage nun<br>Met            | aber in Gro                  | oup = 9.                 |             | <br>.279               |

## SUMMARY OF NUMERICAL RESULTS.

Most of the numerical results given above are summarized in Table 28.

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E

Table 28.

Summary of Results.

| As                       | HOUNT OF E                    | error in Re                      | SERVE                                |        |   |
|--------------------------|-------------------------------|----------------------------------|--------------------------------------|--------|---|
|                          | Метнор                        |                                  |                                      |        |   |
|                          | 111                           | v                                | IX                                   | X      | XIV                                       |
| First Batch Second Batch | <br>- 8·7<br>- 37·7<br>- 46·4 | <br>+ 108·7<br>+ 91·8<br>+ 200·5 | -369·3<br>-120·9<br>-147·6<br>-556·5 | -219·5 | -54·7<br>+55·5<br>-18·5<br>-11·5<br>-39·5 |
| Perc                     | ENTAGE OF                     | ERROR IN I                       | RESERVE                              |        |   |
| Perc                     | EENTAGE OF                    | ERROR IN I                       | RESERVE                              |        |   |
| Perc                     | PENTAGE OF                    | ERROR IN I                       |                                      | X      | XIV                                       |

In all the above numerical illustrations the error has been brought out as a percentage of the reserve. Most of the methods which have been illustrated really consist of obtaining an estimate, not of the reserve, but of (1) the value of the sums assured, and (2) the value of the future premiums; and from a theoretical point of view, it might be considered more proper to examine the errors in these two items separately, instead of the error in their difference, the net liability. The practical object of a valuation, however, being to obtain an estimate of the requisite reserves, it is advisable, before adopting and relying upon any approximate method, to consider what percentage of error in the reserves may be involved thereby, and this is the more necessary, in that all methods of the Second and Third Classes may be expected to bring out errors of opposite sign in the value of the

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sums assured, and the value of the premiums, the errors being therefore additive.

If we regard the errors in the value of the sums assured and the value of the net premiums separately, the five batches of policies which have been dealt with above may be looked upon, I think, as normal cases. But if we regard the percentage of error in the reserves, the five batches form very unfavourable material for the application of approximate methods of valuation, owing to the shortness of the duration of the policies. In the first batch, the average duration is about seven years, and in each of the other four batches about two years only; so that the reserve forms but a small percentage of either the value of the sums assured or the value of the premiums.

In a normal batch comprising policies in all stages of maturity, the error, when reckoned as a percentage of the reserve, would doubtless be less than in the examples given above.

## CONCLUDING REMARKS ON ALL THE METHODS.

Of the 14 methods of valuing endowment assurances in groups which have been discussed, *Methods* IV, V, VIII and XI, are only useful for checking purposes, and of these No. V is probably the most reliable.

Methods IX and X will be, as a rule, more accurate than any of the four just mentioned, but, as shown in the numerical results given above, they may understate the reserves by as much as 5 per-cent if the policies be of short duration, so that they can scarcely be considered sufficiently accurate except for a check valuation.

If, however, some such adjustment as suggested in Method XIV be applied, the margin of possible error will be considerably reduced, and experience may show that the results are sufficiently trustworthy.

Methods XII and XIII bring out results very near the truth. The principal objection to them is the large number of groups entailed, and the consequent labour. Even if the number of future premiums is taken, when fractional, to the nearest integer, we may easily obtain 160 or more groups.

Reliance can also be placed on *Methods* VI and VII, while the number of groups involved is much smaller. All methods, however, of the Second Class, *i.e.*, where the groups depend on selected values of the reversion or annuity, are open to the objection that

they are not adapted to an annual valuation, for, as each year passes, policies originally in the same group move irregularly into other groups and so get separated from one another. For instance, in *Method* VII, two policies with valuation ages 40 and 58 and number of future premiums 10 and 12 respectively would both be in the group corresponding to whole-life age 67, while in the following year they would fall into the groups represented by ages 69 and 68 respectively.

Looking at the broad principles of valuation, in a whole-life series we take the ages nearest birthday and use the corresponding annuities (the reversions need not be considered because they depend directly on the annuities), that is to say, we select some 90 annuity-values and assume the true annuity for each policy to have one or other of these selected values; in other words, we draw the annuity curve and set up ordinates at integral ages, and instead of using the ordinate corresponding to the exact valuation age of each policy we use the nearest of the selected ordinates.

It seems somewhat inconsistent to take some hundreds of groups, or annuity-values, in the valuation of endowment assurances, while we are satisfied with about 90 for whole-term policies, the annuities having practically the same range of value in both classes.

I think it will be admitted that no method of valuing endowment assurances can be considered entirely satisfactory which involves a number of groups wholly disproportionate to the number employed in the valuation of an ordinary whole-life series.

One difficulty of satisfactorily grouping endowment assurances lies in the fact that we have two variables, x and n, to deal with; thus, while in a whole-life series we get only 90 groups by taking equi-distant values  $a_x$ ,  $a_{x+1}$ , . . ., with endowment assurances on the contrary, if we take, as in  $Method\ I$ , equi-distant values  $a_{x:\overline{n}}$ ,  $a_{x+1:\overline{n}}$ , . . . ., n remaining constant, and equi-distant values  $a_{x:\overline{n}}$ ,  $a_{x:\overline{n+1}}$ , . . ., x remaining constant, we may easily obtain 1,000 or more groups, and unnecessary labour is entailed by the overlapping of the annuity-values.

This difficulty may be satisfactorily overcome by taking equidifferent values of the annuity, provided the intervals are not too large, 25 for example, as in *Method VI*, so that the maximum errors possible in the values of individual policies are kept within moderate limits, and this method is no more empirical than the usual method of valuing a whole-life series; in the latter we select equi-distant ordinates, and in the former equi-different ordinates to the annuity curves.

It would be difficult to find an equally simple and more reliable method of valuing endowment assurances quinquennially than *Method* VI, but, as mentioned above, it is open to the objection that it does not readily lend itself to an annual valuation.

On the other hand, Methods III and XIV are both free from this objection, and will, I am inclined to think, bring out sufficiently accurate results, especially if the number of policies dealt with is considerable.

### Discussion.

The President, in extending a cordial welcome to Mr. Thomson as a new contributor, said the very unfortunate development of endowment assurance was a matter of deep concern to the general interests of life assurance, disclosing as it did a significant diversion in the public mind from the conception of prudential assurance to that of insurance involving the element of investment. Undoubtedly that change had been largely aided by the difficulty with which they all had to contend, namely, the continuous fall in the rate of interest and the consequent difficulty in securing remunerative investments elsewhere. But that change of fashion also weighed very heavily upon the solicitude of the actuary in every department of his work, since he not only had to contemplate with very grave concern the heavy defections from premium revenue year by year as the contracts mature, but also the serious difficulty of re-supplying those diminutions of income at a moderate cost, especially if the present stress of competition should continue. The subject possessed also another very perplexing aspect in regard to those modes of equitably apportioning profits between whole-life policies and endowment assurances where the element of excess interest largely entered into the formula. He called upon the referees to be good enough to express their judgments.

Mr. Henry Andras felt it was a great honour, due to the courtesy of his co-referee Mr. Sorley, that he had the pleasure of opening the discussion on so remarkable a paper. It would be useful not only to students but to the actuarial profession generally. As to the ethical side of endowment assurances, there were some gentlemen who thought them an undesirable form of policy; of course they had their disadvantages, but they had, in his opinion, one great advantage to the offices, namely, that whilst they were on the books they contributed more profit than whole-life policies of the same amount and the same standing as to age at entry and duration; he thought that had been demonstrated by several actuaries, and also by the experience on a comparatively large scale of the office keeping its endowment assurance contracts separate from its whole-life contracts

and giving each class its own profits, the resulting bonuses, as published in the Board of Trade returns, being larger on endowment assurance policies than on whole-life policies. He believed this result was chiefly due to very favourable mortality experience in the endowment assurance class. With regard to the valuation factors given in the paper, he himself had been in the habit of using the next anniversary method in valuing endowment assurances individually, and he was inclined to think that, had the author, in calculating his average ages, used the ages next birthday instead of the valuation ages, probably some of the comparatively small resultant errors of his methods would have disappeared. He had himself tested Mr. Thomson's method 3 by valuing a group of about 700 policies with a satisfactory result, except that the resultant percentage error was 5 against Mr. Thomson's '279 for about the same number of policies.

Mr. J. Sorley thought they should be chary in giving anything like an official approval to any of the methods proposed for approximating to the value of endowment assurances. These approximate methods were very valuable for purposes of cheeking, but the basis of facts by which their comparative accuracy was shown was as yet too narrow for them to agree to dispense with the more detailed calculations. If the number of endowment assurances was small in relation to the total business, then the labour of detailed calculations could not be very important, while if they formed a considerable proportion of the total business it was desirable to get at an accurate and certain measure of the reserve. It was well to bear in mind that the approximate methods seemed persistently to err on the side of understating the liability. Looking to the heavy proportional reserves required for this class of business, an understatement of the reserve by even only 1 per-cent might make quite an appreciable difference in the bonus distributed. He thought the labour of valuing endowment assurance policies individually was probably over-estimated. Some years ago he had been employed in sketching a valuation register, in which all the facts were to be recorded and the valuations given in a single line for each policy and without any subsidiary work by a judicious omission of decimals, &c., the values being either got at once by inspection or by a simple reference to Crelle's Tables. The labour of those approximate methods under which they had to find an average age relatively to the sums assured, and then another average age relatively to the premiums payable, could not be very greatly less. Mr. Thomson's suggestion, that it was somewhat inconsistent to take some hundreds of groups for endowment assurances when they were satisfied with about ninety for whole-term policies, would not bear examination. The sub-divisions arose from the nature of the case. There were two variables as against one. A table of joint-life annuities occupied a great deal more space in their actuarial books than a table of single life annuities, although the latter was much more important. The net sums at risk under whole-term policies approximated to a symmetrical curve, while the due dates of endowment assurances tended to cluster round certain ages, fifty, sixty, &c., much in the same way as unadjusted census returns did. Facts such as these constituted the great difficulties in the way of approximate methods, and the investigations

submitted by Mr. Thomson, of their effect on numerical results, were therefore of much value.

Mr. GEO. KING was glad to take an early opportunity in the discussion of joining his congratulations to those of the president and referees upon Mr. Thomson's able paper. A good many years ago he had to make a valuation, in conjunction with two of their most honoured members, of a very large Colonial concern where about one-third of the business was endowment assurances. They had 8,000 of those policies to value. They made a detailed valuation of the whole of them, but wanted a check, and he had set to work to see what could be done to find an easy method of getting it, and had hit upon Mr. Thomson's method No. S. Perhaps the special case to which he referred was unusually favourable for such an application, because not only were there a very large number of endowment assurance policies, but they all ran very uniform in amount and as to their original term. The method was so successful that in that very large mass of business, which had extended over a considerable number of years, and where the reserve was consequently large, they detected an error of £100 in the additions, and that had given him great confidence in the method of grouping. He had been disappointed, however, in applying the method to another case. It broke down simply because the terms varied very much, and the magnitude of the policies differed exceedingly. Nevertheless, for many purposes, method 8 was useful, and it might be remembered that he ventured to suggest it as a method of testing the valuations of companies if they could have it embodied in "The Life Assurance Companies Act." He agreed in the hesitation which had been expressed as to calculating the reserves in the quinquennial valuations for distribution of surplus upon one of those methods of grouping, but he thought they might very well adopt one of them, varying it according to circumstances, for check valuations, which were always exceedingly useful. As a matter of fact, he much preferred to do the work in two entirely independent ways, rather than to check the actual ealeulations, and for such a purpose, not only was the labour probably less, by resorting to a method of grouping, but the fact that they got a complete check was much more satisfactory. From that they might proceed a step further. It was usual to make valuations quinquennially, or at certain periods, but it was very useful for office purposes to make annual valuations indoors which did not go forth to the public. They might thus see how their surplus was growing, and might make preparations for changes, either in the method of valuation, or in the method of division of surplus, which could not be rushed through at the quinquennial valuation itself. If they had a quinquennial individual valuation made of the endowment assurance policies, and tested that by a group valuation, they might, during the remaining years of the valuation period, make their interim valuation purely and simply by the group method, and so save themselves a great deal of trouble. That would give results amply accurate for all practical purposes, and they could periodically make slight alterations in their method of grouping, should such be necessary, by comparing the results with those of the item by item valuation at the quinquennial period, and in that way they would get

really very close approximations throughout the whole five years. The schedule in Table 4 was rather alarming, in that it gave so many different functions according as the premiums were payable quarterly, half-yearly, or annually, but if they made the very convenient assumption that on the average for the fraction there would be in half-yearly cases half of a half-yearly premium still to pay, and, if similarly they took the average of the four lines for quarterly cases, they would come back as regards the sum assured to what they had on the first line of the table. They did not get quite the same result in the case of the annuity for valuing the premiums, but by similarly taking the annuity in the first line of the table they somewhat over-estimated the liability, and seeing that the group methods as a rule had a tendency to under-estimate the liability, they would get a correction which would probably lead to results of close Mr. Thomson had shown, in method 11, that by duplicating the groups they could get rid of the fractions in the functions used in the valuation, and so very much reduce the labour, and they would still remain sufficiently accurate for practical purposes in interim valuations, to which alone he personally should

confine the method at present.

Mr. H. W. MANLY said the author had treated in a masterly manner a subject with which they were all greatly concerned, and which was growing in importance every day. It was possible that the time might come when the business of an insurance office would consist mostly of endowment assurances, and when whole-life assurance would be the exception. Many years ago, he adopted what he understood was Mr. Thomson's 9th method. Some previous speakers had hesitated to adopt a valuation by that method, considering it only as useful for a check. He himself felt absolute confidence in using it. Although he had noticed that the general result produced was slightly less than the result produced by the individual valuation of policies, still, it always seemed to him that the deficiency was so small that it might fairly be neglected, considering the ease and rapidity with which a valuation could be made by that method of grouping. By adopting the method of grouping according to the year of maturity, they could see at a glance how much they had to pay next year, how much the following year, how much the third and fourth years, and so on, and if by their method of valuation they could retain in hand sufficient to provide for those maturing in early years, it did not matter very much whether the valuation for those which would mature in 30 years was 1 per-cent, 5 per-cent, or even 10 per-cent out. He always assumed that in the year of maturity no premiums were receivable, so that if a half-yearly or quarterly premium fell due in the year of maturity, there was a sum in reserve which was not taken credit for, and, therefore, possibly the reserve eventually was somewhat larger than an exact valuation of those policies would produce. had suggested that it would be a convenience if they could make the returns to the Board of Trade of that class of policy in the classified form which had been suggested. On two occasions he had submitted the endowment assurances, where they were very numerous, to the Board of Trade, classified according to the year of maturity, and on

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both occasions they were accepted, so that it might be taken that the Board of Trade was willing to fall in with this view. It was a great convenience to those making the return, and certainly gave a much better opportunity to anyone who wished to make an

independent valuation for arriving at a satisfactory result.

Mr. G. F. HARDY was not inclined to agree with Mr. Andras, that while the endowment assurance policies were on their books they were returning a better profit than the whole-life business. If this had been so in the past, he thought that in the future the tendency would be in the opposite direction. In the face of the loss of profit from falling interest rates they had continued to earn good bonuses in consequence of the profit on mortality. The tendency to displace whole-life business by endowment business, and by endowment business of a shorter average term, would gradually eliminate the greater part of that profit from mortality, and if their whole business were to take the form of endowment assurances it would be much harder in future to earn the bonuses which they had been in the habit of doing. It would not be possible for an office to counteract that effect by materially increasing the loading on its endowment premiums, as the contract no longer became attractive to the public, who would go elsewhere, and the office would lose its business. It appeared to him, therefore, that the gradual displacement of wholelife business by endowment assurance must, in the long run, cause a fall in their rate of bonus. Mr. Sorley considered that an error of 1 per-cent in the estimated reserve at a valuation would be a serious one, and this would undoubtedly be so if the error were fluctuating, and if they were ignorant as to its direction. A method giving a small constant error, always in the same direction, would not, however, be open to the same objection. Of the various methods of grouping described he had been rather attracted to that which might be termed a modification of Mr. Woolhouse's method, in which fixed annuity-values were used. If cards containing particulars for the valuation of the endowment assurance policies were arranged in compartments in a manner corresponding to the table of temporary annuities in Mr. R. P. Hardy's "Valuation Tables", columns representing ages, and rows the number of years to run, and each compartment headed with its appropriate annuity-value to the nearest integer, then a certain definite group of compartments, always an unbroken group, would correspond to each integral annuityvalue. All that would be required at a valuation would be to schedule the cards in each such group of compartments, moving each batch of cards each year diagonally into the contiguous compartment. This would be an easy mode of applying Mr. Woolhouse's method of valuation, and if the number of policies were considerable, would, no doubt, give a very approximate result.

Mr. J. Howard Barnes said it seemed somewhat strange to the younger generation of actuaries that endowment assurances should ever have been granted, subject to n+1 premiums where the difference between the age of maturity and the age at entry was only n years, and they were not surprised that such policies were rapidly becoming fewer and fewer. He ventured to forecast that in the future it would be equally surprising to the actuaries that they

should have charged a man as much for an insurance to run  $20\frac{3}{4}$ years as another man who was nine months older for an insurance to run for 20 years only, and that, as a result of competition and otherwise, endowment assurances maturing on the anniversary of the birthday would be superseded by assurances maturing on the anniversary of the policy. It had now become necessary to group policies, and the methods suggested in Class 1 were most natural, and gave, of course, identical results when properly arranged. even with policies grouped under the ages of maturity they had too many groups, and stress of time, the value of labour, and the necessity for arriving at results quickly after the closing of books and so forth, would compel them in future to become dependent on shorter methods still. The method referred to of ranging the policies round corresponding whole-life ages was employed in a company with which he had been connected, and he had ventured to refer to that two or three years ago when speaking on the subject. It had some very considerable advantages over the methods of Class 1, one of which was that if the actuary had had the whole-life valuation functions printed on sheets for all ages, these sheets could be used for endowment assurances as well, though the method was not quite so accurate as that of taking intervals of 25 in the annuity-value. The disadvantage, of course, was that the schedules were not available from time to time, but only for one valuation. Schedules prepared according to the Class 3 methods were, on the other hand, available from year to year, and were very powerful for the valuation of endowment assurances. With regard to the question of the persistent error, he entirely agreed with Mr. Hardy, and he would far rather face a persistent error if they knew it was there than face an error which was not necessarily persistent when they were uncertain of its direction or magnitude. For instance, he had had occasion to test a whole-life valuation by Mr. Woolhonse's formula, and found there was a difference of over £3,000 in the liability brought out for a large class. Two years later the difference was less than £500—the difference in the valuation of the sums assured alone being less than £20. The reason for this range of difference was that even in a large class the amounts assured at successive ages did not run very smoothly, and while the errors of the approximate methods would sometimes balance, they might pull more one way than another. With many of the assumptions made at valuations—such as the average age—they might unconsciously be introducing errors of great importance though they were unconscious of their presence or The persistent error, therefore, was not a thing to be troubled about, provided they were aware of its existence, and had some means of measuring it. With regard to the numerous formulas Mr. Thomson had given for the valuation of policies, subject to halfyearly or quarterly premiums, he would like to suggest that that difficulty could be got over very easily by scheduling the policies as if they were all subject to annual premiums, but reserving a column for "outstanding premiums", that was, in half-yearly and quarterly cases the portions of the current year's premium falling due after the valuation date. They could take the net premium and adjust the valuation figures so as to obtain a result which would be

substantially correct. For endowment assurances this simple plan would also get over the very unsatisfactory desire suggested in the paper under Class 3 of treating  $n+\frac{1}{4}$  and  $n+\frac{3}{4}$  more premiums as n and n+1 respectively, and  $n+\frac{1}{2}$  as n and n+1 alternately. It was impossible to measure the error which might thus be introduced, and when policies were altered from annual to quarterly, or vice versa, it might be necessary to move them from one group to the next, which was quite got over by the column "Outstanding Premiums." He would also suggest that, in finding the average age at which a valuation was to be made, it was very much easier, where the valuation formula would allow of it, to work with the age at which the policy was to mature than with the age attained, and to correct the result according to the number of future premiums, because the multiplications were simpler, and once they had been worked out they stood at succeeding valuations, so that the labour of re-multiplying year by year for the valuation age was saved.

Mr. F. W. F. HUTCHINSON had had some little experience in the valuation of endowment assurances in one office, and would like to bring the matter before the members. The great majority of the policies matured at quinquennial ages, 45, 50, 55, 60, 65, and 70; very few of them matured at intervening ages, and they were run up or down, as Mr. Thomson suggested in method 2. There were over 8,000 policies altogether, insuring more than three-and-a-half millions. and they were grouped in each of those six ages, and a valuation was made bringing out a reserve of over £500,000. Mr. Thomson had suggested an adjustment which merely affected the average age brought out as in method 9 or 10. He thought it would be very much better if the policies were scheduled under ages such as he had stated, before being brought under one class according to the term to run. There were two reasons for that. The first was that the average age was easier to obtain, because only six multiplications were necessary in that case, whereas, if each individual policy had been multiplied by the age of maturity it would be about 8,000. The second reason, which was the more important, was that due weight would be given to the sums assured on each of those maturing ages in finding out the adjustment. In this example, the adjustment by Mr. Thomson's method was 2.0. whereas the more correct adjustment, got by giving due weight to the sums assured, was 92. He had got out the average age, as explained in method 10, and added on an adjustment of 92, and had done another valuation, and the reserve was only 129 per-cent less than the sum of six valuations taken at those six ages. In that way be thought that by arranging them under six groups, and giving due weight to the sums assured under each of those ages at maturity, and finding the proper adjustment, a valuation could be made in one group, and give practically the same result.

The President said that it might be of interest, though the point was a purely incidental one, to mention that Mr. Pattison was the sole author of the Insurance Act of 1870, and, to his personal remembrance, that Mr. Pattison also prepared the speeches which were delivered from both the Liberal and Conservative benches when the measure was introduced. It was Mr. Pattison who proposed the

insertion of the total premiums paid upon special policies, because, as Mr. Pattison had explained to him, he thought from analogy with the valuation of whole-life policies he was placing a method within the hands of actuaries for assessing approximately the reserve for those special risks, since he assumed that 50 per-cent of the total premiums would represent an H<sup>M</sup> 3 per-cent reserve. He begged to convey the very cordial acknowledgments of the members to Mr. Thomson.

Mr. Thomson, in replying, thanked the members very heartily for the reception they had given to his paper, and also the various speakers for their criticisms. Mr. Andras had pointed out that it would probably get rid of some of the errors dealt with in the paper if they took the age next birthday as the valuation age. That was practically done in Table 21, but the results were not satisfactory. The errors were considerably greater than in some of the other methods—for instance, methods 3 and 14. There seemed to be some difference of opinion as to whether it was safe to value endowment assurances in groups or not. One speaker had objected to an error of 1 per-cent in the reserve, but some of the methods suggested in the paper had, in the numerical results given, brought out errors of only 2 per-cent and 4 per-cent, even when the policies valued were of only two years' average duration; if the average duration had been longer, no doubt the error per-cent would have been less still. He did not quite follow the reasons given for continuing to value endowment assurances individually. He did not see why they should be afraid of the small errors introduced by some of the more reliable methods of valuing endowment assurances in groups. In a valuation of whole-term policies, errors were involved in the reserves on individual policies owing to the errors in the valuation ages, which might be six months out. That might produce an error of 2 per-cent in the reserve on a policy that had been 30 years in existence, 5 percent if 10 years in existence, and 10 per-cent on the reserve if 5 years in existence, and no less than 37 per-cent if only one premium had been paid. In a whole-life valuation they ignored those errors without any scruples, simply because they were equally likely to be + or -, and there was thus a balance of the error, so that the aggregate results for a large number of policies must necessarily be near the truth. He saw no reason why they should be any more afraid of errors in valuing endowment assurances in groups. He was much interested in Mr. King's remarks as to the very close results which method 8 has been known to bring out. He did not think that the method could be relied on, and, as Mr. King had pointed out, it would sometimes break down entirely. He suggested, as one possible explanation of the accuracy of the results in the case mentioned by Mr. King, that the "range" of valuation ages might have been very small in each group. In the extreme case, where the "range" in each group was nil—that is, where all the policies in a group had the same valuation age—there would obviously be no error at all, either in method S or 9. Such a case would not, of course, occur in practice; but if the "range" happened to be small in each group, the results must necessarily be very close to the truth. Another possible explanation was, that the errors in

the reserve for individual groups, though appreciable, might have happened to counter-balance, through being of opposite sign. This was possible for method 8, but not for method 9. Mr. Manly had expressed confidence in method 9. The application of method 14 only added about ten minutes time to a valuation by method 9, and he thought the results must be very much more accurate in all cases. He was glad that Mr. G. F. Hardy approved of valuing endowment assurances in groups, and he was much interested in his explanation showing how the methods of the second class depending upon equidifferent annuity-values could be made continuous, to go on from year to year. One or two speakers had referred to the tables of rather complicated valuation factors on pages 4–5. He sincerely hoped nobody employed them. He certainly did not advocate their use for a moment: they had merely been inserted in order to depict the horrors of an individual valuation if conscientiously carried out.

Some Remarks on the Valuation of Endowment Assurances in groups. By George J. Lidstone, Fellow of the Institute of Actuaries, Assistant Actuary of the Alliance Assurance Company.

[Read before the Institute, 31 January 1898.]

1. ONE of the most striking features in the history of Life Assurance business during the past few years is the marked increase—both absolute and proportional—which has been shown in the number and amount of policies effected under the Endowment Assurance plan. There is every reason to expect that this increase will continue, and that ere long Endowment Assurances will divide honours with whole life policies as regards importance in the periodical valuations. It is consequently very desirable so to improve the methods of valuing such policies as to minimize the amount of labour involved, and it will become more and more necessary to abandon methods which answered very well so long as Endowment Assurances were looked upon as "Special Policies", but which are extremely cumbersome when applied to a large mass of contracts. The shorter methods that have hitherto been suggested, though admirably adapted for use in test or cheek valuations, involve an error which many actuaries will consider too appreciable to be neglected, more especially as the approximate methods nearly always bring out a reserve which

is *smaller* than the true reserve, and it accordingly becomes desirable to seek some process that will be free from this objection.

- 2. The main idea of the methods at present in use is to group together all policies having the same unexpired term, and to use for each group valuation factors based upon a mean age. This process has such obvious advantages that it seems clear that, if possible, the same method of grouping should be adhered to in any modified process that may be suggested, and that any improvement should relate to the method of determining the mean age.
- 3. Mr. King suggested\* (J.I.A. xxix, 517) that the mean age should be found "by adding together the ages of the lives assured and dividing by the number of policies", that is to say, the mean age is the arithmetical average of the individual ages. Mr. Manly referred (J.I.A. xxviii, 279) to the process which he himself adopted, namely, for each group to multiply the sum assured at each age by the age, and divide the sum by the total sum assured, thus weighting the ages in proportion to the amounts assured. This process has been adopted by subsequent writers, and is the plan now generally used when the method is applied in practice. It is but another step to weight the ages separately with respect to the sums assured and the net premiums (since the mean valuation age will not necessarily be the same for both).
- 4. The last-mentioned method would evidently give absolutely accurate results (so far as sums assured and net premiums are concerned) if the annuity-values for any specified term varied with the age by constant first differences, but this is not actually the ease, as may be seen from the following Table.

<sup>\*</sup> Mr. King has pointed out that this method was not intended for valuation purposes, but was suggested by him simply for use in connection with the Valuation Returns to the Board of Trade, with the view of supplying sufficient information to enable an independent opinion to be formed as to the position of a company.

Table I. Showing the Values of  $-\Delta_x a_{x;\overline{n}} = a_{x\overline{n}} - a_{x+1;\overline{n}}$ . [H<sup>M</sup> 3 per-cent.]

|    |      | n =  |      |      |      |      |      |      |      |      |  |  |  |  |
|----|------|------|------|------|------|------|------|------|------|------|--|--|--|--|
| x  | 5    | 10   | 15   | 20   | 25   | 30   | 35   | 40   | 45   | 50   |  |  |  |  |
| 20 | .001 | .005 | .011 | .020 | .030 | .043 | .061 | .081 | ·105 | ·131 |  |  |  |  |
| 25 | .003 | -009 | .018 | .030 | .046 | .066 | .090 | .118 | ·149 | ·179 |  |  |  |  |
| 30 | .002 | .010 | .021 | .037 | .058 | .086 | ·119 | ·155 | ·191 |      |  |  |  |  |
| 35 | .004 | .014 | .030 | .053 | .085 | 124  | .165 | .208 |      |      |  |  |  |  |
| 40 | 1001 | .019 | .042 | .078 | ·123 | ·172 | ·223 |      |      |      |  |  |  |  |
| 45 | .009 | .033 | .070 | ·121 | ·179 | .239 |      |      |      |      |  |  |  |  |
| 50 | .011 | .015 | .099 | .164 | ·236 |      |      |      |      |      |  |  |  |  |
| 55 | .020 | .072 | 145  | .229 |      |      |      |      |      |      |  |  |  |  |
| 60 | .032 | 102  | 197  |      |      |      |      |      |      |      |  |  |  |  |
| 65 | .040 | .139 |      |      |      |      |      |      |      |      |  |  |  |  |

- 5. It will be seen that the differences uniformly increase numerically with the age; hence the mean age will be too low, the mean annuity-value too high, and the calculated reserve too low.
- **6.** It will be useful at this point to investigate mathematically the form of  $\Delta_x a_{x\bar{n}}$ , or rather of  $\frac{d}{dx} \bar{a}_{x\bar{n}}$ , which is equally suitable for the purpose and somewhat easier to handle. We have

$$\begin{split} \frac{d}{dx}\overline{a}_{x\overline{n}} &= \frac{d}{dx}\int_0^{n_t} v_t^t p_x \cdot dt = \frac{d}{dx}\int_0^{n_t} v_t^t \cdot \frac{l_{x+t}}{l_x} \cdot dt = \int_0^{n_t} v_t^t \cdot \frac{d}{dx} \left(\frac{l_{x+t}}{l_x}\right) dt \\ &= -\int_0^{n_t} v_t^t (\mu_{x+t} - \mu_x) \frac{l_{x+t}}{l_x} dt \, . \end{split}$$

Now, on Makcham's hypothesis,  $\mu_{x+t} - \mu_x = \mathbf{B}c^x(c^t - 1)$ , and the differential coefficient reduces to the form  $-\mathbf{B}c^x \int_0^n v^t(c^t - 1)_t p_x dt$ .

- 7. Considering the definite integral we shall see that it represents either
  - (i.) the value of a temporary annuity with increasing payments, or
  - (ii.) the difference between the values of two uniform annuities, one calculated at the ordinary force of interest say  $\delta$ , the other calculated at the force of interest  $\delta \log_{\varepsilon} c$ .

If we make the assumption that the value of the integral decreases in geometrical progression as the age advances (an assumption which, although not theoretically exact, will represent with sufficient accuracy for our purpose the general tendency of the function to diminish with increasing rapidity as the age increases), it follows that the differential coefficients, and therefore the first differences, of the annuity-values are increasing in geometrical progression, with a common ratio r, say, which is less than c. On reference to Table I, it will be seen that this hypothesis does very fairly represent the facts, the first differences being increased by about 50 per-cent every five years, representing an annual increase of about  $8\frac{1}{2}$  per-cent.\*

8. From the hypothesis referred to in the preceding paragraph, it follows that the annuity-values will be approximately in the form  $a-\beta r^x$ . Hence it is evident that the mean age will be found as follows:

Let the sum assured on lives aged x in any group be denoted by  $S_x$ ; then, if the product  $S_x^{x}$  be formed for each age and the sum

<sup>\*</sup> The value 1.085 for the ratio was actually fixed in the manner above mentioned, but it might have been determined approximately as follows. Let x be the mean valuation age and n the mean unexpired term, for the whole business, and t a term of years such that the bulk of the business in any group is included within a range of t years on either side of the mean valuation age for the group. Then r may be taken as approximately equal to

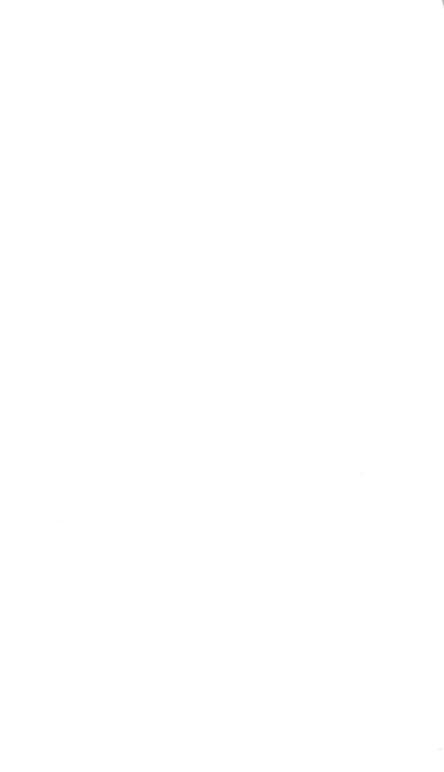
 $<sup>[</sup>a_{x\overline{n}} - a_{x+t,\overline{n}}] \div a_{x-t,\overline{n}} - a_{x\overline{n}}]^{t}$ . Putting x=40, n=20, t=10, and making the calculations on the basis of the Text-Book graduation of the HM experience (in order to avoid the disturbing effect of irregularities), with 3 per-cent interest, it will be found that r=1.083, and practically the same result will be obtained if t be taken =5 or 7. Again, putting x=45, n=15, t=5, 7, or 10, the resulting value of r varies from 1.085 to 1.083, agreeing closely with the former result. In practice, it is recommended that the constant c be taken for the common ratio—see paragraphs 18-20, also Mr. G. F. Hardy's remarks in the discussion.

If the mean and be to, and the common unexperied becale Then \(\Sigma\) = A \(\overline{\text{tr}}\) \(\times\) or \( \left\{ S. (1 - \( \tilde{a}\_{n, \tilde{n}} \) \right\} \) = (1 - \( \tilde{a}\_{n, \tilde{n}} \) \( \tilde{S} \) or  $\sum (S.\bar{a}_{s(h)}) = \bar{a}_{\bar{h},\bar{h}} \times \sum S$ But if any he co, the force x+30" We g & + ∑ {S(x+3c")} = (x+3c").∑S or  $\Sigma S.e^n = e^n. \Sigma S$ 

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of the products be divided by  $\Sigma S_x$ , the quotient will give  $r^y$ , where y is the mean age for the group and is  $=\log [\text{quotient}] + \log r$ .

When r is put equal to unity, the value of y deduced by this process takes the indeterminate form  $\frac{0}{0}$ ; but it is not difficult to show that the limiting value of the indeterminate expression when r=1 is the mean age found by Mr. Manly's process. (See paragraph 3.)

9. In order to test the effect of the new method of calculating the mean ages, several experimental calculations were made, the results of which entirely confirmed the writer's opinion that the method would be much more accurate than any at present in use. It was, therefore, thought desirable to make an extended comparison of the various methods as applied to a large body of Endowment Assurance business, particulars of which are summarized in the following Table.

Table II.
Summary of Particulars.

| Number of<br>Premiums<br>remaining<br>n | Sum Assured | Bonus<br>Additions | Sum Assured<br>and<br>Bonus Additions | Net Premiums<br>II <sup>M</sup> 3 per-cent |
|---|-------------|--------------------|---------------------------------------|--|
|   | £           | = - E              | £                                     | £  |
| 0-4                                     | 26,650      |                    | 26,650                                | 1,124                                      |
| 5- 9                                    | 58,760      | 2,332              | 61,092                                | 3,125                                      |
| 10-14                                   | 104,211     | 3,523              | 107,734                               | 4,731                                      |
| 15-19                                   | 141,900     | 2,608              | 114,508                               | 5,358                                      |
| 20-24                                   | 131,354     | 3,063              | 134,417                               | 4,145                                      |
| 25-29                                   | 107,087     | 1,422              | 108,509                               | 2,806                                      |
| 30-34                                   | 64,550      | 433                | 64,983                                | 1,442                                      |
| 35-36                                   | 13,150      | 93                 | 13,243                                | 269  |
| Total                                   | 647,662     | 13,474             | 661,136                               | 23,000                                     |

- 10. The policies included in the valuation were all payable at one of the ages 50, 55, 60, 65. Assurances maturing at other ages were for convenience excluded, but the total amount of such assurances was relatively so small that their exclusion can have but slightly affected the results of the comparison.
- 11. A valuation of the policies was made on the basis II<sup>M</sup> 3 percent by each of the undermentioned methods.

Method A.—The policies were valued individually, according to the valuation age in each case.

Method B.—The policies were grouped according to the term

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unexpired, and the mean valuation age for each group was found by Mr. Manly's method. (See paragraph 3.)

Method C.—The same grouping was adopted, but the mean valuation age was found separately for the sum assured and the net premiums.

Method D.—The same grouping was used but the mean valuation age for the sum assured was found by the new method (r assumed=1.085), the same age being used for the net premiums.

For the present purpose it is unnecessary to discuss the particular valuation formulæ that were adopted.

Table III.

Summary of Valuation.

|        |  |                             |                  | Error  | Perci                                   | ENTAGE OF 1             | Error   |
|--------|--|-----------------------------|------------------|--|---|-------------------------|---|
| Method | Value of<br>Sum<br>Assured<br>and<br>Bonuses | Value of<br>Net<br>Premiums | Net<br>Liability | Estimated<br>Liability,<br>less<br>True<br>Liability | To<br>Total Sum<br>Assured<br>and Bonus | To<br>True<br>Liability | To<br>Total Sum<br>Assured<br>and Bonus<br>less True<br>Liability |
|        |  |                             | n =              | =0-36  |   |                         |   |
| A      | 405,705                                      | 264,370                     | 141,335          |  |   |                         |   |
| В      | 404,918                                      | 265,266                     | 139,652          | + -1,683   | <b>−</b> ·25                            | -1.19                   | - 32  |
| C      | 404,918                                      | 265,193                     | 139,725          | -1,610   | - 24                                    | -1.14                   | - 31  |
|        | 405,734                                      | 264,395                     | 141,339          | + 4  |   |                         |   |

12. The results of the several valuations are summarized in Table III, and are shown in greater detail in Table IV. These Tables also contain columns showing the amount of the error in the three approximate results, and the percentage of the error as compared with (1) the amount assured (including Bonus additions), (2) the true liability, (3) the amount assured less the true liability, i.e., the "death strain at risk." The percentages, of course, vary with the unexpired term of the assurances, but it will be found that the first and third vary more slowly and within narrower limits than the others, as might have been expected. It would appear that, on the whole, the third form is the best to adopt when comparing results derived from different data, as it enables us to eliminate to the greatest extent the disturbing effect of differences in the constitution of the business.

Table IV.
Summary of Valuation.

|                  |  | 131                                  | ımmary                               | of value   | uuon.                                   |                          |   |
|------------------|--|--------------------------------------|--------------------------------------|--|---|--------------------------|---|
|                  |  |                                      |                                      | Error  | PERCE                                   | NTAGE OF                 | Error   |
| <b>M</b> étho·l  | Value of<br>Sum<br>Assured<br>and<br>Bonuses | Value of<br>Net<br>Premiums          | Net<br>Liability                     | Estimated<br>Liability,<br>less<br>True<br>Liability | To<br>Total Sum<br>Assured<br>and Bonus | To<br>True<br>Liability  | To<br>Total Sum<br>Assured<br>and Bonu-<br>less True<br>Liability |
|                  | -  |                                      | 11 =                                 | = 0 - 4  |   |                          |   |
| A<br>B<br>C<br>D | 24,332<br>24,328<br>24,328<br>24,332         | 2,490<br>2,497<br>2,496<br>2,493     | 21,842<br>21,531<br>21,832<br>21,839 | - 11<br>- 10<br>- 3                                  |   | - ·05<br>- ·05<br>- ·01  | <br>- ·23<br>- ·21<br>- ·06                                       |
|                  |  |                                      | n=                                   | = <b>5 -</b> 9                                       |   |                          |   |
| A<br>B<br>C<br>D | 49,077<br>49,038<br>49,038<br>49,080         | 18,198<br>18,275<br>18,266<br>18,209 | 30,879<br>30,763<br>30,772<br>30,871 | -116<br>-107<br>- 8                                  | - 19<br>- 18<br>- 01                    | - ·38<br>- ·35<br>- ·03  | - · · · · · · · · · · · · · · · · · · ·                           |
|                  |  |                                      | n =                                  | 10-14  |   |                          |   |
| A<br>B<br>C<br>D | 76,906<br>76,804<br>76,804<br>76,906         | 42,453<br>42,625<br>42,589<br>42,486 | 34,453<br>34,179<br>34,215<br>34,420 | -274<br>-238<br>- 33                                 |   | - ·\$0<br>- ·69<br>- ·10 | - · 37<br>- · 32<br>- · 05  |
|                  |  |                                      | n =                                  | 15 <b>-</b> 19                                       |   |                          |   |
| A<br>B<br>C<br>D | 91,115<br>90,913<br>90,913<br>91,101         | 63,700<br>63,949<br>63,952<br>63,722 | 27,415<br>26,964<br>26,961<br>27,379 | -451<br>-454<br>- 36                                 | -:31<br>-:31<br>-:03                    | -1.65<br>-1.66<br>13     | - · 39<br>- · 39<br>- · 03  |
|                  |  |                                      | n =                                  | 20 - 24  |   |                          |   |
| A<br>B<br>C<br>D | 75,049<br>74,834<br>74,834<br>75,063         | 59,880<br>60,097<br>60,078<br>59,866 | 15,169<br>14,737<br>14,756<br>15,197 | -432 $-413$ $+25$                                    | -:32<br>-:31<br>+:02                    | -2·85<br>-2·72<br>+ ·18  | - ·36<br>- ·35<br>- ·02   |
|                  |  |                                      | n =                                  | 25 <b>-</b> 29                                       |   |                          |   |
| A<br>B<br>C<br>D | 54,205<br>54,062<br>54,062<br>54,228         | 46,313<br>46,437<br>46,432<br>46,294 | 7,892<br>7,625<br>7,630<br>7,934     | -267 $-262$ $+42$                                    | - · · · · · · · · · · · · · · · · · · · | -3·38<br>-3·32<br>+ ·53  |   |
|                  |  |                                      | n =                                  | 30-34  |   |                          |   |
| A<br>B<br>C<br>D | 29,435<br>29,362<br>29,362<br>29,438         | 26,160<br>26,203<br>26,198<br>26,149 | 3,275<br>3,159<br>3,164<br>3,259     | -116<br>-111<br>+ 14                                 | 18<br>17<br>+-02                        | -3·54<br>-3·39<br>+ ·42  | - 19<br>- 18<br>+ 02  |
|                  |  |                                      | n =                                  | 35-36  |   |                          |   |
| A<br>B<br>C<br>D | 5,586<br>5,577<br>5,577<br>5,586             | 5,176<br>5,183<br>5,182<br>5,176     | 410<br>394<br>395<br>410             | - 16<br>- 15   | 12<br>11                                | -3.90<br>-3.66           | -·12<br>-·12  |

- 13. On referring to Table III, it will be seen that methods B and C give results which agree very closely in the aggregate, showing that in the ease of the particular business in question no advantage is gained by the use of separate mean ages for the sum assured, and the net premiums. Both methods show an error of upwards of £1,600, representing 1 per-eent on the true liability, 25 per-cent on the total amount assured, or about 3 per-cent on the amount assured less true liability.
  - 14. The new method (D), on the other hand, brings out a result which is practically exact, showing an error of only £4. This is such an extremely close approximation that it seems almost necessary to state that the figures are here given exactly as they originally appeared, without any attempt having been made to adjust them in any way.
  - 15. It can hardly be supposed that the method would give, under all circumstances, results as close to the truth as in this case. The extremely approximate result is no doubt partly accidental, and reference may also be made to the following points:
    - (1) The final error is made up of two errors of opposite sign, which tend to counteract one another; namely, an error in defect in the case of assurances having an unexpired term less than 20 years, and an error in excess in the ease of assurances having more than 20 years still to run. The business included in the valuation embraced a large proportion of recently effected assurances, having more than the average unexpired term; and hence there was a larger proportion of errors in excess, tending to neutralize the errors in defect, than there would be in a business that had reached a stationary condition. It will, however, be found that the total of the first group of errors amounts to only £80, and the total of the second group to £84; so that even if either be considered to exist alone, without any counteraction by errors of opposite sign, the deviation from the true result will only be about 5 per-cent of that produced by the ordinary method (B).
    - (2) The total amount of Bonus additions is small compared with the total sum assured, partly because with-profit and non-profit assurances have been combined, but principally because of the large amount of new assurances. When the Bonus additions are larger in

proportion to the sums assured, the mean age determined from the sums assured alone will probably be somewhat too small, since the proportion of Bonus additions will probably be greatest at the oldest age. This disturbance, however, will apply equally to method B.

- 16. Giving due weight to these considerations, it still appears probable that, under normal conditions, the method can be relied on to give a result which will differ from the truth by not more than about  $\frac{1}{10}$ th of the deviation produced by method (B). Such an approximation is sufficient for the most exacting, and it is, in fact, hardly possible to choose between the approximate result and the so-called "True" result.
- 17. It is regretted that, for want of the necessary data, the new process has not been applied to Endowment Assurances maturing at the expiration of 10, 15, . . . . years from entry, instead of at specified ages, more especially as it is in this case that a method of grouping is most necessary and convenient. It is probable that, with such policies, the valuation ages will vary within a somewhat wider range than when the assurances mature at ages 50, 55, &c.; but, on the other hand, there will be a tendency for the principal weight of the sums assured to fall at and around the mean age, a condition very favourable to the accuracy of the approximation. It is, therefore, probable that the results of the new method would be nearly as good in this case as in that already discussed, and the new plan would certainly compare very favourably with the existing ones. (See paragraph 31.)
- 18. It is evident that an increase in the value of the ratio r will produce a higher mean age, and therefore a larger reserve. Hence, by taking r sufficiently near to its superior limit c, we can make certain of bringing out a reserve which will not be less than the true reserve. There are, as will be seen later, collateral advantages in using Makeham's constant c for the common ratio (see paragraph 34), and it was, accordingly, thought well to test the effects of adopting this value. The mean age for each group was therefore determined on the same principles as before, and it was found that the mean ages so found could be represented very closely indeed by the following expression:—

$$y_{(1.085)} + \frac{1}{10} [y_{(1.085)} - y_{(1)}]$$

where  $y_{(1)}$  and  $y_{(1)085)}$  represent the mean age when r is taken equal to 1 (method B) or 1.085 (method D) respectively.

19. The errors in the reserves will, of course, be in approximately the same relation, and, using a similar notation, we may write with sufficient accuracy for practical purposes

$$\begin{split} \mathbf{E}_{,c,} &= \mathbf{E}_{,1\cdot 085,} + \frac{1}{10} \left[ \mathbf{E}_{,1\cdot 085,} - \mathbf{E}_{,1,} \right] \\ &= 1\cdot 1 \, \mathbf{E}_{\,1\cdot 085,} - \cdot 1 \mathbf{E}_{\,1,} \, . \end{split}$$

- **20.** If we assume that  $E_{1.085}$  lies between zero and  $1E_{11}$  (ride paragraph 16), we shall find that  $E_{c}$  will lie between  $-1E_1$  and  $+01E_1$ ; that is,  $E_{c}$  will have practically the same range as  $E_{1.085}$ , but the opposite sign. If, then, we use c as the common ratio, the approximation to the reserves will be sufficiently close, and the error will be almost certainly on the safe side.
- **21.** Having discussed the new method, and compared it with the existing methods, we may proceed to consider the best way of applying it in practice. Dealing first with the process of determining the mean age, it will be evident that the method referred to in paragraph 8, although the most obvious one, is not very convenient, since it renders it necessary to form the product  $S_x r^x$  afresh at each valuation ( $S_x$  denoting the total sum assured on lives aged x in a specified group). Now instead of  $S_x r^x$  we can use  $S_x r^{x+n}$ , where n is any quantity independent of x. Put n equal to the integral number of years in the unexpired term, that is = [year of maturity] [year of valuation + 1]. Then x+n=[year of maturity] [year of valuation <math>x] 1

= [year of maturity] - [office year of birth] -1

=M-1, say, where M denotes the difference between the year of maturity and the office year of birth, and may be called the "maturity-age." Thus we shall have  $S_M r^{M-1}$ , or multiplying by a constant, say  $r^{-54}$ , to reduce the dimensions of the function,  $S_M r^{M-55} = Z_M$  say, the subscript M being written instead of x because the value of Z corresponding to any given policy is now a function of M only. Then  $Z_{M}$  which answers all the purposes of  $S_x r^x$ , is independent of the year of valuation, and its value may therefore be formed, once and for all, for each policy at the time of issue, and be treated as a valuation constant, in the

same way as a net premium. The work will be much facilitated by calculating an auxiliary Table in the following form\*:—

### TABLE V.

Values of  $S_M r^{M-55} = Z_M$  [r assumed = 1·09561 =  $\log^{-1}$ ·039657, being the value of c employed in the Text-Book graduation of the  $H^M$  Experience].

| Matu-<br>rity-age |     |     |     |     | Sum 2 | Assurei | D     |       |       |       | Δ        |
|-------------------|-----|-----|-----|-----|-------|---------|-------|-------|-------|-------|----------|
| М                 | 100 | 200 | 300 | 400 | 500   | 600     | 700   | 800   | 900   | 1,000 |          |
| :                 | :   | :   | :   | :   | :     | :       | :     | ;     |       | :     | :        |
| :<br>59           | 144 | 288 | 432 | 576 | 720   | 865     | 1.009 | 1,153 | 1.297 | 1,441 | :<br>138 |
| 60                | 158 | 316 | 474 | 631 | 789   | 947     | 1,105 | 1,263 | 1,421 | 1,579 | 151      |
| 61                | 173 | 316 | 519 | 692 | 865   |         |       | 1,384 |       |       | 165      |
| 62                | 190 | 379 | 569 | 758 | 948   | 1,137   | 1,327 | 1,516 | 1,706 | 1,895 | 181      |
| :                 | :   | :   | :   | :   | . :   | 1       | :     | :     |       |       |          |
| :                 |     | :   | :   | :   | :     | :       | :     | 1     | 1 :   | :     | :        |

22. By the aid of such a Table, the values of Z may be taken

out and recorded by a clerk without any actuarial knowledge. If the very convenient system of recording the valuation data on cards be adopted, a space should be provided in which to insert the value of Z. In any case these values will be recorded in the valuation schedules and totalled; then for each group  $\frac{\Sigma Z}{\Sigma S}$  may be found with sufficient accuracy by Crelle's Tables, and thus by reference to our Table we may find by interpolation the mean maturity-age, M, correct to 1 decimal place, and hence the mean valuation age which is equal to M — [integral unexpired term + 1]. The whole process is exemplified in paragraph 29.

23. Endowment Assurances are now very commonly made payable on the policy anniversary next preceding the actual day on which the life assured attains the specified age, i.e., in the case of policies renewable annually, one year after payment of the last annual premium. If, therefore, the policies be grouped according to the calendar year of maturity, the number of premiums remaining to be paid in annual cases will be

[year of maturity] - [year of valuation +1] = n, say,

<sup>\*</sup> An extended table of Z has been prepared and is appended to the Paper. It should be stated that the particular form here given to the function Z has been adopted (in accordance with a suggestion made to the Author) as an improvement on the form originally proposed in the Paper. It is assumed that the assurances are payable, and that the groups are arranged, as explained in paragraph 23: if this be not the ease, the definition of M may have to be slightly modified, according to circumstances.

assuming that the valuation is made as at 31 December. The annuity to be used in the valuation of the net premiums will be

$$_{k} \mathbf{a}_{x\bar{n}} = (1-k) + a_{x:\bar{n-1+k}},$$

k being the average interval to the next renewal date, and the assurance-value will be

$$\mathbf{A}_{x:\overline{n+k}} = \mathbf{1} - d[1 + a_{x:\overline{n-1+k}}].$$

Therefore, by deducting 1-k from the annuity-value used for valuing the net premiums, we obtain the annuity-value by which we must enter the Premium Conversion Tables to obtain the assurance-value. If k be taken  $=\frac{1}{2}$  we shall have

- (a) Annuity-Value  $\frac{1}{2} + a_x \overline{n-\frac{1}{2}}$ (b) Assurance-Value  $1 d \left[ 1 + a_x; \overline{n-\frac{1}{2}} \right]$ found by entering the Tables with the Annuity-Value (a) less  $\frac{1}{2}$ .
- **24.** In practice, the true value of k will generally be nearer  $\frac{2}{3}$  than  $\frac{1}{2}$ , because of the large proportion of new business effected in the closing months of the year. It must, however, be observed that the change in the reserve consequent on a deviation from the true value of k will be much smaller for Endowment Assurances than Whole-Life Assurances. In the case of the latter, an increase of  $\epsilon$  in the value of k will always increase the reserve by  $\epsilon \Sigma P$ , whereas the effect on Endowment Assurances will be to increase the reserves for policies recently effected, but to decrease the reserves for policies which are nearing maturity. It may be worth while to investigate the question a little more closely.

$$\begin{aligned} \text{Let} & \quad V_0 = \Lambda_{x\bar{n}} - \mathbf{a}_{x\bar{n}} P_{y\bar{t}|} = 1 - \left[ P_{y\bar{t}|} + d \right] \mathbf{a}_{x\bar{n}} \\ & \quad V_1 = \Lambda_{x\bar{n}+\bar{1}'} - a_{x\bar{n}} P_{y\bar{t}|} = 1 - \left[ P_{y\bar{t}|} + d \right] \mathbf{a}_{x\bar{n}+\bar{1}|} + P_{y\bar{t}|} \\ & \quad \Delta V_0 = P_{y\bar{t}|} - \left[ P_{y\bar{t}} + d \right]_n \mathbf{E}_x \\ & \quad = \left[ P_{y\bar{t}|} + d \right] \left[ \frac{P_{y\bar{t}}}{P_{y\bar{t}|} + d} -_n \mathbf{E}_x \right] \\ & \quad = \frac{1}{\mathbf{a}_{x\bar{t}}} \left[ \Lambda_{y\bar{t}} -_n \mathbf{E}_x \right] \end{aligned}$$

**25**. The true reserve, when the average interval to the next renewal date is k, will be  $V_k = V_0 + k\Delta V_0$ , and if  $k' = k - \epsilon$  be used instead of k, the calculated reserve will be too small algebraically by  $\epsilon \Delta V_0^*$ .

<sup>\*</sup> Making t and n indefinitely great, we have the case of a whole life assurance, and  ${}_{n}\mathbf{E}_{x}$  vanishes, so that  $\epsilon\Delta\mathbf{V}_{0}=\epsilon\mathbf{P}$ .

The last expression will be +,0, or -, according as  $A_{y\bar{t}} > = <_n E_x$ . There will thus be a turning point, before which the reserves will be diminished, and after which they will be increased, by giving too small a value to k. The position of the turning point will of course vary with the age at entry and the term of the assurance, but for ages at entry 30-40, and endowment ages 50-60 (which will include the major part of the Endowment Assurance business of an ordinary office), the turning point may be taken to be about midway between the dates of entry and maturity. In an ordinary progressive business, the assurances will not on the average have reached that point, and consequently, the reserves will be somewhat too small if too low a value be given to k. It may be shown, however, that the error is much smaller than in the case of whole life assurances. Take for example the following case:

Age at entry 35, policy payable at 60, valuation age 44, number of premiums remaining 15. On the basis of H<sup>M</sup> 3 per-cent we have for a policy of 100

$$\begin{aligned} V_0 &= 67 \cdot 370 - (11 \cdot 203 \times 3 \cdot 366) = 29 \cdot 661 \\ V_1 &= 65 \cdot 936 - (10 \cdot 695 \times 3 \cdot 366) = \underline{29 \cdot 936} \\ \Delta &= + \cdot 275 \end{aligned}$$

The defect in the reserve will therefore be  $275 \times \epsilon$ , whereas for a whole life assurance on a life of the same age, it would be  $2\cdot193 \times \epsilon$ . If the valuation age be 39, and number of premiums remaining be 20, the figures will be:

$$\begin{aligned} V_0 &= 59.841 - (13.788 \times 3.366) = 13.431 \\ V_1 &= 58.667 - (13.191 \times 3.366) = \underbrace{14.266}_{A = +.835} \end{aligned}$$

and the error will be  $\cdot 835\epsilon$  as compared with  $2\cdot 193\epsilon$  for the whole life policy.

26. In half-yearly and quarterly cases the number of years' premiums remaining may be n,  $n + \frac{1}{4}$ ,  $n + \frac{1}{2}$ ,  $n + \frac{2}{4}$ , or  $n + \frac{3}{4}$ . Formulæ similar to those in paragraph 23, will apply if the true half-yearly or quarterly net premiums be used, but the labour of valuation is much increased by the necessity of allowing for the fractional number of payments. It has been shown that practically identical results can be obtained, by valuing the yearly net premiums by the annuity for n payments, and increasing the result by the actual amount of the unpaid portion of the full net premium for the current policy year [vide Mr. J. J. McLauchlan's paper "On some Formulas for use in Life Office Valuations", Transactions of the Actuarial Society of Edinburgh,

Vol. II, page 375; also J.I.A. xxiii, 256, and the Text-Book, Part II, page 347, paragraph 91]. This process is almost absolutely correct if the mly premiums are instalment premiums, and it is shown by Mr. McLauchlan that, in the case of true mly premiums, the error will be quite unimportant for any ordinary amount of half-yearly and quarterly business.

27. As the plan referred to in the preceding paragraph saves an appreciable amount of labour, and is sufficiently accurate, it appears to be the best to adopt. The amount of the unpaid portion of premiums (i.e.  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{2}{4}$  or  $\frac{3}{4}$  of the annual net premium) for any policy will be the same at each valuation, and it should therefore be recorded on the valuation cards or elsewhere. These amounts will then be entered in the valuation schedule, in a column provided for the purpose, and the total amount of the correction will thus be obtained. The correction need not be made for each group of policies separately, but can be applied to the grand total of the class. The values of the office premiums do not in the case of a net premium valuation enter into the final result, and it will be sufficient in their case to find the correction approximately, by increasing the net premium correction in the

ratio  $\stackrel{\Sigma}{\simeq}$  Office Premiums  $\stackrel{\Sigma}{\simeq}$  Net Premiums . [Vide paragraph 29.]

**28.** We may now briefly consider the case of assurances payable on the actual attainment of a specified age, not on the preceding policy anniversary, as assumed in paragraph 23. If it be desired to give effect to this difference, the same method of grouping may be adhered to, but the assurance-value  $A_{x:\overline{n+k+l}}$  will have to be used instead of  $A_{x:\overline{n+k}}$ , t being the average interval between the birthday and the preceding policy anniversary. It should, however, be noted that, in accordance with the principles of a net premium valuation, a corresponding alteration must be made in the premium valued, which should be of the form

$$_{m}P_{y:\overline{m+t}} = \frac{A_{y:\overline{m+t}|}}{\mathbf{a}_{y\overline{m}|}} \left( \text{or more strictly } _{m}P_{y-t:\overline{m+t}} = \frac{A_{y-t:\overline{m+t}|}}{\mathbf{a}_{y-t:\overline{m}|}} \right)$$

instead of  $P_{y\overline{m}}$ , otherwise each policy will have a negative value at the moment of issue. On the whole it would appear to be more satisfactory to value the policies on the assumption that amounts assured are payable as mentioned in paragraph 23, using the ordinary net premiums; by this means we have a zero value at the moment of issue, and the profit due to the benefit being slightly smaller than that provided by the net premium is disclosed at the *end* of the contract instead of the beginning.

M 65° 55 60 45

29. In order to illustrate the processes discussed in paragraphs 21-25, the following schedule has been prepared. Only such columns as are actually necessary for the present purpose have been included, but it must be understood that in practice further columns would usually be required.

### SPECIMEN VALUATION SCHEDULE.

Year of Maturity 1913. Integral Unexpired Term, 15 years.

| Policy<br>Number                     | Sum<br>Assured                | Z                              | Bonus<br>Additions         | Office<br>Premium                   | Net<br>Premium<br>H <sup>u</sup> 3 °, o | Unpaid Portion of Net Premiums for current Policy Year |
|--------------------------------------|-------------------------------|--------------------------------|----------------------------|-------------------------------------|---|--|
| (1)                                  | (2)                           | (3)                            | (4)                        | (5)                                 | (6)                                     | (7)  |
| 15,015<br>15,167<br>17,854<br>19,016 | 500<br>800<br>600<br>100<br>: | 1,246<br>800<br>947<br>40<br>: | 10·65<br><br>12·00<br>1·50 | 21·458<br>31·433<br>25·600<br>2·867 | 17/735<br>24/952<br>21/210<br>2/296     | 12·5<br>5·3<br>  |
| Totals .                             | 10,500                        | 18,540                         | 544.80                     | 418:407                             | 344.470                                 | 68.9   |

#### SPECIMEN OF THE CALCULATIONS.

- Mean value of Z per unit = 18,540 ÷ 10,500 = 1.766. Hence we find, by reference to Table V, mean value of M = 61.2, and hence mean valuation age = 45.2.
- 33. By interpolation from a prepared table of  $\bar{a}_{x\overline{n}}$  we find  $\bar{a}_{45^{\circ}\overline{15}}$  H  $^{\rm M}$  3 per-cent = 10·874. Entering the Premium Conversion Tables with 10·374 we find A = 66871. Hence by multiplication

Value of Sums Assured = 7,021

" Bonus Additions = 364

" Office Premiums = 4,550

" Net " = 3,746

,, 24-5. The correction for the value of the net premiums =  $+ \Sigma$  col. (7) = + 68.9.

Correction for value of office premiums is approximately  $+68.9 \times \frac{2 \text{ col.}(5)}{2 \text{ col.}(6)} = +68.9 \times 418.407 : 344.470 = +83.7$ 

In practice, in a group of this size, the last figure of the Z's need not be recorded in the Valuation Schedule, but care must be taken to make the necessary adjustment when calculating  $\Sigma$   $Z \div \Sigma$  S.

30. Through the courtesy of Mr. L. K. Pagden, Actuary of the Union Assurance Society, the writer has had the opportunity of testing the new method of valuation by applying it to a case in which the usual method B was found to entirely break down, owing to the presence of one large policy maturing at an unusually advanced age (77). The policies in question had an unexpired term of 12 years, and assured a total amount of Mark 462,400, subject to annual net premiums of Mark 23,649. The disturbing policy assured the sum of Mark 100,000 (more than 20 per-cent of the whole) on a life of an age more than 20 years greater than the mean age of the lives assured by the other policies. The following Table shows the results of the several methods of valuation that were employed:—

TABLE VI.

| Metho                                       | d of     | Valu       | ıation        |              |         | Value of<br>Net Premiums | Liability | Error   |
|---|----------|------------|---------------|--------------|---------|--------------------------|-----------|---------|
| Method                                      | A        |            |               |              | 339,430 | 210,777                  | 128,653   |         |
| Method                                      | В        |            | •             |              | 335,878 | 222,158                  | 113,720   | -14,933 |
| Method                                      | C        |            |               |              | 335,878 | 218,823                  | 117,055   | -11,598 |
| Method                                      | D        | •          |               |              | 339,628 | 215,583                  | 124,045   | - 4,608 |
| Method<br>(The same<br>separate<br>used for | as<br>me | D e<br>ean | xcept<br>ages | that<br>were | 339,628 | 210,026                  | 129,602   | + 949   |

31. Here it is seen that method B gives a reserve too small by Mark 14,933, an error which method D at once reduces by more than two-thirds, to Mark 4,608. A truer comparison between the two methods of determining the mean ages will, however, be obtained by considering the results when separate mean ages are used for the sum assured and the net premiums; because, in the case under consideration, the rate of premium on the one special policy was very nearly twice as high as the average rate for the remainder of the group, and, under such conditions, it is inconsistent with the principles of the two methods to use the same mean age for sums assured and premiums. Comparing, then, the results of methods C and E, we see that

the former gives a negative error of 11,598, and the latter a positive error of 949; that is, the *magnitude* of the deviation is reduced by over 90 per-cent, and the *sign* is changed so that the error is on the safe side.

- 32. The new plan is thus shown to possess very considerable elasticity under abnormal conditions. At the same time, the results of the preceding paragraphs show that it will be desirable either to value separately any policies of exceptionally large amount that mature at advanced ages and are subject to an abnormally high rate of premium; or to calculate a separate mean age for the premiums, as in method E, for the groups in which such policies appear.
- 33. When the same age is adopted for both sums assured and premiums, the assumption is tacitly made that the rate of premium is approximately the same whatever the valuation age. If one or two exceptional policies in a group necessitate the calculation of a special mean age, as previously mentioned, the same assumption may nevertheless be made in respect of the remaining policies. The mean valuation age for the premiums may then be found very simply as follows:—Let  $\Sigma S$  and  $\Sigma Z$  be found excluding the special policies, and let the average rate of net premium on the policies included in these totals be P; let the values of S and Z and P for the special policies be respectively S', S''...; Z', Z''...: and  $P(1+\gamma')$ ,  $P(1+\gamma'')$ ...: then the mean valuation age for the premiums will be found from the ratio

 $\frac{\Sigma Z + 1 + \gamma' Z' + \overline{1 + \gamma'' Z'' + \dots}}{\Sigma S + 1 + \gamma' S + 1 + \gamma'' S'' + \dots}$ 

that is, we must increase the values of S and Z for the special policies in the same proportion as the rates of net premiums.

**34.** We may now proceed to show that the use of Makeham's constant c for the common ratio (ride paragraphs 18-20) leads to a very convenient method of calculating expected claims without having to deal with individual ages. This method enables us to avoid one of the disadvantages of a group valuation, namely, that lives of the same age appear in different groups, so that the schedules are not suited for the calculation of expected claims.

Let it be required to find the expected claims for the year Y in respect of a group of policies maturing in the year Y+n. First consider policies whose maturity-age is M; then the age of the lives assured at the beginning of the year will be M-n+1=x say (see paragraphs 21-2.) Let S denote the total sum

assured under such policies at the beginning of the year, S' the similar total at the end of the year, and Z, Z' the corresponding totals of the function Z. Then the expected claims for the year will be

$$\begin{split} & \frac{\mathbf{S} + \mathbf{S}'}{2} \, \mu_{x + \frac{1}{2}} = \frac{\mathbf{S} + \mathbf{S}'}{2} \left[ \mathbf{A} + \mathbf{B} c^{x + \frac{1}{2}} \right] \\ &= \frac{\Lambda}{2} \left[ \mathbf{S} + \mathbf{S}' \right] + \frac{\mathbf{B}}{2} \, c^{x + \frac{1}{2}} [\mathbf{S} + \mathbf{S}'] \\ &= \frac{\mathbf{A}}{2} \left[ \mathbf{S} + \mathbf{S}' \right] + \frac{\mathbf{B}}{2} \, c^{x + \frac{1}{2} - \overline{\mathbf{M}} - 55} [\mathbf{Z} + \mathbf{Z}'], \text{ since } \mathbf{Z}_{\mathbf{M}} = \mathbf{S}_{\mathbf{M}} \cdot c^{\mathbf{M} - 55}. \end{split}$$

Now 
$$x + \frac{1}{2} - \overline{M - 55} = \overline{M - n + 1} + \frac{1}{2} - \overline{M - 55} = 54\frac{1}{2} - n$$
. Thus we have  $\frac{A}{2} [S + S'] + \frac{B}{2} e^{54\frac{1}{2}} [\overline{Z + Z'} | c^{-n}]$ .

This expression is independent of M, and the total expected claims for the whole group will be

$$\frac{A}{2}[{}_{n}S + {}_{n}S'] + \frac{B}{2}c^{54\frac{1}{2}}[{}_{n}\overline{Z + {}_{n}Z'}|c^{-n}],$$

 $_n$ S representing  $\Sigma$ S for the group and so on. Thus for the whole business the expected claims will be

$$\frac{A}{2} \Sigma \left[ {}_{n} \mathbf{S} + {}_{n} \mathbf{S}' \right] + \frac{B}{2} c^{54\frac{1}{2}} \Sigma \left[ {}_{n} \overline{\mathbf{Z} + {}_{n} \mathbf{Z}'} | c^{-n} \right].$$

In the case of the Text-Book Graduation of the  $H^M$  experience the values of the co-efficients will be

$$\frac{A}{2} = .003096; \quad \frac{B}{2} c^{54\frac{1}{2}} = .006962.$$

The same results may, of course, be obtained by multiplying  $\frac{1}{2}\Sigma[{}_{n}S + {}_{n}S']$  by the value of  $\mu$  for the mean valuation-age x, found (by reference to Table A) from the equation

$$c^{x-54} = \frac{\sum \left[ n\overline{Z} + n\overline{Z'} \right] c^{-n}}{\sum \left[ nS + nS' \right]}$$

35. The process described above does not strictly apply to the Bonus additions, as the mean age in their case will generally be slightly higher than that relating to the sums assured. Unless, however, the bonus additions be unusually large, it will be quite sufficient to use the value of  $\mu$  for an age a few years greater than the mean age deduced from the sums assured. For example, if the true mean age for the bonuses be 50, an error of a year will only change the expected claims by about one per mille of the bonus additions, so that even if the estimated mean age be several years out, the error in the expected bonus claims will be quite unimportant.

36. The method of valuation which the writer now submits to the profession is one that undoubtedly possesses in a high degree the desiderata of accuracy and facility of application, and it will be seen, from paragraphs 30–31, that the method very successfully adapts itself to abnormal conditions. It shares with the less exact methods that have been previously proposed, the great practical advantage of saving a large amount of skilled labour, which would have to be performed by persons of experience, at a time of great pressure, substituting work that can be entrusted to any intelligent clerk, and can to a large extent be done once and for all at the time of the issue of a policy. It is, moreover, from the manner in which it has been deduced, an approximate method rather than an empirical one, and as such is entitled to the greater degree of confidence.

### EXPLANATION OF THE TABLES.

Table A gives the values of  $Z_M$  for all maturity-ages from 35 to 75, and for sums assured 100 to 1,000, progressing by 100. The table of proportional parts, calculated with reference to the values relating to sum assured 1,000, shows the amount to be added to the tabular value of Z in respect of each tenth of a year added to M, and by using the table inversely the value of M (corresponding to any given value of  $\frac{\Sigma Z}{\Sigma S}$ ) may be found by inspection correct to 1 decimal place.

The Table has been constructed on the basis of a common ratio equal to 1.09561..., i.e., the value of c used in the Text-Book graduation of the  $H^{M}$  experience.

Table B gives the values of temporary continuous annuities, calculated on the basis of the H<sup>M</sup> Table, at 3 per-cent interest, by the approximate formula  $\overline{a}_{x\overline{n}} = \frac{1}{2} [\mathbf{a}_{x} + a_{x\overline{n}}]$ . The values of n are given at the side, and the values of M (which is equal to x+n+1—see paragraphs 21–2 at the head of the columns.

The column headed "Diff." gives the difference between two successive values in the same row. corresponding to an increase in M, n remaining constant.

The tabulated values were found by interpolation from the values of  $a_{x\bar{n}}$  given in Hardy's Tables, and consequently, in some cases, there may be errors of 1 in the 3rd decimal place.

TABLE A.

| atu-<br>ity<br>ige |     |       | 'alue of Z | M=Sum A<br>un | Assured×1<br>dermentio | [1:09561] <sup>5</sup><br>oned sum: | -м, corre<br>s assured | esponding | to the |
|--------------------|-----|-------|------------|---------------|------------------------|-------------------------------------|------------------------|-----------|--------|
| M                  | 100 | 200   | 300        | 400           | 500                    | 600                                 | 700                    | 800       | 900    |
| 3.5                | 16  | 32    | 48         | 64            | 81                     | 97                                  | 113                    | 129       | 145    |
| 36                 | 18  | 3.5   | 5,3        | 7 1           | 88                     | 106                                 | 123                    | 141       | 159    |
| 37                 | 19  | 39    | 58         | 77            | 97                     | 116                                 | 135                    | 155       | 174    |
| 38                 | 2 I | 42    | 64         | 85            | 106                    | 127                                 | 148                    | 169       | 191    |
| 39                 | 23  | 46    | 70         | 93            | 116                    | 139                                 | 162                    | 186       | 209    |
| ło                 | 2.5 | 51    | 76         | 102           | 127                    | 153                                 | 178                    | 203       | 229    |
| Į I                | 28  | 56    | 84         | 111           | 139                    | 167                                 | 195                    | 223       | 251    |
| <del>1</del> 2     | 31  | 61    | 92         | I 2 2         | 153                    | 183                                 | 214                    | 244       | 275    |
| 13                 | 33  | 67    | 100        | 134           | 167                    | 201                                 | 234                    | 267       | 301    |
| 1-1                | 37  | 7.3   | 110        | 146           | 183                    | 220                                 | 256                    | 293       | 330    |
| 45                 | 40  | 80    | 1 20       | 161           | 201                    | 24 I                                | 281                    | 321       | 361    |
| 46                 | 44  | 88    | 132        | 176           | 220                    | 264                                 | 308                    | 352       | 396    |
| 47                 | 48  | 96    | 145        | 193           | 241                    | 289                                 | 337                    | 385       | 434    |
| 48                 | 53  | 106   | 158        | 211           | 264                    | 317                                 | 369                    | 422       | 475    |
| 19                 | 58  | 116   | 173        | 231           | 289                    | 347                                 | 405                    | 463       | 520    |
| 50                 | 63  | 127   | 190        | 253           | 317                    | 380                                 | 443                    | 507       | 570    |
| 51                 | 69  | 139   | 208        | 278           | 347                    | 416                                 | 486                    | 555       | 625    |
| ; 2                | 76  | 152   | 228        | 304           | 380                    | 456                                 | 532                    | 608       | 684    |
| 53                 | 83  | 167   | 250        | 333           | 417                    | 500                                 | 583                    | 666       | 750    |
| 54                 | 91  | 183   | 274        | 365           | 456                    | 548                                 | 639                    | 730       | 821    |
| 55                 | 100 | 200   | 300        | 400           | 500                    | 600                                 | 700                    | 800       | 900    |
| 56                 | 011 | 219   | 329        | 438           | 548                    | 657                                 | 767                    | 876       | 986    |
| 57                 | 120 | 240   | 360        | 480           | 600                    | 720                                 | 840                    | 960       | 1,080  |
| 58                 | 132 | 263   | 395        | 526           | 658                    | 789                                 | 921                    | 1,052     | 1,184  |
| 9                  | 144 | 288   | 432        | 576           | 720                    | 865                                 | 1,009                  | 1,153     | 1,297  |
| 0                  | 158 | 316   | 474        | 631           | 789                    | 947                                 | 1,105                  | 1,263     | 1,421  |
| ĺΙ                 | 173 | 346   | 519        | 692           | 865                    | 1,038                               | 1,211                  | 1,384     | 1,557  |
| 52                 | 190 | 379   | 569        | 758           | 948                    | 1,137                               | 1,327                  | 1,516     | 1,706  |
| 93                 | 208 | 415   | 623        | 830           | 1,038                  | 1,246                               | 1,453                  | 1,661     | 1,868  |
| 4                  | 227 | 455   | 682        | 910           | 1,137                  | 1,365                               | 1,592                  | 1,820     | 2,047  |
| 55                 | 249 | 498   | 748        | 997           | 1,246                  | 1,495                               | 1,744                  | 1,994     | 2,243  |
| 56                 | 273 | 546   | 819        | 1,092         | 1,365                  | 1,638                               | 1,911                  | 2,184     | 2,457  |
| 57                 | 299 | 598   | 897        | 1,197         | 1,496                  | 1,795                               | 2,094                  | 2,393     | 2,692  |
| 68                 | 328 | 656   | 983        | 1,311         | 1,639                  | 1,967                               | 2,294                  | 2,622     | 2,950  |
| 9                  | 359 | 718   | 1,077      | 1,436         | 1,795                  | 2,155                               | 2,514                  | 2,873     | 3,232  |
| 0                  | 393 | 787   | 1,180      | 1,574         | 1,967                  | 2,360                               | 2.754                  | 3,147     | 3,541  |
| I                  | 431 | 862   | 1,293      | 1,724         | 2,155                  | 2,586                               | 3,017                  | 3,448     | 3,879  |
| 7 2                | 472 | 945   | 1,417      | 1,889         | 2,361                  | 2,834                               | 3,306                  | 3,778     | 4,250  |
| 7.3                | 517 | 1,035 | 1.552      | 2,070         | 2,587                  | 3,104                               | 3,622                  | 4,139     | 4,657  |
| 4                  | 567 | 1,134 | 1,701      | 2,267         | 2,834                  | 3,401                               | 3,968                  | 4,535     | 5,102  |
| 5                  | 621 | 1,242 | 1,863      | 2,484         | 3,105                  | 3,726                               | 4,347                  | 4,969     | 5,590  |
| M                  | 100 | 200   | 300        | 400           | 500                    | 600                                 | 700                    | 800       | 900    |

Table A—continued.

|                |          |          |          | Pro        | portiona | l Parts  |          |          |            | Ma<br>ri<br>ag |
|----------------|----------|----------|----------|------------|----------|----------|----------|----------|------------|----------------|
| 1,000          | 1        | 2        | 3        | 4          | 5        | 6        | 7        | 8        | 9          | 3              |
| 161            | 2        | 3        | 4        | 6          | 8        | 9        | 11       | 1 2      | 13         | 3              |
| 176            | 2        | 3        | 5        | 7          | 9        | 10       | 12       | 14       | 15         | 3              |
| 193            | 2        | 4        |          | 8          | 10       | ΙΙ       | 13       | 15       | 17         | 3              |
| 212            | 2 2      | 4        | 6 7      | 8          | 10       | 12       | 14       | 16<br>18 | 18         | 3              |
| 232            |          | 7        | 1        | •          |          |          |          |          |            | ľ              |
| 254            | 2        | 5        | 78       | I 0<br>I 1 | 12       | 14       | 17       | 19       | 2.2        | 1 1            |
| 278            | 3        | 5        |          | 1 2        | 14       | 17       | 19       | 1        | 24<br>26   | 1 4            |
| 305<br>334     | 3        | 6        | 9        | 13         | 16       | 19       | 22       | 23       | 29         | 4              |
| 366            | 4        | 7        | 10       | 14         | 17       | 21       | 24       | 28       | 31         | 4              |
| 401            | 4        | 8        | 12       | 16         | 20       | 23       | 27       | 31       | 35         | 4              |
| 440            | 4        | 8        | 13       | 17         | 2 I      | 25       | 29       | 34       | 38         |                |
| 482            | 5        | 9        | 14       | 18         | 23       | 28       | 32       | 37       | 41         | 1 4            |
| 528            | 5        | 10       | 15       | 20         | 25       | 30       | 35       | 10       | 4.5        | 4              |
| 578            | 6        | 11       | 17       | 22         | 28       | 33       | 39       | 44       | 50         | 1 4            |
| 633            | 6        | 12       | 18       | 24         | 30       | 37       | 43       | 49       | 5.5        | ] :            |
| 694            | 7        | 13       | 20       | 26         | 33       | 40       | 46       | 53       | 59<br>66   | 1 5            |
| 760            | 7<br>8   | 15       | 22       | 29         | 36       | 44       | 51<br>56 | 58       |            |                |
| 833<br>913     | 9        | 17       | 24<br>26 | 32<br>35   | 40<br>44 | 18<br>52 | 61       | 64<br>70 | 72<br>78   |                |
| 1,000          | 10       | 19       | 29       | 38         | 48       | 58       | 67       | 77       | 86         |                |
| 1,096          | 10       | 21       | 31       | 42         | 52       | 62       | 73       | 83       | 94         | 3              |
| 1,200          | 12       | 23       | 34       | 46         | 58       | 69       | 81       | 92       | 104        |                |
| 1,315          | 13       | 25       | 38       | 50         | 63       | 76       | 88       | 101      | 113        |                |
| 1,441          | 14       | 28       | 41       | 5.5        | 69       | 83       | 97       | 110      | 124        |                |
| 1,579          | 15       | 30       | 45       | 60         | 76       | 91       | 106      | 121      | 136        |                |
| 1,730          | 16       | 33       | 50       | 66         | 82       | 99       | 116      | 132      | 148        |                |
| 1,895          | 18       | 36       | 54       | 7.2        | 90       | 109      | 127      | 145      | 163        |                |
| 2,076          | 20       | 40       | 60<br>65 | 80<br>87   | 100      | 119      | 139      | 159      | 179        |                |
|                |          |          |          |            |          |          | 1        |          | 1          | 1              |
| 2,492<br>2,730 | 24<br>26 | 48<br>52 | 71<br>78 | 95         | 119      | 143      | 167      | 200      | 214        |                |
| 2,730          | 29       | 57       | 86       | 115        | 144      | 157      | 201      | 230      | 235<br>258 |                |
| 3,278          | 31       | 63       | 94       | 125        | 156      | 188      | 210      | 250      | 282        |                |
| 3,591          | 34       | 69       | 103      | 137        | 172      | 206      | 240      | 274      | 309        | 1              |
| 3,934          | 38       | 7.5      | 113      | 150        | 188      | 226      | 263      | 301      | 338        | 1              |
| 4,310          | 41       | 83       | 124      | 165        | 206      | 248      | 289      | 330      | 372        | 1              |
| 4,723          | 45       | 90       | 135      | 180        | 226      | 271      | 316      | 361      | 106        |                |
| 5,174          | 50       | 99       | 148      | 198        | 248      | 297      | 347      | 396      | 445        | 1              |
| 5,669          | 54       | 108      | 163      | 217        | 27 I     | 325      | 379      | 434      | 488        | 1              |
| 6,211          | 59       | 119      | 178      | 238        | 297      | 356      | 416      | 475      | 535        |                |
| 1,000          | 1        | 2        | 3        | 4          | 5        | 6        | 7        | 8        | 9          |                |

HM 3 0/0

### APRIL

TABLE B.

 $\bar{\mathcal{U}}\mathbf{M} = n+1:n$ 

HM 3 0/0 Continuous Temporary Annuities.

MATURITY-AGE, i.e., M. n nDiff. Diff Diff. Diff. Diff. Diff. 48 49 Diff. 45 46 47 50 51 **1**080 **1980** \*a80 ī '979 979 ī '978 078 1 . . . 1.017 2 1.020 2 1'018 T 1,016 2 1'914 1 2 2 1'922 1'920 I 2.823 2.818 2.815 2.826 2.825 2 2 2.821 2.812 3 ī 3 3 36 3 3 3.688 3.684 3.606 3.604 3.692 3.679 3.674 4 4 2 2 4 4 5 5 4.522 4'518 5 4'533 3 4.530 3 4'527 5 4'511 q 4'502 7 5 8 6 6 5°326 5'318 8 5.310 9 5°301 11 5'338 4 5'334 4 5.330 4 78 6.112 6 6.100 6.101 6 6.008 8 6.000 10 6.080 11 13 7 5 8 6.848 8 6.840 8 6.832 6.822 6·8 ró ġ 6.864 6.856 8 10 12 16 7.586 7.536 8.222 7.522 7.567 7.557 8.248 1.1 7.546 16 q g 10 7.576 9 10 10 14 8.236 8.284 Τī 8.273 8.260 12 I 2 14 8.208 17 10 10 13 14 8.002 8.886 8.860 18 8.958 8.944 8.031 8.016 16 11 11 14 13 1.5 1.1 17 16 16 9.562 т8 18 9.526 18 9.508 12 1.2 0.608 14 9.594 9.578 9'544 2 I 16 10'124 10'236 16 10.304 18 10.186 20 10,166 20 10'146 22 22 13 13 10'220 10.842 16 10.826 18 10.808 20 10.788 20 10.768 10.745 10.721 14 23 24 25 14 28 15 11'430 20 11'410 11'391 2 I 11.370 2 I 11'340 11'324 26 11.508 15 10 25 11.956 11.858 16 11,010 11.885 16 12,000 22 11'978 22 22 11'934 24 25 27 30 12.203 24 12.478 12'454 12.426 28 12.398 32 17 12'552 12.528 25 25 24 28 17 12.979 12.951 12.020 ı8 13.087 26 13.061 13'034 28 13.006 27 28 32 18 27 31 13 426 13.604 26 13.578 13.549 29 13.520 3 I 13'489 30 13'459 33 33 10 19 29 14,105 14'077 14,048 14'017 13.984 13'951 13'917 20 20 25 20 31 33 33 34 35 38 36 14.281 14.200 14,466 14'393 2 T 2 I 23 14'558 14'531 31 14'430 37 27 34 38 14.894 22 15'042 20 15'022 14'997 14.066 14'032 14'855 40 22 25 31 34 39 15'466 38 15.384 15.346 23 23 15'487 2 I 22 15'444 28 12,410 32 41 15'305 43 15.896 15.873 15.818 28 15.848 15'783 24 24 15'924 23 25 30 35 41 15'742 44 16.318 16.300 16.263 16.236 16.507 19.197 25 2.5 16.354 36 28 27 27 32 40 44 16.637 36 16.572 42 26 16.784 50 16.734 36 16.608 16.666 29 20 16.608 26 32 64 17.152 17.100 17.060 17.026 16.992 16.062 38 27 27 17'216 52 40 34 31 33 17.644 17.450 28 66 17.206 56 17.408 36 17'372 17.338 36 28 72 17'572 42 34 58 17.786 18.007 76 70 29 17.988 17.844 17'702 37 20 74 17'914 45 17.741 39 78 18.241 48 18.060 18.319 18.160 18,108 30 18:398 72 61 42 30 79 18:416 82 18.636 82 18.480 64 31 31 18.718 18.554 74 52 . . . ... . . . 18.855 18.777 32 10.036 86 18,040 85 78 67 32 . . . . . . . . . 88 82 19.320 90 10'230 19'142 33 3.3 . . . . . . ... . . . . . . 10.600 92 19.208 Q I 34 34 ... . . . ... 19.870 96 35 . **. .** 35 . . . · · · 36 36 ... . . . 37 37 ... . . . . . . . . . . . . . . . . . . ... . . . 38 38 . . . . . . ... ... ... . . . ... . . . . . . . . . . . . . . . ... 39 39 . . . . **. .** . . . . . . ... ... . . . . . . ... . . . ... . . . 40 40 . . . . . . . . . . . . ٠., . . . ... . . . . . . . . . 41 4 I ... ... . . . . . . . . . . . . ... ... . . . . . . 42 42 ... ... . . . ... . . . . . . . . . . . . ... ... . . . . . . 43 ... 43 ... . . . ... . . . . . . ... . . . . . . ... ... . . . ... 44 44 ... . . . . . . . . . ... ... ... ... 45 45 ... ... ... Diff. Diff. 45 Diff. Diff. 47 Diff. 48 Diff. 49 Diff. 50 51 46

## Endowment Assurances in groups.

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HM 3 0/0

# TABLE B-(continued). Continuous Temporary Annuities.

**H**<sup>M</sup> 3 ° 0

|                            |  |                            |  |  |  |                            | $a_{\mathbf{M}-\bar{n}}$                       | ÷1:n                                   |  |                            |  |                              |  |                             |                            |
|----------------------------|--|----------------------------|--|--|--|----------------------------|--|--|--|----------------------------|--|------------------------------|--|-----------------------------|----------------------------|
|                            |  |                            |  |  |  | MA                         | TURITY-A                                       | GE, i.e                                | е., М.   |                            |  |                              |  |                             |                            |
| n                          | 52   | Diff.                      | 53   | Diff.  | 54                                       | Diff.                      | 55   | Diff.                                  | 56   | Diff.                      | 57   | Diff.                        | 58   | Diff.                       | 73                         |
| 1<br>2<br>3<br>4<br>5      | *978<br>1*913<br>2*809<br>3*668<br>4*495       | 3 5                        | .978<br>1.911<br>2.806<br>3.663<br>4.486       | 5  | .977<br>1.910<br>2.802<br>3.658<br>4.478 | <b>3</b>                   | *976<br>1*908<br>2*799<br>3*652<br>4*470       | 3<br>6<br>10                           | .976<br>1.906<br>2.796<br>3.646<br>4.460       | 6 7                        | '975<br>1'904<br>2'790<br>3'639<br>4'452       | 1<br>2<br>4<br>9<br>12       | '974<br>1'902<br>2'786<br>3'630<br>4'440       | <br>6<br>8                  | 1<br>2<br>3<br>4<br>5      |
| 6<br>7<br>8<br>9           | 5.530<br>6.026<br>6.131<br>2.200<br>8.131      | 14<br>16<br>18             | 5'278<br>6'042<br>6'778<br>7'488<br>8'171      | 11<br>16<br>19<br>22<br>23                         | 5.148<br>6.056<br>2.469<br>2.469         | 12<br>15<br>19<br>23<br>26 | 5'255<br>6'011<br>6'740<br>7'443<br>8'122      | 12<br>16<br>20<br>24<br>28             | 5'243<br>5'995<br>6'720<br>7'419<br>8'094      | 13<br>17<br>21<br>25<br>30 | 5°230<br>5°978<br>6°699<br>7°394<br>8°064      | 14<br>17<br>21<br>26<br>30   | 5°216<br>5°961<br>6°678<br>7°368<br>8°934      | 16<br>20<br>24<br>28<br>32  | 6<br>7<br>8<br>9           |
| 11<br>12<br>13<br>14<br>15 | 8.851<br>9.487<br>10.105<br>10.696             | 21<br>22<br>24<br>27<br>28 | 8.830<br>9,462<br>10.669<br>11,545             | 24<br>25<br>27<br>28<br>32                         | 8.806<br>9.440<br>10.021<br>10.021       | 28<br>28<br>29<br>31<br>32 | 8.778<br>9.412<br>10.022<br>10.610<br>11.179   | 30<br>34<br>34<br>35<br>36             | 8.748<br>9.378<br>9.958<br>10.272<br>11.142    | 34<br>36<br>39<br>39       | 8*714<br>9*342<br>9*949<br>10*536<br>11*102    | 36<br>40<br>42<br>45<br>46   | 8.678<br>9.302<br>9.907<br>10.491<br>11.056    | 37<br>42<br>47<br>49<br>51  | 11<br>12<br>13<br>14<br>15 |
| 16<br>17<br>18<br>19<br>20 | 11.828<br>12.366<br>12.888<br>13.393<br>13.882 | 32<br>34<br>36<br>37<br>38 | 11.796<br>12.332<br>12.852<br>13.356           | 34<br>36<br>38<br>40<br>41                         | 13.803<br>13.814<br>13.316<br>11.465     | 35<br>38<br>40<br>42<br>44 | 11.727<br>12.258<br>12.774<br>13.274<br>13.759 | 38<br>49<br>44<br>46<br>47             | 13.218<br>13.258<br>13.558<br>13.218           | 41<br>43<br>44<br>48<br>51 | 11'648<br>12'175<br>12'686<br>13'180<br>13'661 | 46<br>46<br>49<br>50<br>53   | 11.602<br>12.637<br>13.130<br>13.608           | 5 5 5 5 5 5                 | 16<br>17<br>18<br>19<br>20 |
| 21<br>22<br>23<br>24<br>25 | 14'355<br>14'815<br>15'262<br>15'698<br>16'120 | 39<br>41<br>44<br>47<br>48 | 14'316<br>14'774<br>15'218<br>15'651<br>16'072 | 42<br>44<br>46<br>48<br>50                         | 14'274<br>15'603<br>15'172<br>15'603     | 45<br>46<br>46<br>50<br>52 | 14.684<br>12.139<br>14.531                     | 49<br>49<br>52<br>51<br>54             | 14.180<br>14.180<br>14.180                     | 52<br>54<br>53<br>56<br>56 | 14'128<br>14'581<br>15'021<br>15'446<br>15'860 | 56<br>57<br>59<br>59<br>61   | 14'072<br>14'524<br>14'962<br>15'387<br>15'799 | 59<br>62<br>64<br>65<br>65  | 21<br>22<br>23<br>24<br>25 |
| 26<br>27<br>28<br>29<br>30 | 16.230<br>16.924<br>17.805<br>18.018           | 48<br>46<br>42<br>39<br>40 | 16.482<br>16.878<br>17.260<br>17.626           | 5 <sup>2</sup><br>5 <sup>2</sup><br>50<br>46<br>43 | 16:430<br>16:826<br>17:580<br>17:935     | 54<br>56<br>54<br>52<br>49 | 16.376<br>16.770<br>17.156<br>17.528<br>17.886 | 58<br>60<br>60<br>56                   | 16.319<br>16.215<br>17.830<br>17.830           | 59<br>61<br>64<br>64<br>64 | 16.560<br>16.621<br>17.404<br>17.766           | 60<br>63<br>66<br>68<br>69   | 16.588<br>16.388<br>17.336<br>17.697           | 66<br>68<br>70<br>73        | 26<br>27<br>28<br>29<br>30 |
| 31<br>32<br>33<br>34<br>35 | 18'364<br>18'710<br>19'060<br>19'417<br>19'774 | 45<br>55<br>70<br>85<br>94 | 18.319<br>18.652<br>19.332<br>19.680           | 43<br>49<br>58<br>74<br>90                         | 18.276<br>18.606<br>18.932<br>19.258     | 46<br>46<br>52<br>61<br>76 | 18.530<br>18.560<br>19.19.<br>19.514           | 5 <sup>2</sup><br>49<br>49<br>55<br>65 | 18.178<br>18.831<br>19.142<br>19.449           | 61<br>57<br>53<br>54<br>59 | 18.117<br>18.424<br>18.728<br>19.088           | 68<br>64<br>60<br>56<br>57   | 18.049<br>18.390<br>18.218<br>19.333           | 73<br>72<br>70<br>65<br>61  | 31<br>32<br>33<br>34<br>35 |
| 36<br>17<br>18<br>9        | 20'128   |                            | 20'028   | 98<br>103<br>                                      | 19'930<br>20'270<br>20'608<br>           |                            | 19.838<br>20.168<br>20.502<br>20.831           | 72<br>96<br>106<br>110                 | 19.756<br>20.021<br>20.236<br>20.231<br>21.044 | 68<br>84<br>100<br>109     |  | 62<br>72<br>89<br>104<br>113 | 19.626<br>19.916<br>20.207<br>20.508<br>20.817 | 61<br>66<br>75<br>92<br>108 | 36<br>37<br>38<br>39<br>40 |
| 1 2 3 4 4 5 5              |  |                            |  |  |  |                            |  |  |  |                            | 21.246   | 117                          | 21'129<br>21'438<br><br>                       |                             | 41<br>42<br>43<br>44<br>45 |
|                            | 52   | Diff.                      | 53   | Diff.  | 54                                       | Diff.                      | 55   | Diff.                                  | <b>5</b> 6                                     | Diff.                      | 57   | Diff.                        | 58   | Diff.                       |                            |

# [APRIL

## TABLE B—(continued).

**H**<sup>M</sup> 3 °/0

# Continuous Temporary Annuities.

**H**<sup>M</sup> 3 °/0

 $\bar{a}_{\mathbf{M}-n+1}:\overline{n}$ 

|                            |  |                            |   |                            |  | MATU                       | RITY-AGE                                       | E, i.c.,                      | , М.   |                              |  |                              |  |                               |                            |
|----------------------------|--|----------------------------|---|----------------------------|--|----------------------------|--|-------------------------------|--|------------------------------|--|------------------------------|--|-------------------------------|----------------------------|
| n                          | 59   | Diff,                      | 60  | Diff.                      | 61   | Diff.                      | 62   | Diff.                         | 63   | Diff.                        | 64   | Diff.                        | 65   | Diff.                         | n                          |
| 1<br>2<br>3<br>4<br>5      | '974<br>1'898<br>2'780<br>3'622<br>4'427       | 1<br>2<br>6<br>10          | '973<br>1'896<br>2'774<br>3'612<br>4'413                | 1<br>4<br>7<br>12<br>15    | '972<br>1'892<br>2'767<br>3'600<br>4'398       | 1<br>3<br>7<br>11<br>18    | .971<br>1.889<br>2.760<br>3.589<br>4.380       | 5<br>8<br>13<br>18            | '970<br>1'884<br>2'752<br>3'576<br>4'362       | 2<br>4<br>10<br>15<br>20     | '968<br>1'880<br>2'742<br>3'561<br>4'342       | 5<br>10<br>16<br>23          | '967<br>1'875<br>2'732<br>3'545<br>4'319       | 5<br>11<br>17<br>25           | 1<br>2<br>3<br>4<br>5      |
| 6<br>7<br>8<br>9           | 5°200<br>5°941<br>6°654<br>7°340<br>8°002      | 18<br>22<br>26<br>30<br>35 | 5.182<br>5.919<br>6.628<br>7.310<br>7.967               | 21<br>25<br>29<br>33<br>37 | 5.161<br>5.894<br>6.599<br>7.277<br>7.930      | 22<br>28<br>33<br>37<br>41 | 5.866<br>5.866<br>6.566<br>7.240<br>7.889      | 25<br>30<br>36<br>41<br>45    | 5.114<br>5.836<br>6.530<br>7.199<br>7.844      | 26<br>32<br>39<br>45<br>51   | 5.088<br>5.804<br>6.491<br>7.154<br>7.793      | 40<br>37<br>43<br>50<br>57   | 5.058<br>5.767<br>6.448<br>7.104<br>7.736      | 32<br>39<br>47<br>54<br>60    | 6<br>7<br>8<br>9           |
| 11<br>12<br>13<br>14<br>15 | 8.641<br>9.260<br>9.860<br>10.442<br>11.002    | 39<br>44<br>50<br>54<br>56 | 8.602<br>9.216<br>9.810<br>10.388<br>10.949             | 42<br>46<br>51<br>57<br>61 | 8.560<br>9.170<br>9.759<br>10.331<br>10.888    | 45<br>51<br>55<br>59<br>66 | 8.515<br>9.119<br>9.704<br>10.272<br>10.822    | 50<br>53<br>59<br>64<br>68    | 8.465<br>9.066<br>9.645<br>10.208              | 55<br>60<br>63<br>69<br>74   | 8.410<br>9.006<br>9.582<br>10.139<br>10.680    | 62<br>66<br>70<br>73<br>78   | 8.348<br>8.940<br>9.512<br>10.066<br>10.602    | 68<br>74<br>78<br>82<br>86    | 11<br>12<br>13<br>14<br>15 |
| 16<br>17<br>18<br>19<br>20 | 11°550<br>12°076<br>12°584<br>13°075<br>13°551 | 58<br>58<br>58<br>59<br>61 | 11'492<br>12'526<br>13'16<br>13'492                     | 64<br>66<br>66<br>66       | 11'428<br>11'952<br>12'460<br>12'950           | 70<br>72<br>74<br>74<br>74 | 11.358<br>11.880<br>12.386<br>12.876<br>13.350 | 74<br>78<br>81<br>82<br>82    | 11'284<br>11'802<br>12'305<br>12'794<br>13'268 | 78<br>84<br>87<br>90<br>91   | 11'206<br>11'718<br>12'218<br>12'704<br>13'177 | 84<br>88<br>94<br>98<br>99   | 11'122<br>11'630<br>12'124<br>12'606<br>13'078 | 89<br>95<br>100<br>104<br>108 | 16<br>17<br>18<br>19<br>20 |
| 21<br>22<br>23<br>24<br>25 | 14.013<br>14.462<br>14.898<br>15.322<br>15.734 | 63<br>66<br>68<br>68<br>70 | 13'950<br>14'396<br>14'830<br>15'254<br>15'664          | 68<br>70<br>72<br>75<br>75 | 13.882<br>14.326<br>14.758<br>15.179<br>15.589 | 74<br>74<br>76<br>79<br>81 | 13.808<br>14.252<br>14.682<br>15.100<br>15.508 | 82<br>82<br>82<br>84<br>86    | 13'726<br>14'170<br>14'600<br>15'016<br>15'422 | 90<br>90<br>90<br>90         | 13.636<br>14.080<br>14.510<br>14.926<br>15.330 | 100<br>100<br>99<br>98<br>98 | 13.536<br>13.980<br>14.411<br>14.828<br>15.232 | 110<br>110<br>109<br>108      | 21<br>22<br>23<br>24<br>25 |
| 26<br>27<br>28<br>29<br>30 | 16.134<br>16.522<br>16.898<br>17.266<br>17.624 | 70<br>71<br>72<br>74<br>75 | 16 <b>'0</b> 64<br>16'451<br>16'826<br>17'192<br>17'549 | 77<br>77<br>76<br>77<br>79 | 15.987<br>16.374<br>16.750<br>17.115<br>17.470 | 81<br>82<br>82<br>82<br>83 | 15'906<br>16'292<br>16'668<br>17'033           | 89<br>89<br>90<br>89<br>89    | 15'817<br>16'203<br>16'578<br>16'944<br>17'298 | 93<br>95<br>96<br>96<br>96   | 16.485<br>16.848                               | 102<br>102<br>104            | 15.624<br>16.006<br>16.380<br>16.744<br>17.100 | 110                           | 26<br>27<br>28<br>29<br>30 |
| 31<br>32<br>33<br>34<br>35 | 17'976<br>18'318<br>18'648<br>18'967<br>19'272 | 78<br>79<br>76<br>74<br>69 | 17.898<br>18.239<br>18.572<br>18.893<br>19.203          | 81<br>83<br>84<br>81<br>79 | 17.817<br>18.156<br>18.488<br>18.812<br>19.124 | 85<br>86<br>89<br>90<br>86 | 17.732<br>18.070<br>18.399<br>18.722<br>19.038 | 89<br>92<br>92<br>94<br>95    | 17.643<br>17.978<br>18.307<br>18.628<br>18.943 | 95<br>94<br>97<br>98<br>100  |  | 113                          | 17'445<br>17'781<br>18'108<br>18'426<br>18'739 | 111                           | 31<br>32<br>33<br>34<br>35 |
| 36<br>37<br>38<br>39<br>40 | 19.565<br>19.850<br>20.132<br>20.416<br>20.709 | 65<br>66<br>71<br>80<br>96 | 19°500<br>19°784<br>20°061<br>20°336<br>20°613          | 74<br>68<br>69<br>74<br>84 | 19'426<br>19'716<br>19'992<br>20'262<br>20'529 | 83<br>79<br>74<br>74<br>79 | 19'343<br>19'637<br>19'918<br>20'188<br>20'450 | 93<br>89<br>83<br>78<br>78    | 19°250<br>19°548<br>19°835<br>20°110<br>20°372 | 98<br>94<br>89<br>84         | 19°150<br>19°450<br>19°741<br>20°021<br>20°288 |                              | 19°044<br>19°638<br>19°638<br>19°195           | 112                           | 36<br>37<br>38<br>39<br>40 |
| 41<br>42<br>43<br>44<br>45 | 21'012<br>21'318<br>21'621<br>                 | 112<br>122<br>125<br>      | 20'900<br>21'196<br>21'796<br>                          | 116                        | 20.800<br>21.080<br>21.370<br>21.664<br>21.956 | I 20<br>I 30               | 20'712<br>20'975<br>21'250<br>21'534<br>21'823 | 84<br>92<br>110<br>124<br>134 | 20.628<br>20.883<br>21.140<br>21.410<br>21.689 | 84<br>89<br>96<br>114<br>129 | 20°544<br>20°794<br>21°044<br>21°560           |                              | 20°456<br>20°706<br>20°950<br>21°194<br>21°441 | 92<br>98                      | 41<br>42<br>43<br>44<br>45 |
|                            | 59   | Diff.                      | 60  | Diff.                      | 61   | Diff.                      | 62   | Diff,                         | 63   | Diff.                        | 64   | Diff.                        | 65   | Diff.                         |                            |

### Discussion.

Mr. G. F. HARDY said that the fact that the Institute had had the pleasure on two meeting nights in succession of listening to papers on the valuation of endowment assurances was sufficient evidence that at all events the subject occupied the minds of many members, and he thought it might be also taken as an evidence that it was a subject which was proving itself to be one of considerable practical importance. Mr. Thomson had given them an exhaustive analysis and criticism of both the principles and procedure of the existing methods of grouping endowment assurances for valuation purposes. Mr. Lidstone had produced a new method, which, it would be seen at once, had very much to recommend it, and would, no doubt, prove as effective in practice as it was ingenious in conception. The method was based on the fact that the differences of temporary annuities, when the term was constant and the age varied, were nearly in geometrical progression, and that the method of finding the mean age was different from that which would be required on the rougher assumption of arithmetical progression. That fact Mr. Lidstone had established partly by experiment—as shown by the figures in his first table—and partly by an analytical investigation. If they remembered that a temporary life annuity differed from an annuity certain of the same term by the fact that the force of interest was supplemented by the force of mortality, and that the latter consisted, according to Makeham's theory, of a constant and a term involving as a factor cx, it would be seen at once that they could express the value of an annuity for n years on a life x in the form  $f(n, c^x)$ , which could be expanded in a series of ascending powers of the quantity  $c^x$ . In this way they obtained the series

$$\bar{a}_{x\bar{n}} = a_0 + c^x \log g(a_1 - a_0) + c^{2x} (\log g)^2 \frac{(a_2 - 2a_1 + a_0)}{1 \cdot 2} + \&c.$$

where (log g) entered symmetrically with  $c^x$ , and the quantities  $a_0$ ,  $a_1$ ,  $a_2$ , &c., represented the values of annuities certain for n years at rates of interest corresponding to

$$v_0 = vs$$
;  $v_1 = vsc$ ;  $v_2 = vsc^2$ ; &c.

This series would be found to be rapidly convergent for all annuities that usually occur in the valuation of endowment assurance policies. For example, if n=20 they found by the Text-Book Tables at 3 per-cent

$$\bar{a}_{x:\overline{20}} = 14.288 - 0.02356c^x + 0.000346c^{2x} - \&c.$$

and 
$$\bar{a}_{x;\overline{20}} - \bar{a}_{x+1;\overline{20}} = 02356(c-1)c^x - 0000346(c^2-1)c^{2x} + \&c.$$
  
=  $002252c^x(1-00308c^x + \&c.)$ 

So long, therefore, as  $c^x$  was not large, the successive values of these quantities would approximate to a geometrical series with the common ratio c. Approaching the matter from the point of view of general reasoning, since the values of temporary life annuities having the same terms differed *inter se* only in consequence of differences in the

force of mortality, and small changes in the value of the latter might be assumed to produce proportionate changes in the value of the annuity—hence, the differences in the force of mortality being in geometrical progression, the differences in the annuity-values would approximately follow the same law. In methods B and C, which were the methods at present ordinarily in use, they found about the same error, judging by the illustration given in the paper, an error of from 1 to  $1\frac{1}{4}$  per-cent in the value of the net liability. In method B, which was described as Mr. Manly's method, the policies being grouped according to the term unexpired, the mean age for each group was found by simply multiplying the sum assured by the age and dividing by the total sum assured. The reversionary bonuses were not brought into account, nor were the premiums. Mr. Lidstone suggested that the reversionary bonuses should be brought into account, and probably a slightly more accurate result would be If, however, both the premiums and the reversionary bounses were left out of account, the resulting errors would partly neutralize one another.

Mr. Lidstone said that the suggestion was only for expected

claims, not for the valuation itself.

Mr. HARDY said that the point would naturally arise in determining the average age as to whether a separate calculation should be made for the premiums, and whether in computing the average age for the sums assured the bonuses should be included, and it might, therefore, be as well to notice that if a separate calculation were not made in respect of the premiums a better result would then be obtained by using sums assured only, and not sums assured plus bonus additions. The reason for that was that, given policies having a uniform term to run, those policies which had been longest on the books, and which, therefore, would have the largest reversionary bonuses, would, of necessity, have the smallest premiums, and as the effect of the bonuses acted in the opposite direction, there would be a counteracting influence, and the error in computing the ages would be less by taking simply the sums assured, as Mr. Manly did, than by taking the sums assured and the bonuses. Coming to the practical application of Mr. Lidstone's method, they would all agree that a very great advantage would be obtained by employing not the actual average rate of increase in the differences of the annuity-values, say  $S_{\frac{1}{2}}$  per-cent, which the author found from his initial table, but the somewhat higher rate of increase, about 9½ per-cent, which correspended to the value of the constant c in Makeham's formula. effect of that was to give an error, which, while still remaining very small, was almost certain to be on the right side, and also to provide a very simple method of computing the expected claims; hence they had a double reason for fixing upon that constant as representing the ratio in question. Once set in motion, of course, Mr. Lidstone's method required practically very little more labour—they might almost say no more labour-than the ordinary method, as they might just as easily multiply the sums assured by an arbitrary factor, determined once for all when the policy was set up as by any other factor such as the age. It might very well happen, however, that they had valuation books in use which were not immediately suited

to that particular form of valuation. If that were the case, and if they had been in the habit of valuing their endowment assurances by using simply the average age, as ordinarily understood, he thought that Mr. Lidstone's Table 4 suggested to them a simple means of arriving at an approximate estimate of the error they had been Turning to the central column of that table, showing the error in each group of policies, i.e., the estimated liability, less the true liability, they would see that, as they passed through the different groups, they started with a small error, reaching a maximum about the middle group, n = 15 - 19, the error falling off again till it became insignificant in the last group. The reason for that, of course, was that in the early groups, where the term was very short, an error in determining the average age made little difference in the annuity-value. Where the term was 0 it of course made no difference, and when it was only one year the difference was extremely slight. In the same way, towards the close of the Table, where all policies had a great many years to run, the error was also small, on the one hand, because the total sums assured and reserves in these groups were small, and, on the other hand, because the ages for those long terms must all be comparatively young, and near to one another, the annuity-values more nearly approaching an arithmetical progression than they would for shorter terms. Hence the errors in the successive groups formed a curve very similar to the normal curve of error, starting at zero, for duration 0, rising up to a maximum towards the middle of the whole Table, and then falling away to zero again at the conclusion. Hence, apart from irregularities in the curve, the area of the whole curve might be found approximately by taking the central term and multiplying it by one half of the number of terms in the entire series. As a matter of fact, in the quinquennial group, n = 15 - 19, they had an error of 454—he was dealing with method C.—and in n = 20-24there would be an error of 413. They might, therefore, infer an error of about 92 for the single central group, n = 18. Taking that value, 92, and multiplying it by half the number of single year groups in the entire Table, this would give them 15 times 92, or about 1660, as the total error in the valuation by method C, which was almost exactly equivalent to the sixteen hundred odd by which Mr. Lidstone showed that valuation was in error. He mentioned that, not in any way as a substitute for Mr. Lidstone's more exact method. but as showing how they might rectify their present somewhat rougher methods, and know approximately what sort of error they were making in treating the annuity-values as an arithmetical series instead of a geometrical one. If they had a set of endowment assurance policies which were, in almost all cases, maturing at quinquennial ages, as was the ease in a great many offices, of course a detailed valuation was much less laborious than in the case in which n might have any value whatever. Mr. Lidstone's method, however, had the great advantage that it was a continuous method, which, once started, flowed on without the slightest difficulty to anyone concerned. It was not necessary that the operator should be a scientific actuary, or even know the rudiments of the principles which he was applying, and there was the further advantage that, although offices

might at the present moment issue endowment assurances maturing at quinquennial ages, it was impossible to say how long they might find it convenient to continue that system. It was becoming fashionable to issue endowment assurances maturing after a fixed term of years, and then the whole of the accuracy and simplicity which resulted from the arrangement of the policies in groups maturing at 45, 50, 55, 60, and so on, would break down. To avoid this difficulty, they simply required two more columns in their valuation book, and practically no more addition to their labours when the valuation came round, than under the most rough and ready system at present in use. It had been a great pleasure to him to hear another paper by Mr. Lidstone, and they must all congratulate him on having shown in his present contribution both originality and

sound judgment.

Mr. F. Schooling admired exceedingly the masterly way in which Makeham's hypothesis had been manipulated. In paragraph 2, he thought it would be better to use the words "the same number of future payments" in preference to "the same unexpired term", as the latter expression might include policies with a different number of future payments. In describing C and D, reference was made to finding the mean age for the net premium. Of course that did not mean that the net premiums were found from the mean age, but that the annuity used for the purpose of finding the value of the future premiums was that for the average mean age as found from the net premiums. It was generally admitted that in any system of grouping the exact net premiums must be used, and not the net premium for the mean age. He thought Mr. Lidstone had been rather hard on method B. He had assumed that only one group could be used for each number of future payments. In the paper he (Mr. Schooling) had read before the Institute, he pointed out, practically agreeing with Mr. Lidstone, that for the purpose of a test valuation these groups were all that were necessary, but in an actual valuation the following method of grouping before dividing into the number of future payments could be adopted, as many groups being formed as was found desirable. All policies maturing under age n would form group 1; and between ages n and m group 2; the general rule being that the older the age of maturity the smaller the groups, because it was in these groups that the question of mortality became of increasing importance. If such sub-grouping were adopted, all cases of a given class which had the same number of future payments could only differ in age to a certain limited extent. Again, in paragraph 10, it was pointed out that assurances maturing at other ages were for convenience omitted, and that only policies maturing within the narrow range of 15 years were included. But in a later paragraph method B was said to have entirely broken down, because one very large policy maturing at age 77 was included. Surely the first duty of an actuary in arranging groups for any purpose was to see that the policies forming the group were fairly homogenous, and to cast out isolated cases for separate valuation. He regretted that the methods were not tested for policies maturing after a given number of years instead of at given ages, because in the former class, where policies mature at every age from 35 to 75,

some system of grouping was absolutely necessary, but as Mr. Hardy pointed out, in the case in question, if the policies were only payable at ages 50, 55, 60, and 65, there really was not very much necessity if the numbers were at all large for any grouping, but they might adopt the ordinary method of treating each class by itself—that was, form four primary groups for the four fixed ages, and then all policies in each group which had the same number of future payments must

have approximately the same age. Mr. H. W. MANLY said that some years ago he had devised the scheme to which Mr. Lidstone had been good enough to attach his name, and he believed it gave a sufficiently close approximation to the desired results. If they kept out of the grouping any exceptional policies it was evident that his method would not go very far wrong, particularly with reference to those policies which matured within the next 15 years. He was not one who believed in extreme accuracy in valuing a policy which matured 30 or 40 years hence, as there was a great probability that very few of those policies would ever mature. If they could get the values of those long-term policies within 3 or 4 per-cent of the true value, which Mr. Lidstone had shown to be about the range, they would not be far out, particularly if the rate of interest used in the valuation was considerably less than the rate of interest which the company happened to be earning at the time. Mr. Lidstone's was an extremely neat formula, and he had no doubt that the method could be applied with great rapidity if their classification books were properly constructed for its use. Those who had not vet adopted this useful mode of grouping would no doubt do so now, and adapt their classification books in a manner to enable them to use the author's Table V. He did not know how Mr. Lidstone had valued the policies upon what he was pleased to call his (Mr. Manly's) method, whether he had attempted to make an adjustment for the actual due dates of the policies and premiums or not. His method was to classify the policies according to the year of maturity, to assume first that the sums assured and the bonuses would be payable at the end of the year of maturity, and that the premiums would fall due at the end of each year. That was for the sake of convenience and rapidity of valuing, because all that was necessary, having got the average age, was to obtain the annuity for the term n-1, and then enter Orchard's Tables for the value of the endowment assurance at the end of the term. Having done that they would see that the assumption was in both instances six months wrong, and consequently he made an adjustment by increasing the reserve value by a half-year's interest, which at 3 per-cent meant an addition of  $1\frac{1}{2}$  per-cent. He did not know how far that correction entered into the figures which Mr. Lidstone had given, but it seemed to him that by that method he got very close indeed to the results of an individual valuation.

Mr. Joseph Burn particularly admired the way in which the expected claims might be calculated by Mr. Lidstone's method. He thought that writers of valuable papers such as the present were very apt to forget that the majority of the readers of the paper in the future would be students, and that being so, he thought they were entitled to some consideration. He therefore advocated the introduction of explanations here and there, which could be put in

brackets, so as not to interfere with the continuity of ideas. He did not mean to imply that Mr. Lidstone's paper would give students particular trouble, but he knew from personal experience that some papers in the past had necessitated students spending what he

considered a great deal of unnecessary time.

Mr H. A. Thomson said that Mr. Lidstone had added a very ingenious method of valuing endowment assurance policies to those already in existence, and his assumption that the first differences of the temporary annuities formed a geometrical progression was evidently very close to the truth; at any rate, at the central ages of the mortality table—that is, the age with which they were chiefly concerned in endowment assurance business. From that hypothesis was deduced a very elegant scheme of reducing to inconsiderable dimensions the persistent negative errors involved in method B. Those errors occurred in each constituent group, and were due to the tacit assumption that the temporary annuities were in arithmetical progression, or, in other words, that they formed a straight line instead of a curve, as was actually the case. In the last method described in the paper read in December (method 14) an attempt, based on geometrical considerations, was made to counteract those errors, the suggested adjustment consisting of additions to the average ages brought out by method B, such additions depending on the range of valuation ages in each group, and a table of suggested additions was given. On perusing Mr. Lidstone's paper, and seeing how accurately his plan had worked in the case to which he applied it, he was interested to see what addition to the average valuation ages had been involved by the application of his method. Mr. Lidstone had kindly allowed him to glance over the schedules, and he saw that each group (with two unimportant exceptions) contained policies maturing at both 50 and 65, and therefore had a range of 15 years. His additions to the average ages brought out by method B varied from 34 to 1.69, but most of them lay between 5 and 1.1, the average addition being 86. Method 14 would have involved an addition of '7 only in each group, so that it seemed probable that, were that method to be applied to the batch of policies in question, the reserve brought out would be slightly too small. Turning to Table 6, which gave the results of the valuation of a group of policies containing one exceptional policy, Mr. Pagden had kindly lent him a schedule giving particulars of the policies, and he had valued them by method 14; using one age, as in method D, the error was-4182, and using two ages, as in method E, it was +1252. Valuing them by the rough method of taking the annuities to the nearest integer, (similar to Mr. Woolhouse's method of taking the assurances to the first decimal place), the error brought out was -4486, and if the exceptional policy had been omitted that error would have been halved. He did not think that much importance should be attached to the valuation results of that batch of policies, because, as Mr. Lidstone had pointed out in paragraph 32, and most of the speakers had agreed, exceptional policies of the kind therein contained should be omitted from the valuation, and dealt with separately. The figures given in Table 2 related to policies whose ages at maturity extended over a range of 15 years only, namely, from 50 to 65. It would be interesting to see the results of the application of Mr. Lidstone's method to policies possessing a wider range than 15 years. It was largely the policies maturing at ages above 65, and to a smaller extent also the policies maturing at the younger ages, 35 to 50, which threw out of gear some of the methods of valuing endowment assurances in groups. It certainly looked, however, as if the present method would apply with sufficient accuracy for all ordinary ages at maturity. If they dealt only with policies maturing between 50 and 65, a very simple method of valuing them was to take some central age, say 58, and assume all policies to mature at that age, and alter the net premiums accordingly, so that a policy taken out at 30 and maturing at 65, would be treated throughout as if effected at 23, and maturing at 58, and so for all the policies. In each case the change in the premium valued would tend to counteract the effect of the change in the reversion and annuity due to the increase or decrease of the valuation age, and the error in the reserve resulting from the three-fold change would be very small. For policies maturing between 50 and 65, the largest possible individual error caused by this method would be about 8 on a policy of £100. In most cases the alterations in the reserve caused by moving the maturity age from 50 or 65, to 58 would be much less than this amount, perhaps 3 or 4, and could, he thought, safely be left to balance each other. In valuing whole-life policies they allowed a larger range of error than that. The error in an ordinary whole-life valuation might be unity on a policy of £100, simply through taking the valuation age half a year too high or too low. If the age at entry be high, the error on individual policies might be as much as  $1\frac{1}{2}$  per £100 assured. These errors were assumed to balance one another. It seemed, then, a fair deduction to make that when they valued endowment assurances at some central age the results would, in most eases, be not less accurate than in an ordinary whole-life valuation, provided the true maturity ages did not extend over more than about 15 years, say, from 45 to 60, or from 50 to 65. The method was very simple, and avoided the calculation of any average ages, or any adjustment at all.

The President said, that with all his sympathy with the students, he could hardly advise the writers of papers to construct them precisely on the lines which had been suggested by Mr. Burn, for the papers were designed, not to save trouble to the students, but rather to stimulate them to take trouble for themselves. In conveying the cordial acknowledgments of the members and himself to Mr. Lidstone, he desired to add that the present paper had been in his hands several months, and was an entirely separate investigation

from Mr. Thomson's into the same subject.

Mr. Lidstone, in reply, said he felt much indebted to the members for the cordial way in which they had received his paper. Judging from the discussion on Mr. Thomson's paper, there seemed to be considerable difference of opinion as to how far these broad methods of grouping were desirable. Some actuaries, principally, he thought, those who had to do with endowment assurances in small numbers, considered such methods superfluous and even undesirable, whereas Mr. Manly spoke quite cheerfully of methods producing errors of 3,

4, or 5 per-cent. He found himself in a middle position between these extremes. On the one hand, although Mr. Hardy had shown that "systematic" errors were less objectionable than errors that were not "systematic", an error of 1 per-cent was rather more than he cared to have. On the other hand, with the rapid increase of endowment assurances it seemed to him that some wide grouping was, or was becoming, absolutely necessary. For it must be remembered that, in the case of a progressive office, about 50 or 60 per-cent of the policies now written were endowment assurance policies. The paper was the outcome of an attempt to devise a method which would preserve the advantages of grouping and at the same time reduce the error to a minimum. It was a pleasure to him to find that the referees were substantially in accord with his method, especially in the case of Mr. George Hardy, not only because his judgment had such weight, but because he (Mr. Lidstone), as well as many other younger members, had such a very grateful remembrance of the time when they had the advantage of his tuition. Mr. Hardy had developed the mathematical side of the paper in his own masterly way, and he hoped he would be induced to write a note as an adjunct to the paper. He was entirely in accord with Mr. Hardy when he suggested that c should be used rather than any other ratio, even if one could be found more accurate. It should be a little less than c, but he certainly thought the best thing to do was to use c, knowing very well it produced but a small error, and one on the safe side. Mr. Schooling had commented on one or two points of detail. He could not quite agree with his objection to the phrase "unexpired term", because, as a matter of fact, he had not classified the policies according to the number of premiums remaining to be paid. In the latter part of the paper he suggested that policies in respect of which there remained  $19\frac{1}{4}$ ,  $19\frac{1}{2}$ , and  $19\frac{3}{4}$  years' premiums should all be brought into one broad group. Mr. Schooling said they could divide their broad groups into as many sub-groups as might be thought desirable. His (Mr. Lidstone's) point was that it was not desirable to sub-group at all if they could find a method which would produce quite as good results. Sub-grouping meant additional labour and anxiety at the time of the valuation. Perhaps his language was not so clear as it might be with regard to what he called the mean age for the net premiums, but he should hardly have thought it could be read in the way Mr. Schooling suggested. Of course, he meant net premiums calculated in the ordinary way for the true ages at entry. Mr. Schooling said such a method as Mr. Manly's should not be applied to a group of policies like that referred to later in the paper. Of course he should not propose to do so himself, but one did occasionally try experiments. Mr. Pagden had kindly allowed him to see the results of such an experiment, and it appeared to him that such a case gave the best possible test of how far the new method would grapple with abnormal conditions. He was inclined to think that the result was distinctly satisfactory, and that was why he had exhibited it in the paper, although he would not suggest that any approximate method should be actually adopted in such an abnormal case. He could not at all agree with the remarks that had fallen from Mr. Burn, because if one went too much into points of detail,

the general argument of the paper was liable to be confused. As his method had met with some approval, he proposed to publish an extended table of the function which he called Z, in order that the method might be easily applied. He certainly hoped that actuaries who had the opportunity would apply it, and communicate the results, so that they might see whether, under ordinary conditions, the method would give good results. Of course, he did not attach any importance to the extremely small error of £4 brought out in the case referred to in the paper; at the same time he was distinctly convinced that under ordinary conditions the method would give very accurate results, and certainly very much more accurate results than the approximate methods previously employed.

On the Compound Survivorship Annuity. By George F. Hardy, F.I.A., Actuary of the English and Scottish Law Life Assurance Association.

THE following is a method of obtaining an approximate value of the annuity  $\overline{a}_{yz}$ , based on the assumption that the mortality follows Makeham's formula. The numerical results will be found sufficiently close for practical purposes.

The annuity may be expressed, as is well known, in the form

$$a_{yz,x}^{-1} = \int_{0}^{\infty} r^{t}_{t} p_{xyz}.\mu_{z+t}.\overline{a}_{x+t}.dt \quad . \quad . \quad . \quad (1)$$

Assume for convenience that z>y, as in the alternative case we may write

$$a_{yz|r}^{-1} = a_{yz|x} - \bar{a}_{yz|x}^{1}.$$
Let 
$$\mu_{y+t} = \Lambda + Bc^{y+t}; \quad \mu_{z+t} = \Lambda + Bc^{z+t}$$

$$\therefore \quad \mu_{z+t} - \Lambda = \frac{c^{z}}{c^{y} + c^{z}} \cdot \{\mu_{y+t} + \mu_{z+t} - 2\Lambda\}$$
and 
$$\mu_{z+t} = \frac{1}{c^{y-z} + 1} \{\mu_{y+t} + \mu_{z+t} - (1 - c^{y-z})\Lambda\} \quad . \quad (2)$$
write 
$$c^{y-z} = \kappa, \text{ and } (1 - c^{y-z}) \Lambda = (1 - \kappa) \Lambda = \phi.$$

Then 
$$\mu_{z+t} = \frac{1}{1+\kappa} (\mu_{y+t} + \mu_{z+t} - \phi)$$
 . . . (3)

and, since  $v^t = e^{-t\delta}$ , where  $\delta$  is the force of discount, we have from (1)

$$\overline{a}_{yz,x}^{1} = \frac{1}{1+\kappa} \int_{0}^{\infty} e^{-t\delta_{t}} p_{xyz}(\mu_{y+t} + \mu_{z+t} - \phi) \overline{a}_{x+t} dt \quad . \quad (4)$$

Let y' be a life of the same age as y, but subject throughout life to a force of mortality less than that of y by the constant  $\phi$ ; in other words, let  $\mu_{y'+t} = \mu_{y+t} - \phi$ ; whence, of course,  $t p_y = e^{-t\phi_t} p_{y'}$ .

Then from (4)

$$\overline{a}_{yz,\epsilon}^{1} = \frac{1}{1+\kappa} \int_{0}^{\infty} e^{-t(\delta+\phi)_{t}} p_{xy'z}(\mu_{y'+t} + \mu_{z+t}) \overline{a}_{x+t} dt$$

which may be put in the alternative forms

$$\frac{1}{1+\kappa} \int_0^\infty e^{-t\delta_t} p_{xy'z}(\mu_{y'+t} + \mu_{z+t}) \overline{u}^{\delta}_{x+t} [e^{-t\phi}] dt \qquad . \tag{5}$$

or, 
$$\frac{1}{1+\kappa} \int_0^\infty e^{-t(\delta+\phi)} t p_{xy'z}(\mu_{y'+t} + \mu_{z+t}) \overline{a}_{x+t}^{\delta+\phi} \left[ \frac{\overline{a}^{\delta}_{x+t}}{\overline{a}_{x+t}^{\delta+\phi}} \right] dt \quad (6)$$

where  $\bar{a}^{\delta}$  and  $\bar{a}^{\delta+\phi}$  are the values of annuities when the force of discount is  $\delta$  and  $\delta + \phi$  respectively.

The integrals in (5) and (6) could obviously be evaluated if the factors in the brackets [] were constants. They could also be evaluated, as will be shown, if the values of the factor  $\left\lceil \frac{\tilde{a}_{x+t}^{\delta}}{\tilde{a}^{\delta+\phi}} \right\rceil$  formed a G. P. diminishing at a different rate from

 $e^{-t\phi}$ . For both (5) and (6) are identical with the expression

$$\frac{1}{1+\kappa} \int_0^\infty e^{-t(\delta+\psi)} t^{\mu} p_{xy'z} (\mu_{y'+t} + \mu_{z+t}) \bar{a}_{x+t}^{\delta+\psi} \left[ \frac{\bar{a}_{x+t}^{\delta}}{\bar{a}_{x+t}^{\delta+\psi}} \cdot e^{t(\psi-\phi)} \right] dt \quad (7)$$

and assuming, for the moment, the values of  $\frac{\bar{a}_{x+t}^{\delta}}{\bar{a}_{x+t}^{\delta+\psi}}$  to be in G.P. a value of  $\psi$  can be found which will make that factor diminish with t at the same rate as the factor  $e^{t(\psi-\phi)}$  increases. In other words, for all values of t, by properly selecting  $\psi$ , we shall have

$$\frac{d}{dt}\left(\log\frac{\overline{a}^{\delta}_{x+t}}{\overline{a}^{\delta+\psi}_{x+t}}\right) = \frac{d}{dx}\left(\log\frac{\overline{a}^{\delta}_{x}}{\overline{a}^{\delta+\psi}_{x}}\right) = \phi - \psi = \text{a constant},$$

and therefore  $\frac{\bar{a}^{\delta}_{x+t}}{\bar{a}^{\delta+\psi}_{x+t}} \cdot e^{t(\psi-\phi)} = \frac{\bar{a}^{\delta}_{x}}{\bar{a}^{\delta-\psi}_{x-t}} = a \text{ constant.}$ 

If this constant is substituted in (7) for the factor in the bracket [], the whole expression at once reduces to

$$\begin{split} \bar{a}_{yz\,x}^{\delta_{1}} &= \frac{1}{1+\kappa} \cdot \bar{a}_{y\,z\,x}^{\delta+\psi} \cdot \left[ \frac{\bar{a}_{x}^{\delta}}{\bar{a}_{x}^{\delta+\psi}} \right] \\ &= \frac{1}{1+\kappa} \left( \bar{a}_{x}^{\delta+\psi} - \bar{a}_{xy\,z}^{\delta+\psi} \right) \left[ \frac{\bar{a}_{x}^{\delta}}{\bar{a}_{x}^{\delta+\psi}} \right] \end{split}$$

which, since  $\mu_{y'} = \mu_y - \phi$ ,

$$= \frac{1}{1+\kappa} \left( \bar{a}_x^{\delta+\psi} - \bar{a}_{xyz}^{\delta-\psi-\phi} \right) \begin{bmatrix} \bar{a}_x^{\delta} \\ \bar{a}_z^{\delta+\psi} \end{bmatrix} \quad . \quad (8)$$

It now remains to be shown that the condition on which this result has been obtained (viz.: that the values of  $\frac{\tilde{a}_{x+!}^{\delta}}{\tilde{a}_{x+!}^{\delta+\cup}}$  form a G. P.) is approximately true in practice.

As  $\psi$  will be found to be about one-fourth greater than  $\phi$ , and  $\phi = (1 - \kappa)A$ , we may conveniently take A (the constant portion of the force of mortality) as somewhere near the maximum value of  $\psi$ .

Taking as illustrations the  $H^M$  Table at 3 per-cent and 4 percent interest (Text-Book graduation), and the Carlisle Table at 3 per-cent (King and Hardy's graduation, J.I.A., vol. xxii, p. 221, &c.), we have the following table, which sufficiently explains itself.

|  | i  |                                   | Ни Л                                   | ABLE  |   |  | CARL  | ISLE TABI   | LE   |
|--|--|-----------------------------------|--|---|---|--|---|---|--|
| x                                      | 3  | per-cent                          |  | 4   | per-cent  |  | 3   | per-cent  |  |
|  | $\log \frac{\bar{a}_x^{\delta}}{\bar{a}_x^{\delta+\psi}}$                                      | log G.P.<br>Series                | Error<br>in 4th<br>place               | $\log \frac{\bar{a}_x^{\delta}}{\bar{a}_x^{\delta+\psi}}$   | log G.P.<br>Series  | Error<br>in 4th<br>place                 | $\log \frac{\bar{a}_x^{\delta}}{\bar{a}_x^{\delta+\psi}}$   | log G.P.<br>Series  | Error<br>in 4th<br>place                   |
| 25<br>35<br>45<br>55<br>65<br>75<br>85 | ·0436<br>·0378<br>·0313<br>·0244<br>·0177<br>·0117<br>·0067                                    | ·0247<br>·0183<br>·0119           | + 3<br>- 2<br>+ 3<br>+ 6<br>+ 2<br>-12 | ·0397<br>·0348<br>·0293<br>·0232<br>·0171<br>·0113<br>·0066 | ·0403<br>·0346<br>·0289<br>·0232<br>·0175<br>·0118<br>·0061 | + 6<br>- 2<br>- 4<br>- + 4<br>+ 5<br>- 5 | ·0564<br>·0494<br>·0415<br>·0330<br>·0243<br>·0163<br>·0098 | ·0570<br>·0490<br>·0410<br>·0330<br>·0250<br>·0170<br>·0090 | + 6<br>- 4<br>- 5<br><br>+ 7<br>+ 7<br>- 8 |
| $-\frac{d}{dx}$                        | $-\log \frac{\bar{a}_x^{\delta}}{\bar{a}_x^{\delta+\psi}} \left\{ \log e^{\psi-\phi} \right\}$ | = .0006                           | 4                                      |   | ·00057  |  |   | ·00080  |  |
| $\log e$                               | $\Psi(=\log e^{\Lambda})$  | =:0026                            | 9                                      |   | .00269  |  |   | .00351  |  |
|  | $\therefore \log e^{q}$  | b = ·0020                         | 5                                      |   | .00212  |  |   | 00271   |  |
|  | ∴ ↓  | $r = \frac{269}{205}  \mathrm{c}$ | Þ                                      |   | $rac{269}{212} \phi$                                       |  |   | $\frac{351}{271}\phi$                                       |  |
|  |  | $=1.3 \phi$                       | nearly.                                | _   | - ½ φ nea   | arly.                                    | =   | 13φ1  | nearly.                                    |

Hence, for the H<sup>M</sup> Table and the Carlisle Table  $\psi=1.3\phi$  when the rate of interest is 3 per-cent, and  $\frac{5}{4}\phi$  for the H<sup>M</sup> at 4 per-cent; indeed,  $\psi$  might probably be taken, without very serious error, at  $\frac{9}{4}\phi$  in general.

As an example of the application of the formula, take the annuity  $\overline{a}_{45,00,30}$ , Text-Book Tables, at 4 per-cent.

Here, making  $\psi = \frac{5}{4}\phi$ , and  $\psi - \phi = \frac{1}{4}\phi$  as above, equation (8) becomes

$$\bar{a}_{45,60|30}^{\delta} = \frac{1}{1+\kappa} (\bar{a}_{30}^{\delta+\frac{5}{4}\phi} - \bar{a}_{30,45,60}^{\delta+\frac{1}{4}\phi}) \frac{\bar{a}_{30}^{\delta}}{a_{30}^{\delta+\frac{5}{4}\phi}}$$

$$\kappa = c^{-15} = \cdot 2542$$

$$1-\kappa = \cdot 7458$$

$$(1-\kappa)\Lambda = \phi = \cdot 00462$$

$$\frac{1}{4}\phi = \cdot 00116$$

$$\frac{5}{1}\phi = \cdot 00577$$

Here

$$a_{30}^{\delta} = 17.655$$

Therefore, result = 
$$\frac{17.655 \times 8.100}{1.2542 \times 16.303} = 6.994$$

Value by summation formula 39a—See Text-Book, p. 266—6.996,, 33, taking n=3 6.996

As a second example, I will calculate the value, by same table and rate of interest, of the annuity

Here 
$$\kappa = c^{-30} = .06461$$

$$1 - \kappa = .93539$$

$$(1 - \kappa) A = \phi = .00579$$

$$\frac{\phi}{4} = .00145$$

$$\frac{5}{10} \phi = .00724$$

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$$03922 + 00724 = 04646$$

$$i = 045 \delta = 04402$$

$$i = 05 \delta = 01879$$

$$03922 + 00724 = 04646$$

$$i_{25} = 17\cdot183$$

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Therefore result = 
$$\frac{18.449 \times 9.208}{10.646 \times 16.608} = 9.608$$

Value by formula 33, taking n=3 9.614

The work in these examples would, of course, be shortened by use of a table of the value of  $\kappa$  and  $\phi$  for various values of z-y, and still further by working with rates of interest in lieu of forces of interest. The table appended gives the values of  $\kappa$  for all values of z-y from 1 to 40 for both the H<sup>M</sup> and Carlisle Tables, and at the same time the rates of interest corresponding to the forces of interest  $\delta + \psi$  and  $\delta + \psi - \phi$  when  $\delta$  corresponds to 3 per-cent and 4 per-cent respectively.

As a final example, we will find the value of  $\tilde{a}_{30.79|60}$  by the Carlisle Table at 3 per-cent.

Referring to the table following, we find

$$1 + \kappa = 1.0270,$$

rate of interest corresponding to  $\delta + \psi = 4.055$  per-cent

$$\delta + \psi - \phi = 3.241$$
 per-cent

hence 
$$\tilde{a}_{30,70,60}^{3,9/9} = \frac{1}{1.0270} (\tilde{a}_{60}^{4.055,9/9} - \bar{a}_{30,70,60}^{3.241,9/9}) \frac{\tilde{a}_{60}^{3,9/9}}{\tilde{a}_{60}^{4.055,9/9}}$$

In the paper (J.I.A., vol. xxii, p. 191) only 3 per-cent annuities are given, but remembering that  $a_{\omega}^{\delta+\Lambda} = a_{x,x}^{\delta}$  where  $2c^x = c^{\omega}$  or  $x = \omega - \frac{\log 2}{\log c}$  [Text-Book, Part II, chap. xii, § 43], we may obtain from the 2 and 3 life annuities the values of the single life annuities when  $\delta$  becomes  $\delta+\Lambda$  and  $\delta+2\Lambda$ , that is to say, at 3.836 per-cent and 4.680 per-cent respectively.

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We have then the following data:

whence 
$$\tilde{a}_{30.7060}^{30.7060} = \frac{1}{1.0270} (10.286 - 5.838) \frac{11.170}{10.286} = 4.704$$

the value by the summation formula 33 being 4713.

It will be observed that the formula just given in each case under-estimates the value of the annuity required. This is because the approximation  $\bar{a} = \frac{1}{2} + a$  has been employed throughout. If the true value of  $\bar{a}$  for both single and three-life annuities had been used, in the last example for instance, the value would have come out 4.711 instead of 4.704; correspondingly, the use of the approximation  $\frac{1}{2} + a$  in the summation formula over-estimates the value, and had the true values of  $\bar{a}$  been used, there would practically be no difference between the values as given by the present formula and those by the summation formulas 39a or 33 of the Text-Book. As it is, the differences average only 006, and the error of the formula given above, when used, as it would be in practice, with the value  $\frac{1}{2} + a$  for  $\bar{a}$  may be taken on the average, say, at about 004, in the final annuity-value.

Values of  $c^{-n}$ , and corresponding Rates of Interest.

 $\mathbf{H^{M}}$ .

|              |               | <b>3</b> PEI  | R-CENT          | <b>4</b> PEI | R-CENT          |     |
|--------------|---------------|---------------|-----------------|--------------|-----------------|-----|
| n            | c-n           | $i_{\psi}$    | $i_{\psi-\phi}$ | $i_{\psi}$   | $i_{\psi-\phi}$ | n   |
| 0            | 1.00000       | 3.000         | 3.000           | 4.000        | 4.000           | 0   |
| 1            | ·91273        | .072          | .017            | .070         | .014            | 1   |
| 2            | ·83309        | .139          | .032            | ·13 t        | .027            | 2 3 |
| 3            | .76038        | 199           | .046            | .193         | .038            |     |
| 4            | 69402         | .254          | .059            | .247         | .049            | 4   |
| 5            | ·63346        | .304          | .070            | 295          | .059            | 5   |
| 6            | .57818        | .320          | .081            | .340         | .068            | 6   |
| 7            | .52772        | ·392          | .080            | ·381         | .076            | 7   |
| $\mathbf{s}$ | .48167        | ·43I          | .099            | .418         | .084            | 8   |
| 9            | •43963        | •466          | .107            | .452         | .090            | 9   |
| 10           | .40127        | · <b>4</b> 98 | .115            | .483         | .096            | 10  |
| 11           | 36624         | .527          | .121            | •511         | .102            | 11  |
| 12           | ·33129        | $\cdot 554$   | ·127            | .537         | .107            | 12  |
| 13           | 30511         | .578          | .133            | .261         | .112            | 13  |
| 14           | ·27848        | .600          | .138            | .582         | ·116            | 14  |
| 15           | .25418        | .620          | .143            | .602         | .120            | 15  |
| 16           | .23200        | .639          | ·147            | ·620         | .124            | 16  |
| 17           | $\cdot 21175$ | .656          | .151            | .636         | ·127            | 17  |
| 18           | 19328         | ·671          | $\cdot 154$     | .651         | .130            | 18  |
| 19           | 17641         | .685          | .158            | 665          | .133            | 19  |
| 20           | .16101        | .698          | ·161            | .677         | .135            | 20  |
| 21           | ·14696        | .710          | .163            | .689         | .137            | 21  |
| 22           | 13414         | .720          | ·166            | .699         | .139            | 22  |
| 23           | .12243        | 730           | ·168            | .709         | ·141            | 23  |
| 24           | ·11175        | · <b>7</b> 39 | ·170            | .717         | ·143            | 24  |
| 25           | .10200        | .747          | ·172            | .725         | .145            | 25  |
| 26           | .09309        | .755          | .174            | .732         | ·146            | 26  |
| 27           | .08497        | .761          | .175            | .739         | ·147            | 27  |
| 23           | .07756        | .767          | ·177            | .745         | ·148            | 28  |
| 29           | .07079        | .773          | ·178            | .750         | .150            | 29  |
| 30           | .06461        | .778          | ·179            | .756         | .151            | 30  |
| 31           | .05897        | .783          | .180            | ·760         | .152            | 31  |
| 32           | .05382        | .787          | 181             | 761          | .152            | 32  |
| 33           | .04913        | .791          | .182            | .768         | .153            | 33  |
| 34           | .01484        | ·795          | ·183            | ·772         | .154            | 34  |
| 35           | .04093        | ·798          | .184            | .775         | .154            | 35  |
| 36           | 03736         | .801          | ·184            | .778         | .155            | 36  |
| 37           | .03410        | .801          | 185             | .780         | .156            | 37  |
| 38           | .03112        | .806          | .186            | .783         | .156            | 38  |
| 39           | 02840         | ·809          | ·186            | .785         | ·157            | 39  |
| 40           | .02593        | ·s11          | ·187            | .787         | .157            | 40  |

Values of  $c^{-n}$ , and corresponding Rates of Interest. Carlisle. Carlisle.

|         |               | 3 рег       | R-CENT          | 4 PEF         | -CENT           |     |
|---------|---------------|-------------|-----------------|---------------|-----------------|-----|
| n       | $c^{-n}$      | $i_{\psi}$  | $i_{\psi-\phi}$ | $i_{\psi}$    | $i_{\psi-\phi}$ | n   |
| 0       | 1.00000       | 3.000       | 3.000           | 4.000         | 4.000           | 0   |
| 1       | ·91368        | .094        | .022            | $\cdot 092$   | .018            | 1   |
| $^{2}$  | ·S34S2        | .181        | .041            | .176          | .035            | 2 3 |
| 3       | ·76275        | .259        | .059            | .253          | .021            |     |
| 4       | ·69691        | •331        | .076            | •323          | .065            | 4   |
| 5       | •63675        | .397        | .090            | ·387          | .077            | 5   |
| 6       | ·58179        | .457        | .104            | .445          | .089            | 6   |
| 7       | ·53157        | .511        | .117            | .498          | .100            | 7   |
| 8       | 48568         | .561        | .128            | .547          | .109            | 8   |
| 9       | 44376         | .607        | 138             | •591          | ·118            | 9   |
| 10      | 40545         | .618        | .148            | .632          | .126            | 10  |
| 11      | ·37045        | .686        | .156            | .669          | .134            | 11  |
| 12      | ·33847        | .721        | .164            | .702          | .140            | 12  |
| 13      | *30926        | .753        | .172            | ·733          | ·147            | 13  |
| 14      | .28256        | .781        | ·178            | .761          | ·152            | 14  |
| 15      | ·25817        | ·808        | .184            | ·787          | ·157            | 15  |
| 16      | .23589        | $\cdot 832$ | .190            | ·810          | $\cdot 162$     | 16  |
| 17      | ·21552        | .854        | .195            | $\cdot 832$   | .166            | 17  |
| 18      | ·19692        | .874        | .199            | .851          | .170            | 18  |
| 19      | ·17992        | .892        | .203            | .869          | ·174            | 19  |
| 20      | 16439         | •909        | .207            | .885          | ·177            | 20  |
| 21      | -15020        | .924        | .211            | .900          | .180            | 21  |
| 22      | $\cdot 13724$ | .538        | .214            | .914          | ·183            | 22  |
| 23      | $\cdot 12539$ | .950        | .217            | •926          | $\cdot 185$     | 23  |
| 24      | ·11457        | .962        | ·219            | .937          | ·187            | 24  |
| 25      | ·10468        | .972        | .222            | ·947          | .189            | 25  |
| 26      | •09564        | $\cdot 982$ | .224            | .957          | .191            | 26  |
| $^{27}$ | .08739        | .991        | .226            | ·965          | .193            | 27  |
| 28      | 07984         | .999        | .228            | .973          | .195            | 28  |
| 29      | .07295        | 4.006       | .229            | .980          | .196            | 29  |
| 30      | .06665        | .013        | .231            | ·987          | ·197            | 30  |
| 31      | .06090        | .019        | .232            | .993          | ·199            | 31  |
| 32      | .05564        | .025        | .234            | .998          | ·200            | 32  |
| 33      | 05084         | .030        | .235            | 5.003         | .201            | 33  |
| 34      | .04642        | .035        | .236            | .008          | ·201            | 34  |
| 35      | 04244         | .039        | ·237            | .012          | .202            | 35  |
| 36      | .03878        | .043        | .238            | .016          | .203            | 36  |
| 37      | 03543         | .046        | .239            | .019          | .204            | 37  |
| 38      | .03237        | 049         | .239            | 022           | 204             | 38  |
| 39      | ·02958        | $\cdot 052$ | 240             | .025          | .205            | 39  |
| 40      | .02702        | .055        | ·241            | ·0 <b>2</b> 8 | .205            | 40  |

## ACTUARIAL NOTES.

## By ALEXANDER FRASER, F.F.A.

## I .- Double Endowment Assurances.

THE reserve value of a double endowment assurance policy can be readily obtained from the following formula, if values of temporary annuities and ordinary endowment assurance policy values are tabulated.

Taking age at entry x, age at maturity x+t, and duration of policy n years, we have the value of the double endowment assurance policy

$$= {}_{n}V_{x\bar{t}} + {}_{t-n}E_{x+n} - \pi_{x\bar{t}}^{\frac{1}{2}}(1 + a_{x+n}; \overline{t-n-1})$$

$$= {}_{n}V_{x\bar{t}} + {}_{t-n}E_{x+n} - {}_{t}E_{x} \frac{1 + a_{x+n}; \overline{t-n-1}}{1 + a_{x}; \overline{t-1}}$$

$$= {}_{n}V_{x\bar{t}} + {}_{t-n}E_{x+n} - {}_{t}E_{x}(1 - {}_{n}V_{x\bar{t}})$$

$$= {}_{n}V_{x\bar{t}}(1 + {}_{t}E_{x}) + ({}_{t-n}E_{x+n} - {}_{t}E_{x}) . . . . (1)$$

That is, the policy may be treated as an endowment assurance of  $(1+_tE_x)$ , if to the result brought out is added the increase in value of the pure endowment portion.

Similarly, the value of a half-endowment policy of 1, payable at age x+t, or 2, at death if previous=

$$_{n}V_{\bar{x}\bar{t}}(2-_{t}E_{x})-(_{t-n}E_{x+n}-_{t}E_{x})$$
 . . . (2)

### II.—Annuities-Certain and Income Tax.

It sometimes happens that an office is asked to quote for the purchase of an annuity-certain where income tax is deductible from the whole annual payment instead of from the interest portion alone, and it is required to find the reduced price the office can afford to allow.

Assume i to be the rate of interest realized and t the proportion of payments deducted for tax, or one-thirtieth when income tax is at 8d, in the £1.

An obvious and very close approximation to the deduction

from the ordinary price is to deduct the present value of all that would be paid as tax on the capital. Since the successive repayments of capital are  $v^n$ ,  $v^{n-1}$ ,  $v^{n-2}$ , &c., and these have to be discounted by v,  $v^2$ ,  $v^3$ , &c., it follows that the value of each payment of capital is  $v^{n+1}$ , and the total value of the tax is=

$$tnv^{n+1}$$
 . . . . . . . . . (3)

This does not, however, take into account the altered capital invested, and the correct formula is obtained as follows: where A is the price and the accented values are calculated at rate j = (1-t)i.

The interest in the first payment is iA; of the balance (1-iA) the tax accounts for t(1-iA), and (1-t)(1-iA) goes to repay the capital. Similarly, in the second year the repayment of capital is  $(1-t)(1-iA)\{1+(1-t)i\}$ , or (1-t)(1-iA)(1+j), and we have

$$(1-t) (1-iA)s'_{\bar{n}} = A$$

$$A = \frac{(1-t)s'_{\bar{n}}}{1+(1-t)is'_{\bar{n}}}$$

$$= \frac{(1-t)s'_{\bar{n}}}{1+js'_{\bar{n}}}$$

$$= (1-t)a'_{\bar{n}} . . . . . . . . (4)$$

Taking the case of annuity for 20 years at 5 per-cent, with tax at 8d., the exact value is 12·2188 and the approximate value 12·2229, a difference of about '034 per-cent.

#### THEINSTITUTE OF ACTUARIES.

## Mr. Chisholm's Prize.

At the monthly meeting of the Institute of Actuaries, held on the 31 January, the President made the following announcement in reference to this matter:

The meeting will remember that Mr. James Chisholm was good enough, a short time ago, to offer three prizes for essays dealing with the appropriate relations between the State and the actuarial profession, regarded both from an historical and a prospective aspect. Unfortunately, only two essays have been presented, and the adjudicators were regretfully unable to award more than the first That prize was secured by Mr. John Nicoll, of the Life Association of Scotland, Edinburgh, for an essay displaying eareful enquiry and thoughtful treatment; and, in making this public announcement, I very gladly, on behalf of the Institute and of Mr. Chisholm, congratulate Mr. Nicoll upon his success, and I accompany that congratulation with the confident hope and prediction that the essay marks but the commencement of a career honourable to himself and of profit and usefulness to the profession in which he has gained this coveted distinction. To the gentleman who failed, with whose name I am necessarily unacquainted, I cordially bid him be of good hope and courage for the future, for no honest work is ever lost. The apparently difficult character of the subject, requiring some original thought and research, lying somewhat remote from the ordinary range of actuarial work, no doubt explains to some extent the paucity of essays; still, with every allowance, I cannot avoid expressing my deep disappointment that the younger members, with this opportunity of distinction before them, did not at all events make a larger attempt. An honourable failure is often as beneficial to the progress of capacity, and certainly as helpful by way of moral and mental discipline, as success. I venture very earnestly to urge upon the younger members the vast importance of seizing opportunities of this description, and entering into such competitions, since they afford a valuable means of securing a distinctive status in the profession, and will aid them—looking especially to the broadening and deepening stress which the future development of assurance business will entail upon the actuary—to cultivate a wider scheme of study than mere mathematical processes, and thus fit them to become, in due time, larger-minded and more capable professional advisers.

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES.

Some Observations on Industrial Assurance. By C. H. E. Rea, Actuary to the Pearl Life Assurance Company, Limited.

[Read before the Institute, 28 March 1898.]

THE almost phenomenal advances that have recently been made in the business of industrial assurance have brought this branch of our provident system into a position of national importance. Ten years ago, the business of our industrial companies was, roughly speaking, as under:—

Policies in Force. 9,000,000 Sum Assured, £83,750,000 Annual Premium. £4,000,000

whereas to-day it is, approximately,

17,500,000

£165,500,000

£7,500,000

Beyond the business of these companies, a very large number of industrial assurance contracts exist in the societies, clubs, and various orders registered under the Friendly Societies Act of 1875. Much useful information on this subject is contained in Mr. Sutton's report of 1896, on "The Sickness and Mortality Experience of Friendly Societies", and the monetary tables, which have been based upon this experience, should prove of considerable value to those engaged in industrial assurance.

The business, as now transacted by our industrial companies, differs in almost every important feature from that

of the ordinary life offices. Applying, as it does, to the poorer classes of the community, the policies are almost invariably taken out as a provision against funeral expenses, and the amounts assured are, comparatively-speaking, very small. The benefit per policy is known to vary in different places directly with the local cost of funerals, but, taking the facts altogether, the average present benefit, per policy in force, approximates to £9. The premiums on these policies are payable weekly, and, being naturally of very small amount, it is found necessary in the tables of rates to make the sum assured the varying factor, instead of the premium as in the case of ordinary assurances. The premiums being collected weekly, it follows that all the routine of the work, both as affecting the indoor and the outdoor staff, has to be conducted under the same periodical arrangement. weekly system of business appears to be absolutely necessary to the requirements of the classes concerned, as all attempts that have been made to simplify the labour in this respect have been unsuccessful, and this essential feature industrial assurance remains the one cause of the seemingly high rate of expense attending the conduct of the business. But the assured have the matter in their own hands, and, in the face of the more favourable rates offered where premiums are collectible at less frequent intervals, and all manner of persuasion, they either cannot, or will not, relieve the companies of trouble and expense, and enjoy the corresponding increase of their own assurance benefits: so long as this condition is maintained, there can be but little hope for any material reduction in the expense rate as now existing. Compared with ordinary assurance, the very great difference in the labour occasioned may be estimated from the fact that the average industrial weekly premium is, approximately, 2d., as compared with an average yearly premium of £12 per ordinary policy; an amount that it would require an industrial agent to make, probably, 1,500 separate calls to collect. Consolidate the business, and arrange the calls how you may, it is impossible to bring the expenditure on the two classes of business into anything like uniformity, and it is unreasonable to aver that the industrial collector is more than moderately paid in the performance of an arduous and necessary duty.

One of the most prominent and unavoidable characteristics

of industrial assurance is the exceptionally heavy rate of secession. The average assurant alters his mind many times before finally completing and maintaining a contract. The agent, too, in his anxiety to improve his business, sometimes falls into the error of over-solicitation, and in the early policy years the lapse rate will often vary with the personal equation as associated with the agent: but as the success of his business depends on his being smart, enterprising, and positive in his canvass, there is every excuse for an exceptional filtration at the outset.

The following unadjusted table, taken from an experience I recently examined, demonstrates generally, though roughly, the effect of these secessions; and considering the general facilities for re-joining under industrial tables, I think it is shown that, after the first year, the business rapidly assumes a stable and satisfactory condition.

Percentage of Total Secessions (including Deaths) at Various Ages, according to Duration of Policy—unadjusted.

| Present<br>Age        |      |      | Cos  | IPLETEI | DURA | TION OF | Polic | A IN L | EARS |      |     |
|-----------------------|------|------|------|---------|------|---------|-------|--------|------|------|-----|
| (Central<br>of Group) | 0    | 1    | 2    | 3       | 4    | 5       | 6     | 7      | 8    | 9    | 10  |
| 5                     | 62.6 | 35.2 | 19.9 | 17.6    | 10.6 | .,.     |       |        |      |      |     |
| 10                    | 57.2 | 30.3 | 21.4 | 15.1    | 12.5 | 9.6     | 7.9   | 6.8    | 6.8  | 5.3  |     |
| 15                    | 54.3 | 23.7 | 15.3 | 10.5    | 9.0  | 7.5     | 7:4   | 6.0    | 5.8  | 4.8  | 4.1 |
| 20                    | 52.9 | 24.3 | 16.9 | 11.3    | 9:1  | 8.5     | 8.0   | 6.5    | 7.1  | 5.2  | 5.8 |
| 25                    | 52.7 | 24.7 | 15.2 | 12.2    | 9.1  | 7·8     | 7.8   | 7.5    | 6.8  | 5.5  | 5.  |
| 30                    | 56.5 | 24.7 | 16.0 | 11.2    | 7.6  | 8.2     | 7.5   | 6.8    | 7.8  | 6.2  | 7:  |
| 35                    | 52.7 | 27.4 | 14.9 | 11.9    | 8.5  | 10.3    | 6.7   | 7.6    | 6.3  | 5.3  | 6:  |
| 40                    | 54.1 | 26.3 | 17.3 | 13.8    | 11.3 | 7.8     | 7.9   | 7.0    | 3.9  | 10.0 | 5.7 |
| 45                    | 50.7 | 21.7 | 13.6 | 10.8    | 10.3 | 8.8     | 8.6   | 5.8    | 4.7  | 9.2  | 5.6 |
| 50                    | 46.3 | 21.4 | 13.9 | 10.2    | 10.0 | 8.6     | 8.1   | 7.9    | 4.9  | 7.2  | 8.2 |
| 55                    | 43.1 | 19.6 | 12.5 | 9.6     | 7.6  | 10.7    | 7.7   | 7.3    | 8.2  | 4.1  | 7.0 |
| 60                    | 43.8 | 18.1 | 13.4 | 11.3    | 10.9 | 10.6    | 8.0   | 8.8    | 6.8  | 5.6  | 8.6 |
| 65                    | 37.8 | 19.0 | 12.5 | 11.6    | 11.5 | 12.3    | 10.4  | 10.4   | 7.4  | 8.3  | 7.3 |
| 70                    | 35.8 | 18.8 | 16.2 | 12.6    | 13.9 | 11.6    | 13.9  | 12.5   | 15.1 | 9.8  | 12% |

It has been suggested that industrial companies were indifferent to lapsing, or even encouraged it as a matter of profit to themselves; but the most superficial view of such an argument will suffice to show its unsoundness. The serious rate of lapsing occurs in the early period of assurance, when the office expenses are at their maximum and there cannot possibly have accrued anything in the nature of a profit to the

companies. This is well recognised by all who are responsible for the conduct of industrial assurance, and every influence is brought to bear to prevent the discontinuance of any contract; indeed, the agents have to make good their lapses in account, and their remuneration is specially regulated by the solidity of their business: this disposes of the argument with respect to early lapsing. In the case of more matured assurances, a policyholder is, in most industrial companies, entitled to a paid-up surrender-value policy, which, taking it generally, is equivalent to a present value of about 30 per cent. of the premiums that have been paid; so that the suggestion of a profit accruing to the companies on the lapses of industrial assurance is without foundation.

A large proportion of the industrial assurances now in force is on the lives of infants, the amount assurable in these cases being strictly limited under the provisions of "The Friendly Societies Act, 1875." Generally speaking, there is no benefit under three months, when the amount starts at a very small figure and gradually increases up to £6 at age 6, and to £10 at age 10. These are the maximum benefits permitted to be assured on any infant's life by any one company, or any combination of companies. This system of increasing assurance in infantile cases was initiated by the Prudential Company many years ago, and, as meeting all requirements, it has been followed by other companies, and continues in operation with singular success. An attempt was made some time ago to gain credibility for an unsupported statement that industrial assurance was conducive to childmurder. In 1890, a Select Committee of the House of Lords considered a Bill presented to them on this subject; but the Bill was withdrawn for want of trustworthy support; indeed, all the substantial evidence of experts went to establish a direct negative. Subsequently Dr. Hugh R. Jones, in the Howard Medal Prize Essay, "On the Perils and Protection of Infant Life", read before the Royal Statistical Society in December 1893, showed conclusively that drunkenness is the one chief cause for any marked increase in the rate of infantile mortality. The diagram given in his paper shows that the curves, as applying to "apprehensions for drunkenness", and to "infants' death from suffocation in bed", are remarkable in their significant identity

throughout each day of the week; arriving at a pronounced maximum on the Saturday night. The thrifty industrial classes are widely distinct from these drunkards, and if those professing an interest in the moral condition of the lower classes would direct their energies to alleviate those social conditions which tend to encourage this vice, instead of vainly continuing a feeble attempt to brand our working classes with attributes baser than those of the brute creation, they would be doing some service to the country.

Medical examinations are dispensed with in nearly all cases of industrial assurance, as they would necessarily have to be hasty and superficial, and considering the heavy lapse rate in the first years of assurance and the nature of the deferred benefits, the result would involve an expense far out of proportion to any value they could possibly afford. In fact, the cost, even at the lowest possible rate, would provide for a very considerable increase in the death claims over the first five years; and quite apart from this, the delay and inconvenience that would be occasioned would be fatal to the extension of the business. So long as a company is protected from absolute fraud by the personal attention of an officer whose interests are entirely dependent on his judgment and discretion, the effect of medical examination in industrial assurance is reduced to a minimum: indeed, it is shown by experience that the opinion of a well-trained and expert official is an excellent, if not a perfect, substitute for medical examination in these small cases.

I have just made an examination of some 10,000 life claims as part of an experience falling in the year 1897, and taken quite by hazard from various parts of the country, afterwards being grouped into nearest District centres as shown below:

10,188 Claims in 1897 taken at random, and afterwards summarized as under:

| Name of Nearest<br>Central Division  | No, of<br>Claims  | Total   | Durati        | on  | Aver                      | age Di                       | ıration  |
|--|---|---|---------------|---|---------------------------|------------------------------|--|
| North London . South ,, . Portsmouth . Plymouth . Cardiff Norwich . Birmingham . Liverpool . Leeds . Newcastle-on-Tyne Glasgow . | 2,316<br>1,501<br>189<br>766<br>835<br>462<br>1,141<br>812<br>771<br>862<br>533 | Years<br>16,230<br>9,716<br>771<br>4,001<br>3,715<br>2,748<br>5,035<br>4,473<br>4,010<br>4,726<br>2,736 | $\frac{9}{2}$ | Days<br>12<br>18<br>12<br>11<br>11<br>11<br>17<br>12<br>14<br>29<br>23<br>6 | Years 7 6 4 5 4 5 5 5 5 5 | M'ths 0 3 0 2 5 11 3 6 2 5 1 | Days<br>2·8<br>8·5<br>28·9<br>20·5<br>11·7<br>12·1<br>27·1<br>3·3<br>4·8<br>23·9<br>18·1 |

The average duration in connection with these particular cases is found to be 5.71 years; whereas the average duration of the policies in force in the same company is found to be 5.24 years. The very slight difference existing between these two average durations is somewhat curious and interesting as compared with the experience of ordinary assurance where the average duration of claims would, no doubt, be far in excess of the average duration of the whole contracts.

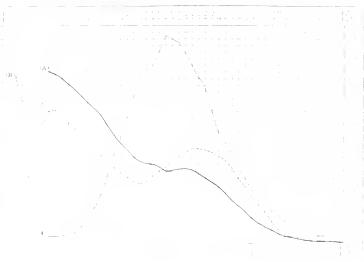
In a valuable paper read before the Institute as far back as the 24th of April 1871, Sir Henry Harben (then Mr. Harben) gave a most interesting diagram, showing the percentage of sums assured at different periods of age, both as relating to the experience of his own company and that of six ordinary offices. The distinct features, as shown by the curves, are remarkable, and I have since made several comparisons on the same lines. In the diagram here following I have taken industrial assurance of English companies generally, and also several ordinary offices combined; and the results follow a close resemblance in their characters to the diagram of 1871.

It is a somewhat curious fact, as I noticed in the several distinct curves formed from the numbers at "entry age" of the individual offices at different dates, that in their earlier

—Total Sum £62,871,537).

| Present<br>Ages taken<br>Two<br>Together. | \begin{cases} \frac{1}{\pi} & \frac{1}{2} & | 4 7              |    | 70<br>-<br>70 | \$1<br>\$2 | \$8<br>-<br>54 | \$ - 5         | \7<br>-<br>>> | \$9<br>-<br>90 | 11<br>12 | 98<br>-<br>94 | .0<br>+<br>96 | 97  | 90  <br>100 | Present<br>Ages taken<br>Two<br>Together, |
|---|---|------------------|----|---------------|------------|----------------|----------------|---------------|----------------|----------|---------------|---------------|-----|-------------|---|
| 6.5                                       |   | -                |    |               |            |                |                |               |                |          |               |               |     |             | 6/2                                       |
| 6.0                                       |   |                  |    |               |            |                |                |               | -              |          |               |               |     |             | 6.0                                       |
| 5 5                                       |   | +                |    |               |            |                |                |               | -              |          |               |               |     |             | 5.  |
| -6  | -+  |                  |    | _:            |            |                |                |               | -              |          |               |               |     |             | 16  |
| .4  |   | -                |    |               |            |                |                | _             |                |          |               | _             |     |             | •4  |
| •2  |   | -                |    |               |            |                |                |               |                |          |               |               |     |             |   |
| 5.0                                       | (D)   |                  |    |               |            |                |                |               | 4              |          |               |               |     |             | 510                                       |
| 4.8                                       | (B.)  | 4                | -  |               |            |                |                |               |                |          |               |               |     |             | 4.4                                       |
| •6  | 1   |                  |    |               |            |                |                |               | 1              |          |               |               |     |             | 16  |
| .4  | : \   | 1                |    |               |            |                |                |               | 1              |          |               |               |     |             | .4  |
| -2  | '   |                  |    |               |            |                |                |               |                |          |               |               |     |             | -2  |
| 4.0                                       |   | 7                |    |               |            |                |                |               |                |          |               |               |     |             | 40  |
| 5.8                                       |   | 1                |    |               |            |                |                |               |                |          |               |               |     |             | 318                                       |
| ·6  |   | 4                |    |               |            |                |                |               | 1              |          |               | •             |     |             | -6  |
| -4  |   | -                |    |               |            |                |                |               | Ĭ.             |          |               |               |     |             |   |
| -2  |   |                  |    |               |            |                |                |               |                |          | -             |               |     |             | -4  |
| 3.0                                       |   |                  | -  |               |            |                |                |               | 1              |          |               |               |     |             | -2  |
| 215                                       |   |                  |    |               |            |                |                | •             | -              |          |               |               | i   |             | 2.0                                       |
| **;                                       |   |                  |    | -             |            |                |                | -             |                |          |               | -             |     |             | 211                                       |
| .4  |   |                  |    |               |            |                |                | -             | •              |          |               |               |     |             | પ્  |
| -2  |   |                  |    |               |            |                |                |               | -              |          |               | -             |     |             | - 1                                       |
| 2.0                                       |   |                  |    |               | -          |                | -              | _             |                | -        |               | _             |     |             | -2  |
| 1.8                                       |   |                  |    |               |            | -              | -              | -             |                |          | -             | -             | -   |             | 210                                       |
| .0  |   |                  |    |               |            | -4             | -              | İ             | +-             |          | -             |               | -   |             | 1 `                                       |
|   |   | -                | -+ | -             | -          | -              |                | -             | +              | +        |               | -             | -   |             | 16  |
| *4  | 1   | +                |    |               |            |                | -              |               |                |          | -             | -             |     |             | . 1                                       |
| -2  |   | -+               |    | -             |            | -              |                |               |                |          |               |               |     |             | -3  |
| 1.0                                       | -   | +                | 1  |               |            |                |                |               |                |          | -             | -             |     |             | 1.0                                       |
| .8  |   | 1 +-             | 1  | 1             | -          |                |                | -             | e mayor        | -        | -             |               |     |             | ->  |
| -6  |   | -                |    | ,             | 1          |                | -              | -             | -              |          |               | -             | -   |             | 113                                       |
| -4  | +++1  | >                |    | 1             |            |                |                |               | -              | -        |               |               |     |             | •4  |
| -2  | -+  |                  |    | 1             | ,          |                | /              |               |                |          |               |               |     | _           | 1   |
| -   |   |                  |    |               | /          | -              | ==             | _             | 1              | -        | -             | 7 -           | -   |             |   |
| Dungana                                   | - 1   |                  |    |               |            |                |                |               | -              | -        | 7.            | _             | -   |             |   |
| Present<br>ges taken<br>Two<br>Together.  |   | 3 75<br><br>4 76 |    | 79<br>+<br>50 | 1 - 1      | \$3<br>+<br>8± | \$5<br>-<br>86 | ;;            | S.J.           | 91<br>92 | 901<br><br>94 | 95<br>        | :57 | 99  <br>10e | Present<br>Ages taken<br>Two<br>Together, |

- A rate Industrial Wisher Life Associances (Combaniel Experience of Fire Companies Tetra Sum C137-217-28 of
- 1. 4. . . . be Gisterry Whole Life Assurances (Combined Experience of Eight Companies Lotal Size Vo2871 5.7)



history there is a marked disturbance at the decennial ages 20, 30, 40, &c., and at 50 especially. It is well known that the industrial classes have a preference for round figures in these matters. This was brought to my mind by what was told me some years ago by an energetic worker in the cause of industrial assurance. He said it was not uncommon, after having successfully demonstrated to a matron the advisability of assuring, and having further satisfied her mind with regard to the special advantages of his own office in particular, that in filling in the proposal card at the question of age she would ask, "How much do you get for age 55?" Reply, so much. "Then what would you get at age 50?" and the reply would, of course, be somewhat more. "Oh, then I think you had better put me down at 50 please", would be the prompt reply. Again, it is possible that at these particular periods there is an awakening to the fact of life's shortening, and a corresponding strengthening of the opportunities and arguments that suggest themselves to the agent. Moreover, it has been known for females to be told that they look considerably younger than their age, more with a view to compliment than a strict regard for veracity; and these people become at once convinced of a desirable adjustment in their age from an insurance point of view: the adjustment generally consisting of a rejection of the odd units. agent, being a courteous and careful man, does not endanger the situation and his prospects by word or sign of misgiving; and so in the earlier experience of industrial companies the average age error was probably of measurable consideration From a recent observation of claims paid, however, I found an age error per adult policy of under two months, which I think points to the more exact and careful working of recent years.

The irregularity in the curves to which I before referred, although still existing, is reduced in the combined results almost to the point of extinction except about the age 50, and probably much may be explained by the fact that at this period there is a tendency to the same character and law as is seen to relate to ordinary assurance. It will be noticed that I have not completed the drawing as extending to the sums assured on infantile lives, as these are in the nature of small temporarily increasing assurances, and the results would convey no satisfactory meaning.

In the case of an office transacting a large industrial assurance business, the matter of keeping the most satisfactory classification at the least cost is one that has called, and will continue to call, for the best consideration of the practical actuary engaged in this class of business. The details of the various methods of arrangement for the different purposes of the office requirements would be far too voluminous to embody in these observations. I will therefore simply and briefly refer to some methods of classification for valuation purposes.

Some time ago I had the requisite particulars copied from the original proposals on to small cards, and dealt with these in the usual manner of classification. In another case, where the proposals themselves were upon cards, I tried to employ the original data, the difficulty at the outset resting in the fact that the office classification was under what may be called a topographical or agency order, which could not be disturbed without serious consequences and a risk in getting the proposal cards back again into their proper order. However, acting on a suggestion, I tried a system in the general registry department of having a small label (smaller than an ordinary postage stamp) stuck on the corner of the proposal cards, indicative of their present agency positions. These particulars are invariably given at the outset, but the labels were brought into use for transfers and removals: and with this means of sorting back into agency classification, I dealt with the original proposals direct, but, although the process of summarising was performed very expeditiously, the experiment was not an unqualified success.

The method I last adopted for continuous classification, where the amount assured was constant and exactly *pro rata* with the premium, was briefly as follows:

Starting from the data of a valuation, the classification is made weekly. Proposals are received in card form, and after they have passed through the various stages of the office up to the point of acceptance and issue of the policies, they then immediately go through a distinct valuation department. Here they are divided into classes of assurance according to the Company's tables, and then sub-sorted in pigeon-holes according to age next birthday, as shown on proposals, this being practically exact age at end of year. These sub-

divisions are then totalled and checked, the results being entered on card slips and transferred to register "A," as shown hereunder.

## Exact Age at December 1897

Register (" $\Lambda$ ").

New Business and Secessions (New) for the Year 1897.

|             | NEW BU                | SINI     | 288       |   |             | ]          | REINST.               | ATEM | ENTS          | OF | , NE | W         | SECES                 | SIONS FR           | OM NEV      |
|-------------|-----------------------|----------|-----------|---|-------------|------------|-----------------------|------|---------------|----|------|-----------|-----------------------|--------------------|-------------|
| 'ARTICULARS | No.<br>of<br>Policies |          | ekly      |   | Sum         | _          | No.<br>of<br>Policies |      | ekly<br>niums |    | Sum  | s-1<br>—— | No.<br>of<br>Policies | Weekly<br>Premiums | Sum         |
| euding:     |                       | ε.<br>—— | <u>d.</u> | ŧ | s. <i>c</i> | <i>l</i> . |                       | 8.   | <i>d</i> .    | £  | 8.   | d.<br>-   |                       | s. d.              | & <u>s.</u> |

The sum assured is filled in at the time of summation, it being at once determined from the weekly premium and the entry age. The column here exists only in case of any exceptional note to be recorded.

The total premium at all ages of entry is then checked, as against the new premium debit of accounts, and the number of policies is further verified by the record of the policy stamping machine.

These entries having been duly made, the cards are again put into numerical order and forwarded to the general registry department for proper distribution under their topographical system, it being found strongly advisable to keep each agent's business under strict review in the manner that this arrangement affords. The proposal cards once in this department, remain there throughout their existence, only altering positions to correspond with any change of residence on the part of the assured.

The lapses of each week's account are extracted regularly from the latter agency registration, and these, too, are sent through the valuation department, classified and entered under age and year of entry, as per register B hereunder. Age next at Entry.

Register ("B").

SECESSIONS and DEATHS during the year 1897 in respect of POLICIES in force 31 December 1896.

|  |          | Year    | of E<br>1896 | ntry       |    |          | Year    | of E<br>1895 | Intry      | 7  |          | Year    | of E<br>1894 | ntry       | 7  |          | Year    | of E<br>1893 | ntry       | У          | &c. |
|--|----------|---------|--------------|------------|----|----------|---------|--------------|------------|----|----------|---------|--------------|------------|----|----------|---------|--------------|------------|------------|-----|
| Date of<br>Secession                                 | of Poles | y Prems |              | OUR<br>SUR |    | of Poles | y Prems |              | 10U<br>SUR |    | of Poles | r Prems |              | IOU<br>SUR |    | of Poles | 7 Prems |              | IOU<br>SUF | NT<br>RED  | &c. |
| January 1 , 8  | No. (    | Wkly    | £            | s.         | d. | No. (    | Wkly    | £            | s.         | d. | No. (    | Wkly    | £            | 8.         | d. | No. 6    | Wkly    | £            | s.         | <i>d</i> . | &c. |
| ,, 15<br>,, 22<br>,, 29<br>February 5<br>&c. &c. &c. |          |         |              |            |    |          |         |              |            |    |          |         |              |            |    |          | 1       |              |            |            |     |

It will be noted that the lapsing of first year's business goes through register A in all cases. The totals of these registers are brought forward weekly, and the final results entered in register "C," which is shown to give the particulars at each valuation age, as also with respect to each year of entry:

Register ("C").

30 Exact Age December 1896.

Exact Age December 1897.

31

SECESSIONS AND DEATHS OF OLD Policies in Force, 31 December 1896, Brought forward from last Valuation OLD POLICIES REMAINING IN FORCE, 31 December 1897 Business in 1897 No. of Weekly Sum No. of Weekly Sum No. of Weekly Sum Duration Policies Premiums Assured Policies 4 8 1 Premiums Assured Policies Premiums Assured (n) 0 1 3 4 5 &c.

It will be observed from this latter classification that there is every facility for simple and continuous registration: duration 0 for age x being brought forward in the following year's book as duration 1, age x + 1, and so on; the new figures

for duration 0 in all cases coming direct from the new business register "A."

From these results, assuming that the method of valuation adopted is that by which a fixed percentage of the office-premium is reserved for future expenses and profits, we can at once determine the years of negative value as follows:

Policy value 
$$\begin{aligned} &= \mathbf{A}_{x+n} - \mathbf{P}_x (1 + a_{x+n}) \text{ or, } \bar{\mathbf{A}}_{x+n} - \bar{\mathbf{P}}_x \cdot a_{x+n} \text{ &c.} \\ &\quad \mathbf{A}_x - \mathbf{P}_x (1 + a_x) = 0 \\ &\quad \mathbf{A}_x - \mathbf{P}'_x (1 + a_x) = -\mathbf{Q} \\ &\quad \mathbf{A}_{x+n} - \mathbf{P}'_x (1 + a_{x+n}) \text{ is negative} \\ &\quad \mathbf{So long as} \end{aligned}$$
 so long as

where  $P'_{x}$  is the portion of office premium under valuation.

By drawing a dividing line as now found between the positive and the negative portions of the business, which, under present rates for whole-life industrial assurances, comes generally after the fifth year of assurance, and summing the respective totals, we form final valuation schedules with respect to these two great divisions of the business; and, in the result the effect of the negative values is satisfactorily and entirely eliminated.

A complete set of books is kept for each class of business, the registers only varying as regards the special features and requirements of the contracts.

Mr. Schooling's method of substituting joint lives of equal ages in all cases of joint lives is, as far as my experience takes me, the most applicable where the contracts are extremely numerous, and it is important to keep up a continuous system of registration. It is a somewhat curious fact that, in following Makeham's law some time ago, I drew out and adopted a table for equating two joint lives of unequal ages to two joint lives of equal ages exactly similar to that given by Mr. Schooling in his paper.

In cases where the joint lives are comparatively few in number, it may be found more expeditions to employ an equivalent single life according to the table here following, where the older age is given at the top, the younger age in the body, and the single life equivalent in the margin.

[July

Older Age at top.

Single Life Equivalents

| 11   | 12   | 13  | 14  | 15  | 16   | 17   | 18  |
|--|--|---|---|---|--|--|---|
| 10-11   29<br> <br>10 }   29<br>   | 10-12   30   | 10-11   30<br><br>12-13   31<br><br>  | 10 30<br>11-12 31<br>13-14 32<br>   | 10-11 31<br>12-14 32<br>15 33<br>   | 10   31<br>11-12   32<br>13-15   33<br>16   34<br>   | 10-11   32<br>12-14   33<br>15-16   34<br>17   35<br>  | 10   32<br>11-13   33<br>14-15   34<br>16-17   35<br>18   36<br>  |
| 27   | 28   | 29  | 30  | 31  | 32   | 33   | 34  |
| 10-12   38<br>13-16   39<br>17-19   40<br>20-22   41<br>23-24   42<br>25-27   43<br>   | 10 38<br>11-14 39<br>15-17 40<br>18-20 41<br>21-23 42<br>24-26 43<br>27-28 44  | 10-11 39<br>12-15 40<br>16-19 41<br>20-22 42<br>23-25 43<br>26-28 44<br>29 45                         | $\begin{array}{c cccc} 10-13 & 40 \\ 14-17 & 41 \\ 18-20 & 42 \\ 21-24 & 43 \\ 25-27 & 44 \\ 28-29 & 45 \\ 30 & 46 \\ \end{array}$                              | $\begin{array}{c cccc} 10 & 40 \\ 11-15 & 41 \\ 16-19 & 42 \\ 20-22 & 43 \\ 23-25 & 44 \\ 26-28 & 45 \\ 29-31 & 46 \\ \end{array}$                              | 10-12 41<br>13-17 42<br>18-21 43<br>22-24 44<br>25-27 45<br>28-30 46<br>31-32 47   | $\begin{array}{c cccc} 10-14 & 42 \\ 15-19 & 43 \\ 20-23 & 44 \\ 24-26 & 45 \\ 27-29 & 46 \\ 30-32 & 47 \\ 33 & 48 \\ \end{array}$                               | 10-11   42<br>12-16   43<br>17-21   44<br>22-24   45<br>25-28   46<br>29-31   47<br>32-34   48  |
| 43   | 44   | 45  | 46  | 47  | 48   | 49   | 50  |
| 10-16 48<br>17-23 49<br>24-28 50<br>29-33 51<br>34-38 52<br>39-42 53<br>43 54<br>  | 10 48<br>11-19 49<br>20-25 50<br>26-31 51<br>32-36 52<br>37-40 53<br>41-44 54<br>  | 10-13 49<br>14-21 50<br>22-28 51<br>29-33 52<br>34-39 53<br>40-42 54<br>43-45 55<br>                  | 10-16 50<br>17-24 51<br>25-30 52<br>31-36 53<br>37-41 54<br>42-44 55<br>45-46 56<br>  | 10-19 51<br>20-27 52<br>28-33 53<br>34-39 54<br>40-43 55<br>44-46 56<br>47 57<br>   | $\begin{array}{c cccc} 10-12 & 51 \\ 13-22 & 52 \\ 23-29 & 53 \\ 30-36 & 54 \\ 37-41 & 55 \\ 42-44 & 56 \\ 45-47 & 57 \\ 48 & 58 \end{array}$                  | $\begin{array}{ccccc} 10-15 & 52 \\ 16-24 & 53 \\ 25-32 & 54 \\ 33-39 & 55 \\ 40-43 & 56 \\ 44-46 & 57 \\ 47-49 & 58 \\ & \cdots & \cdots \end{array}$           | 10-18 53<br>19-27 54<br>28-35 55<br>36-41 56<br>42-45 57<br>46-47 58<br>48-50 59  |
| 59   | 60   | 61  | 62  | 63  | 64   | 65   | 66  |
| 10-24 61<br>25-37 62<br>38-45 63<br>46-49 64<br>50-52 65<br>53-55 66<br>56-57 67<br>58-59 68<br>   | 10-26 62<br>27-40 63<br>41-46 64<br>47-50 65<br>51-53 66<br>54-56 67<br>57-58 68<br>59-60 69<br>   | 10 62<br>11-28 63<br>29-42 64<br>43-48 65<br>49-52 66<br>53-54 67<br>55-57 68<br>58-59 69<br>60-61 70 | 10-13 63<br>14-30 64<br>31-44 65<br>45-49 66<br>50-53 67<br>54-56 68<br>57-58 69<br>59-60 70<br>61-62 71  | 10-15   64<br>16-33   65<br>34-45   66<br>46-50   67<br>51-54   68<br>55-57   69<br>58-59   70<br>60-62   71<br>63   72   | 10-18 65<br>19-36 66<br>37-47 67<br>48-52 68<br>53-55 69<br>56-58 70<br>59-60 71<br>61-63 72<br>64 73  | $ \begin{array}{c cccc} 10-20 & 66 \\ 21-39 & 67 \\ 40-49 & 68 \\ 50-53 & 69 \\ 54-56 & 70 \\ 57-59 & 71 \\ 60-61 & 72 \\ 62-64 & 73 \\ 65 & 74 \\ \end{array} $ | $\begin{array}{c} 10-22 & 67 \\ 23-42 & 68 \\ 43-50 & 69 \\ 51-54 & 70 \\ 55-57 & 71 \\ 58-60 & 72 \\ 61-62 & 73 \\ 63-65 & 74 \\ 66 & 75 \\ \end{array}$ |
| 75   | 76   | 77  | 78  | 79  | 80   | 81   | 82  |
| $\begin{array}{c cccc} 10-44 & 76 \\ 45-54 & 77 \\ 55-59 & 78 \\ 60-63 & 79 \\ 64-66 & 80 \\ 67-69 & 81 \\ 70-72 & 82 \\ 73-74 & 83 \\ 75 & 84 \\ \end{array}$ | $\begin{array}{c cccc} 10-46 & 77 \\ 47-55 & 78 \\ 56-60 & 79 \\ 61-64 & 80 \\ 65-67 & 81 \\ 68-70 & 82 \\ 71-73 & 83 \\ 74-75 & 84 \\ 76 & 85 \\ \end{array}$ | 10-48 78<br>49-56 79<br>57-61 80<br>62-65 81<br>66-69 82<br>70-72 83<br>73-74 84<br>75-77 85<br>      | $\begin{array}{cccc} 10-11 & 78 \\ 12-49 & 79 \\ 50-57 & 80 \\ 58-63 & 81 \\ 64-67 & 82 \\ 68-70 & 83 \\ 71-73 & 84 \\ 74-75 & 85 \\ 76-78 & 86 \\ \end{array}$ | $\begin{array}{cccc} 10-13 & 79 \\ 14-50 & 80 \\ 51-59 & 81 \\ 60-64 & 82 \\ 65-68 & 83 \\ 69-71 & 84 \\ 72-74 & 85 \\ 75-77 & 86 \\ 78-79 & 87 \\ \end{array}$ | $\begin{array}{c cccc} 10-17 & 80 \\ 18-52 & 81 \\ 53-60 & 82 \\ 61-65 & 83 \\ 66-69 & 84 \\ 70-73 & 85 \\ 74-76 & 86 \\ 77-79 & 87 \\ 80 & 88 \\ \end{array}$ | $\begin{array}{c cccc} 10-21 & 81 \\ 22-54 & 82 \\ 55-62 & 83 \\ 63-67 & 84 \\ 68-71 & 85 \\ 72-74 & 86 \\ 75-77 & 87 \\ 78-80 & 88 \\ 81 & 89 \\ \end{array}$   | 10-25 82<br>26-56 83<br>57-63 84<br>64-68 85<br>69-72 86<br>73-76 87<br>77-79 88<br>80-82 89<br>  |

for Two Joint Lives.

Older Age at top.

| 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  |
|---|---|---|---|---|---|---|---|
| 10-11 33<br>12-14 34<br>15-16 35<br>17-19 36  | 10 33<br>11-12 34<br>13-15 35<br>16-18 36<br>19-20 37   | 10-11 34<br>12-14 35<br>15-16 36<br>17-19 37<br>20-21 38                                      | 10-12 35<br>13-15 36<br>16-18 37<br>19-20 38<br>21-22 39<br>  | 10 35<br>11-13 36<br>14-16 37<br>17-19 38<br>20-21 39<br>22-23 40<br>                                 | 10-11 36<br>12-15 37<br>16-18 38<br>19-20 39<br>21-23 40<br>24 41<br>                                 | $\begin{array}{cccc} 10 & 36 \\ 11-13 & 37 \\ 14-16 & 38 \\ 17-19 & 39 \\ 20-22 & 40 \\ 23-24 & 41 \\ 25 & 42 \\ \end{array}$ | 10-11 37<br>12-14 38<br>15-17 39<br>18-20 40<br>21-23 41<br>24-26 42                                  |
| 35  | 36  | 37  | 38  | 39  | 40  | 41  | 42  |
| 10-14 43<br>15-19 44<br>20-23 45<br>24-26 46<br>27-30 47<br>31-33 48<br>34-35 49                      | 10-11 43<br>12-16 44<br>17-21 45<br>22-25 46<br>26-29 47<br>30-32 48<br>33-36 49  | 10-13 44<br>14-18 45<br>19-23 46<br>24-27 47<br>28-31 48<br>32-35 49<br>36-37 50              | 10-16 45<br>17-21 46<br>22-25 47<br>26-29 48<br>30-33 49<br>34-37 50<br>38 51   | 10-12 45<br>13-18 46<br>19-23 47<br>24-28 48<br>29-32 49<br>33-36 50<br>37-39 51                      | 10-15 46<br>16-21 47<br>22-26 48<br>27-30 49<br>31-35 50<br>36-38 51<br>39-40 52                      | 10-11 46<br>12-18 47<br>19-23 48<br>24-28 49<br>29-33 50<br>34-37 51<br>38-41 52  | 10-13 47<br>14-20 48<br>21-26 49<br>27-31 50<br>32-35 51<br>36-39 52<br>40-12 53                      |
| 51  | 52  | 53  | 54  | 55  | 56  | 57  | 58  |
| 10-20 54<br>21-29 55<br>30-37 56<br>38-42 57<br>43-16 58<br>47-49 59<br>50-51 60                      | $\begin{array}{ccccc} 10 & 54 \\ 11-22 & 55 \\ 23-31 & 56 \\ 32-39 & 57 \\ 40-44 & 58 \\ 45-47 & 59 \\ 48-50 & 60 \\ 51-52 & 61 \\ \end{array}$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{cccccccccccccccccccccccccccccccccccc$  | 10-16 57<br>17-28 58<br>29-38 59<br>39-44 60<br>45-48 61<br>49-51 62<br>52-53 63<br>54-55 64          | $\begin{array}{cccccccccccccccccccccccccccccccccccc$  | 10-20 59<br>21-32 60<br>33-42 61<br>43-47 62<br>48-50 63<br>51-53 64<br>54-55 65<br>56-57 66                                  | 10-22 60<br>23-34 61<br>35-43 62<br>44-48 63<br>49-51 64<br>52-54 65<br>55-56 66<br>57-58 67          |
| 67  | 68  | 69  | 70  | 71  | 72  | 73  | 74  |
| 10-24 68<br>25-44 69<br>45-51 70<br>52-55 71<br>56-58 72<br>59-61 73<br>62-63 74<br>64-66 75<br>67 76 | 10-27 69<br>28-46 70<br>47-52 71<br>53-56 72<br>57-59 73<br>60-62 74<br>63-64 75<br>65-67 76<br>68 77   | $\begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{ccccc} 10-30 & 71 \\ 31-49 & 72 \\ 50-54 & 73 \\ 55-58 & 74 \\ 59-61 & 75 \\ 62-64 & 76 \\ 65-66 & 77 \\ 67-69 & 78 \\ 70 & 79 \\ \end{array}$ | 10-33 72<br>34-50 73<br>51-55 74<br>56-59 75<br>60-62 76<br>63-65 77<br>66-67 78<br>68-70 79<br>71 80 | 10–36 73<br>37–51 74<br>52–56 75<br>57–60 76<br>61–63 77<br>64–66 78<br>67–68 79<br>69–71 80<br>72 81 | 10-39 74<br>40-52 75<br>53-57 76<br>58-61 77<br>62-64 78<br>65-67 79<br>68-69 80<br>70-72 81<br>73 82                         | 10-42 75<br>13-53 76<br>51-58 77<br>59-62 78<br>63-65 79<br>66-68 80<br>69-70 81<br>71-73 82<br>74 83 |
| 83  | 84  | 85  | 86  | 87  | 88  | 89  | 90  |
| 10-26 83<br>27-57 84<br>58-64 85<br>65-70 86<br>71-74 87<br>75-77 88<br>78-80 89<br>81-83 90<br>      | 10-29 84<br>30-58 85<br>59-66 86<br>67-71 87<br>72-75 88<br>76-79 89<br>80-82 90<br>83-84 91  | 10-32 85<br>33-60 86<br>61-68 87<br>69-73 88<br>74-77 89<br>78-81 90<br>82-84 91<br>85 92<br> | 10-39 86<br>40-62 87<br>63-70 88<br>71-75 89<br>76-79 90<br>80-83 91<br>84-86 92<br>  | 10-46 87<br>47-64 88<br>65-71 89<br>72-77 90<br>78-82 91<br>83-85 92<br>86-87 93<br>                  | 10-49 88<br>50-65 89<br>66-73 90<br>74-79 91<br>80-83 92<br>81-87 93<br>88 94<br>                     | 10-51 89<br>52-68 90<br>69-76 91<br>77-81 92<br>82-85 93<br>86-89 94<br>  | 10-55 90<br>56-71 91<br>72-78 92<br>79-84 93<br>85-87 94<br>88-90 95                                  |

With regard to endowment assurances generally, I have followed Mr. Schooling's method of taking the average age under each period to run: with results that have in all cases been extremely satisfactory. In dealing, however, with endowment assurances where the contracts are not so great in number, I sometimes found it convenient to classify according to an equivalent single whole-life age; and a table as exhibiting the relation of these equivalents is given hereunder:—

Single Whole-Life, equivalent to Endowment Assurance.

| æ   |          |                 | MATURING | AGE OF E | NDOWMENT |                 |     |
|-----|----------|-----------------|----------|----------|----------|-----------------|-----|
|     | 70       | 65              | 60       | 55       | 50       | 45              | 40  |
| 15  | 18       | 20              | 24       | 28       | 33       | 39              | 45  |
| 16  | 19       | 22              | 25       | 29       | 34       | 40              | 47  |
| 17  | 20       | 23              | 26       | 30       | 36       | $\frac{10}{42}$ | 48  |
| 18  | 21       | 24              | 27       | 31       | 37       | 43              | 50  |
| 19  | 22       | 25              | 28       | 33       | 39       | 45              | 51  |
| 20  | 23       | 26              | 29       | 34       | 40       | 46              | 52  |
| 21  | 24       | 27              | 31       | 36       | 41       | 47              | 54  |
| 22  | 26       | 28              | 32       | 37       | 43       | 49              | 55  |
| 23  | 27       | 29              | 33       | 38       | 11       | 50              | 57  |
|     |          | 31              |          |          |          |                 |     |
| 24  | 28<br>29 | $\frac{31}{32}$ | 35       | 40<br>41 | 46       | 51<br>53        | 58  |
| 25  |          |                 | 36       |          | 47       |                 | 60  |
| 26  | 30       | 33              | 37       | 43       | 48       | 54              | 61  |
| 27  | 31       | 34              | 39       | 44       | 50       | 56              | 63  |
| 28  | 32       | 36              | 40       | 45       | 51       | 57              | 65  |
| 29  | 33       | 37              | 41       | 46       | 52       | 59              | 67  |
| 30  | 34       | 38              | 43       | 48       | 53       | 60              | 69  |
| 31  | 36       | 39              | 11       | 49       | 55       | 62              | 71  |
| 32  | 37       | 41              | 45       | 50       | 56       | 64              | 74  |
| 33  | 38       | 42              | 46       | 51       | 57       | 65              | 76  |
| 34  | 39       | 43              | 47       | 53       | 59       | 67              | 79  |
| 35  | 40       | 44              | 48       | 54       | 60       | 69              | 83  |
| 36  | 41       | 45              | 50       | 55       | 62       | 71              | 86  |
| 37  | 42       | 46              | 51       | 56       | 64       | 74              | 90  |
| 38  | 43       | 47              | 52       | 58       | 65       | 76              | 95  |
| 39  | 45       | 48              | 53       | 59       | 67       | 79              | 99  |
| 40  | 46       | 49              | 54       | 61       | 69       | 83              | 100 |
| 41  | 47       | 51              | 55       | 62       | 71       | 86              |     |
| 42  | 48       | 52              | 57       | 64       | 74       | 91              |     |
| 43  | 49       | 53              | 58       | 66       | 76       | 95              |     |
| 44  | 50       | 54              | 60       | 67       | 79       | 99              |     |
| 45  | 51       | 55              | 61       | 69       | 83       | 100             |     |
| 46  | 52       | 56              | 63       | 72       | 86       |                 |     |
| 47  | 53       | 58              | 64       | 74       | 91       |                 |     |
| 48  | 54       | 59              | 66       | 77       | 95       |                 | l   |
| 49  | 55       | 60              | 68       | 80       | 99       |                 |     |
| 50  | 57       | 62              | 70       | 83       | 100      |                 |     |
| 51  | 58       | 63              | 72       | 86       |          |                 |     |
| 52  | 59       | 65              | 74       | 91       |          |                 |     |
| 53  | 61       | 67              | 77       | 95       |          |                 |     |
| 54  | 62       | 69              | 80       | 99       |          |                 |     |
| o r |          | O.              | 00       | 0.0      |          |                 | ,   |

Single Whole-Life, equivalent to Endowment Assurance—continued.

|      |     |     | MATURING | AGE OF E | NDOWMENT |    |    |
|------|-----|-----|----------|----------|----------|----|----|
| x    | 70  | 65  | 60       | 55       | 50       | 45 | 40 |
| 55   | 63  | 71  | 83       | 100      |          |    |    |
| 56   | 65  | 73  | 87       |          |          |    |    |
| 57   | 67  | 75  | 91       |          |          |    |    |
| 58   | 68  | 77  | 95       |          |          |    |    |
| 59   | 70  | 80  | 99       |          |          |    |    |
| 60   | 72  | 83  | 100      |          |          |    |    |
| 61   | 7-4 | 87  |          |          |          |    |    |
| 62   | 76  | 91  |          |          |          |    |    |
| 63   | 78  | 95  |          |          |          |    |    |
| 64   | 81  | 99  |          |          |          |    |    |
| 65   | 84  | 100 |          |          |          |    |    |
| 66   | 87  |     |          |          |          |    |    |
| 67   | 91  |     |          |          |          |    | 1  |
| 68   | 96  |     |          |          |          |    |    |
| 69   | 99  |     |          |          |          |    |    |
| 70 - | 100 |     |          |          |          |    |    |

I may now mention, as probably attaching some special value to these two latter tables for equivalents, that they are copied direct from the manuscripts calculated and used somewhat extensively by my late grandfather, W. S. B. Woolhouse. The methods of construction are not given in the MS., but the equivalents appear to have been derived from the continuous annuity-values calculated on the English Life Table (No. 3), at 3 per-cent. The values of the continuous life annuities, premiums, and assurances, with their respective logarithms, I have myself calculated on the basis of this experience; and, as the results are of some value in connection with industrial assurance, I give them at the end of this paper.

Recently I have made a departure in connection with the claim records. When each claim comes to the office, the proposal card is taken out of its place in the registry department and attached to the claim papers, and, at the same time, a copy card, called a "Claim card", with a black corner instead of the corner cut off, is filled in with certain particulars, as shown in this diagram:

| D                                 | <u>D</u>                               |
|-----------------------------------|--|
| VALUATION DEPARTMENT.             | CLAIM CARD.                            |
| Name                              |  |
| Policy No.                        | Age at Entry                           |
| Weekly ( Sum<br>Preminm ) Assured |  |
| Date of Decease                   | Cause Primary                          |
| Date of Entry                     | Secondary                              |
| Duration                          | Date of Lapse \( \) from \( \pi/c^2 \) |

As soon as the assurance has been removed from the agent's account and register (which is not always immediately the case, in the event of a claim being sent direct to the chief office without notice to the district representative) these cards are taken from the box and again transferred to the valuation department, where they are sorted, to ascertain durations of policies, mortality experience, and the causes of death. Probably, other features of value may present themselves in the course of the work, and possibly the results of these classifications may prove of interest in the future.

Industrial assurance has generally been regarded as having a close relation to term assurance, and in fact, the contracts have been frequently compared to fire rather than to ordinary life risks. This generalisation is, no doubt, sound in the early life of a company where the new assurances bear a large ratio to the total assurances in force, but with an increasing average policy duration, the argument tends to give way, and industrial companies are under as great a necessity to make a strict reserve for their liabilities as ordinary offices.

The outdoor staff now engaged in this class of business throughout the kingdom comprises men of intelligence and unquestionable character. The honest, straightforward, and indefatigable manner in which they perform their duties is certainly one of the principal factors in the enormous results and successes that are achieved in industrial assurance at the present day.

One cannot be closely associated with the business, and hear the constant voice of gratitude from the masses in their time of trouble, without appreciating the vast amount of good that is daily being effected through the medium of our industrial institutions.

I have so far avoided making mention of individual companies, but it is impossible to conclude any observations on the subject of this paper without referring to the immense advantage that all concerned have derived from the wide experience of the Prudential, and the courtesy of its officials.

In conclusion, I venture to express a hope that some members of the Institute may extend further consideration to the scientific side of this important subject. For my own part, I shall be only too happy to continue my enquiries in any direction that may be thought of interest or value.

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English Life Table (No. 3). 3 per-cent Interest.

| x               | $a + (\cdot 5 - \frac{\mu + \delta}{2})$                 |                    |                    |                      |                         |                      |                           |
|-----------------|--|--------------------|--------------------|----------------------|-------------------------|----------------------|---------------------------|
|                 | $a_x + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | P                  | Ā                  | ā                    | $\overline{\mathbf{P}}$ | Ā                    | x                         |
| 1               | 21.8385  | .016232            | *35448             | 3392229              | ·2103720                | .5495917             | 1                         |
| $\hat{2}$       | 22.9968  | .013926            | .32025             | 3616674              | 1438264                 | 5054891              | 2                         |
| 3               | 23.5266  | 012946             | .30458             | 3715592              | 1121356                 | 4837014              |                           |
| 4               | 23.7980  | 012462             | 29657              | 3765405              | 0955877                 | 4721272              | Ì                         |
| 5               | 23.9329  | .012225            | 29258              | 3789953              | 0872489                 | .4662446             | ١ ;                       |
| 6               | 23.9689  | .012162            | .29151             | 3796481              | .0850050                | 4646535              | l (                       |
| 7               | 23.9377  | 012216             | .29242             | 3790824              | .0869290                | 4660071              | 1 :                       |
| 8               | 23.8642  | 012345             | •29460             | 3777468              | 0914911                 | 4692327              |                           |
| 9               | 23.7505  | 012546             | 29797              | 3756728              | 0985053                 | 4741725              |                           |
| 10              | 23.6041  | .012807            | .30230             | 3729874              | 1074474                 | 4804381              | 10                        |
| 11              | 23.4318  | .013118            | .30738             | 3698057              | 1178676                 | 4876756              | 1:                        |
| 12              | 23.2400  | .013471            | 31307              | 3662361              | 1293998                 | 4956415              | 1                         |
| 13              | 23.03.45   | 013471             | 31912              | 3623788              | 1415752                 | 5039540              | l i                       |
| 14              | 22.8205  | 014261             | 32544              | 3583252              | 1541500                 | 5124709              | 1                         |
| 15              | 22.6023  | 014201             | 33189              | 3541527              | 1668444                 | 5209942              | 1                         |
| 16              |  | 015116             | 33835              | 3499338              | 1794369                 | r5293662             | 1                         |
|                 | 22:3838  | 015551             | 31473              | 3457265              | 1917583                 | 5374791              | 12                        |
| 17              | 22.1680  |                    | 35097              | 3415809              | 2036855                 | 5452700              | 1                         |
| $\frac{18}{19}$ | $21.9574 \\ 21.7537$                                     | $015984 \\ 016410$ | 35698              | 3375331              | 2151086                 | 5526439              | 19                        |
|                 |  |                    |                    | 1                    |                         |                      | 20                        |
| 20              | 21.5581  | 016828             | 36278              | 3336105              | .2260325                | 5596433              | 20                        |
| 21              | 21.3711  | 017233             | *36829             | 3298268              | 2363609                 | .5661899             | 2:                        |
| 22              | 21.1809  | .017653            | ·37391             | 3259445              | 2468185                 | 5727671              | 2:                        |
| $\frac{23}{24}$ | 20·9872<br>20·7897                                       | ·018089<br>·018542 | 37964<br>38548     | 3219546<br>3178482   | 2574146<br>2681566      | ·5793720<br>·5860019 | 24                        |
|                 |  |                    |                    | 3136163              | 2790506                 |                      | 2:                        |
| 25              | 20.5881  | .019013            | 39144              | 3092489              | 2790306                 | 5926652              | $\frac{2\epsilon}{2\ell}$ |
| 26              | 20.3821  | 019504             | 39753              |                      |                         | 5993699              |                           |
| 27              | 20.1715  | .020016            | 410375             | 3017382              | 3013773                 | 6061125              | $\frac{27}{28}$           |
| 28 .<br>29 .    | 19·9563<br>19·7361                                       | $020551 \\ 021110$ | $-41012 \\ -41663$ | ·3000800<br>·2952613 | ·3128330<br>·3244882    | ·6129109<br>·6197505 | 20                        |
|                 |  |                    | .42327             | 2902795              | 13363396                | 6266175              | 30                        |
| 30              | 19.5110  | 021694             | 43009              | 2851228              | 13484412                | 6335593              | 31                        |
| 31              | 19:2807  | *022307            |                    |                      | 3607448                 |                      | 32                        |
| 32              | 19.0453  | 022948             | 43705              | 2797878              | 3732799                 | 6405311              | 38                        |
| $\frac{33}{34}$ | 18·8044<br>18·5584                                       | ·023620<br>·024325 | 44416 $45143$      | ·2742595<br>·2685406 | 3860528                 | 6475394<br>6545904   | 34                        |
|                 | i i  |                    |                    |                      | 3990677                 |                      | 38                        |
| $\frac{35}{36}$ | 18·3071<br>18·0504                                       | 025065<br>025842   | -15887 $-16646$    | ·2626196<br>·2564868 | 4123261                 | 6616897<br>6688144   | 36                        |
|                 | 17.7884  | 026658             | 47421              | 2501369              | 4258276                 | 6759707              | 37                        |
| 37<br>38        |  | 027516             | 48211              | 2435589              | 4395853                 | 6831461              | 38                        |
| 39              | $\begin{array}{c c} 17.5210 \\ 17.2486 \end{array}$      | 027310             | 49015              | 2367538              | +535782                 | 6903290              | 39                        |
| 40              | 16.9709  | 029366             | 49836              | 2297048              | 4678448                 | 6975432              | 40                        |
| 41              | 16.6880  | 029366             | 50673              | 2224043              | 4823733                 | .7047766             | 40                        |
| 42              | 16.4001  | 031416             | 51523              | 2148465              | 4971509                 | 7047700              | 42                        |
| 43              | 16.1072  | 032525             | .52389             | 2070201              | 5122173                 | .7192401             | 43                        |
| 41              | 15.8092  | .033696            | 53271              | 1989099              | 5275783                 | 7264908              | 4-                        |
| 45              | 15.5065  | .034930            | .54164             | 1905138              | .5431986                | 7337107              | 48                        |
| 46              | 15.1988  | .036236            | 55074              | 1818093              | .5591403                | .7409466             | 46                        |
| 47              | 14.8864  | .037617            | .55999             | 1727897              | 5753842                 | 7481803              | 47                        |
| 48              | 14.5691  | 039080             | .56936             | 1634327              | 5919546                 | 7553870              | 48                        |
| 49              | 14.2471  | .010631            | .57887             | 1537265              | 6088575                 | 7625810              | 49                        |

English Life Table (No. 3). 3 per-cent Interest.

| x        | $a_x + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | P                  | $ar{\Lambda}$    | Logarithms           |                      |                      |          |
|----------|--|--------------------|------------------|----------------------|----------------------|----------------------|----------|
|          |  |                    |                  | ιĩ                   | P                    | Ā                    | x        |
| 50       | 13.9202  | 012279             | .28823           | 1436454              | 6261247              | 7697686              | 50       |
| 51       | 13.5884  | 0 1403 1           | *59835           | 1331683              | 6437881              | 7769553              | 51       |
| 52       | 13:2594  | -0122825           | - 60806          | 1225238              | 6611216              | 7839464              | 52       |
| 53       | 12:9272  | .047798            | 61789            | 1115044              | 6791097              | 7909112              | 53       |
| 54       | 12.5916  | .049859            | 62780            | 1000810              | 6977436              | 7978213              | 54       |
| 55       | 12:2525  | .052057            | 63783            | 0882248              | 7164791              | 8047049              | 55       |
| 56       | 11.9099  | .054405            | 64796            | 0759081              | 7356388              | 8115482              | 56       |
| 57       | 11:5640  | .056917            | 65819            | 0631081              | 7552120              | 8183513              | 57       |
| 58       | 11.2155  | .059604            | .66849           | 0498187              | .7752754             | 8250919              | 58       |
| 59       | 10.8647  | .062482            | 67885            | 0360178              | 7957549              | .8317738             | 59       |
| 60       | 10.5125  | .065566            | ·68926           | 0217062              | .8166787             | 8383831              | 60       |
| 61       | 10.1597  | 068869             | 69969            | 0068809              | ·S3S023S             | 8449057              | 61       |
| 62       | 9.8070   | .072409            | .71012           | 9915362              | ·S597925             | .8513317             | 62       |
| 63       | 9.4555   | 076200             | 72051            | 9756845              | 8819550              | 8576400              | 63       |
| 61       | 9.1059   | 080260             | ·73084           | 9593229              | 9014992              | \$635223             | 61       |
| 65       | 8:7595   | 084603             | .74108           | 9124793              | 9273858              |                      | 65       |
| 66       | 8.4168   | 054505             | 75121            | 9251170              | 9506131              | 8098651              | 66       |
| 67       | 8:0789   | 094220             | ·76119           | 9073522              | 9741431              | 8757614<br>8814931   | 67       |
| 68       | 7:7467   | 099528             | .77101           | 8591167              | 9979453              | ·SS70600             | 68       |
| 69       | 7:1208   | 105198             | 78065            | 5704507              | 0220075              | 5925464              | 69       |
| - 1      |  |                    |                  |                      |                      |                      | 1        |
| 70       | 7.1020   | 111247             | ·79008           | ·S513S07             | 0462884              | 8976711              | 70       |
| 71<br>72 | 6.7908   | ·117699<br>·121570 | ·79927<br>·80822 | 8319209              | 10707729             | 19026935             | 71       |
| 73       | 6·4881<br>6·1940   | 131888             | -81691           | SI21175<br>7919712   | ·0954135<br>·1202052 | 9075296              | 72       |
| 74       | 5:9092   | 139669             | .82533           | 7515712              | 1151000              | 19121742<br>19166276 | 73<br>74 |
|          |  |                    |                  |                      |                      | -                    | ı        |
| 75       | 5.6337   | 117914             | ·83347           | 7507937              | 1700974              | 9208900              | 75       |
| 76       | 5:3680   | 156730             | -81133<br>-81889 | 7298125              | 1951521              | 92 19664             | 76       |
| 77<br>78 | 5.1122   | ·166052<br>·175936 | ·85616           | -7086078<br>-6871989 | ·2202441<br>·2453547 | 9288514              | 77       |
| 79       | 4·8663<br>4·6304   | 186405             | ·86313           | 6656185              | 2704576              | +9325549<br>+9360762 | 78<br>79 |
|          |  |                    |                  |                      |                      |                      | i        |
| 80       | 4.4015   | 197482             | 86981            | 6438966              | 2955275              | 9304244              | 80       |
| 81       | 4.1883   | -209202            |                  | 6220378              | 3205659              | 9426032              | 81       |
| 82       | 3.9819   | .221578            | *88230           | 6000003              | 3455267              | 9456163              | S2       |
| 83       | 3·7849<br>3·5973   | ·234649<br>·245428 | *88812<br>*89367 | 5780541              | *3704187             | 9481717              | 83       |
| - 1      |  |                    |                  | 5559767              | 3952005              | 9511772              | 84       |
| 85       | 3.4187   | 262950             | .89892           | 5338610              | 4198732              | 9537355              | 85       |
| 86       | 3.2489   | 278238             | 90397            |                      | 4444165              | 9561540              | 86       |
| 87       | 3.0876   | 294317             | 90873            | 4896210              | 4688154              | .0281310             | 87       |
| 88       | 2.9345   | 311215             | 91326            | 1675341              | 4930605              | 9605914              | 88       |
| 89       | 2:7891   | *328980            | 91756            | 1454641              | .5171695             | .9626345             | 89       |
| 90       | 2.6514   | 347600             | ·92163           | 4231753              | 5410798              | 9645566              | 90       |
| 91       | 2.5210   | 367109             | 92548            | 4015728              | 5617950              | 9663670              | 91       |
| 92       | 2.3973   | 387577             | 92914            | 3797221              | 5883580              | 9650812              | 92       |
| 93       | 2.2802   | 408999             | 93260            | 3579729              | 6117222              | 9696954              | 93       |
| 94       | 2.1695   | 431377             | .93557           | 3363597              | .63 18569            | 9712155              | 94       |
| 95       | 2.0646   | ·454797            | .93897           | 3148359              | 6578176              | 9726517              | 95       |
| 96       | 1.9653   | 479269             | 16116.           | 2931259              | 6805793              | 9740091              | 96       |
| 97       | 1.8713   | 504829             | 91169            | 2721434              | ·7031443             | 9752893              | 97       |
| 98       | 1.7823   | 531514             | 94732            | 2509808              | $\cdot 7255147$      | 9761967              | 98       |
| 99       | 1.6981   | 559335             | ·94981           | 2299633              | ·7476720             | 9776367              | -99      |
| 00       | 1.6182   | .588412            | 95217            | 2090322              | .7696815             | 9787145              | 100      |
| 1        |  |                    |                  |                      |                      |                      | 1~~~     |
|          |  |                    |                  |                      |                      |                      |          |

English Life Table (No. 3).  $3\frac{1}{4}$  per-cent Interest.

| x               | $a + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | $ar{	ext{P}}$  | Ā             | Logarithms      |                 |                 |                 |
|-----------------|--|----------------|---------------|-----------------|-----------------|-----------------|-----------------|
|                 |  |                |               | ã               | Ē               | Ā               | x               |
| 1               | 20.7922  | .016112        | 33500         | .3179005        | 2071495         | .5250448        | 1               |
| 2               | 21.8994  | $\cdot 013680$ | 29958         | $\cdot 3404322$ | 1360861         | 4765128         | 2               |
| 3               | 22.4102  | .012640        | $\cdot 28326$ | .3504458        | 1017471         | 4521853         | 3               |
| 4.              | 22.6760  | $\cdot 012116$ | .27475        | 3555664         | .0833593        | $\cdot 4389377$ | 4               |
| 5               | 22.8128  | $\cdot 011852$ | .27038        | 3581786         | .0737916        | 4319746         | 5               |
| 6               | 22.8559  | $\cdot 011770$ | .26901        | 3589983         | 0707765         | $\cdot 4297684$ | 6               |
| 7               | 22.8356  | .011808        | ·26964        | $\cdot 3586124$ | 0721763         | 4307843         | 7               |
| 8               | 22.7747  | 011925         | .27159        | $\cdot 3574527$ | 0764584         | .4339138        | 8               |
| 9               | 22.6758  | 012117         | $\cdot 27476$ | 3555626         | 0833951         | $\cdot 4389535$ | 9               |
| 10              | 22.5456  | 012372         | .27893        | 3530618         | 0924399         | -4454952        | 10              |
| 11              | 22.3909  | 012678         | .28387        | .3500716        | $\cdot 1030507$ | 4531195         | 11              |
| 12              | 22.2175  | 013027         | .28943        | 3466952         | 1118444         | .4615435        | 12              |
| 13              | 22.0308  | 013408         | 29539         | •3430303        | 1273610         | 4703958         | 13              |
| 14              | 21.8359  | .013813        | 30162         | 3391711         | 1402880         | 4794601         | 14              |
| 15              | 21.6369  | 014234         | ·30798        | .3351950        | 1533270         | 4885225         | 15              |
| 16              | 21.4374  | .014665        | 31438         | 3311721         | 1662821         | 1974549         | 16              |
| 17              | 21.2403  | 015097         | .32066        | 3271607         | 1788907         | 5060448         | 17              |
| 18              | 21.0480  | 015527         | 32682         | 3232108         | 1910876         | .5143086        | 18              |
| 19              | 20.8624  | 015950         | .33276        | 3193643         | 2027607         | 5221311         | 19              |
| 20              | 20.6844  | .016363        | .33846        | .3156429        | 2138629         | .5295074        | 20              |
| 21              | 20.5115  | 016763         | 34388         | 3120610         | 2243517         | .5361069        | 21              |
| 22              | 20:3417  | .017177        | 34941         | .3083872        | 2349473         | .5433353        | 22              |
| 23              | 20.1653  | 017607         | 35505         | 3046047         | 2456854         | .5502895        | 23              |
| 24              | 19.9851  | 018054         | 36082         | 3007064         | 2565734         | 5572906         | $\frac{23}{24}$ |
| 25              | 19:8010  | .018519        | 36670         | 2966871         | 2676175         | 5643109         | 25              |
| 26              | 19:6127  | 019005         | 37274         | 2925374         | 2788679         | 5714060         | 26              |
| 27              | 19:4199  | .019511        | 37890         | 2882471         | 2902795         | 5785246         | 27              |
| 28              | 19.2223  | 020040         | 38521         | 2838054         | 3018977         | 5856976         | 28              |
| 29              | 19.0201  | .020593        | 39168         | 2792128         | 3137196         | 5930423         | 29              |
| 30              | 18.8129  | .021172        | .39831        | 2744559         | 3257619         | 6002212         | 30              |
| 31              | 18.6005  | 021779         | 40510         | 2695246         | 3380379         | 6075622         | 31              |
| 32              | 18.3831  | 022415         | 41206         | 2644188         | 3505387         | 6149605         | 32              |
| 33              | 18.1605  | .023082        | 41918         | 2591278         | 3632734         | 6224006         | 33              |
| 34              | 17.9326  | 023782         | 42647         | 2536433         | 3762484         | 6298885         | 34              |
| 35              | 17:6993  | .024516        | .43392        | 2479561         | 3894496         | 6374097         | 35              |
| 36              | 17.4607  | 025289         | .44156        | 2420617         | 4029317         | 6449897         | 36              |
| 37              | 17.2170  | .026099        | 44935         | 2359575         | 4166239         | 6525847         | 37              |
| 38              | 16.9677  | .026953        | 45733         | 2296230         | 4306071         | 6602297         | 38              |
| 39              | 16.7133  | .027850        | 46547         | 2230622         | 1418252         | 6678917         | 39              |
| 40              | 16.4535  | .028794        | 47376         | 2162583         | 4593020         | 6755584         | 40              |
| 41              | 16.1886  | 029789         | 48224         | 2092093         | 4740559         | 6832632         | 41              |
| 42              | 15.9184  | 030838         | 49089         | 2018994         | 4890862         | 6909842         | 42              |
| 43              | 15.6432  | 031943         | 49969         | 1943256         | 5043757         | 6987007         | 43              |
| 44              | 15.3629  | .033109        | .50865        | 1864732         | 5199461         | 7064190         | 44              |
| 45              | 15.0776  | .034341        | .51778        | 1783322         | .5358129        | .7141453        | 45              |
| 46              | 14.7873  | 035643         | 52706         | 1698889         | 5519743         | 7218601         | 46              |
| 47              | 14:4917  | 037022         | .53651        | 16111194        | 5684599         | .7295778        | 47              |
| $\overline{48}$ | 14.1914  | 038482         | .24611        | 1520252         | 5852576         | 7372801         | 48              |
| 49              | 13.8859  | 040033         | .55589        | 1425742         | 6024181         | 7449889         | 49              |
| 20              | 10 0000  | 0.10000        | 00000         | 1120132         | 0024101         | 1110000         | 1               |

English Life Table (No. 3).  $3\frac{1}{4}$  per-cent Interest.

|     | ā  | _             | _       |           | LOGARITHM          | 5          |     |
|-----|--|---------------|---------|-----------|--------------------|------------|-----|
| x   | $a + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | $ar{	ext{P}}$ | Ā       | ιĩ        | $\vec{\mathrm{P}}$ | Ã          | x   |
| 50  | 13:5754  | ·041650       | ·56582  | ·1327526  | ·6199277           | *7526783   | 50  |
| 51  | 13:2599  | ·013433       | ·57592  | ·1225401  | ·6378198           | *7603622   | 51  |
| 52  | 12:9467  | ·045257       | ·58593  | ·1121591  | ·6556858           | *7678457   | 52  |
| 53  | 12:6301  | ·047193       | ·59606  | ·1014065  | ·6738776           | *7752900   | 53  |
| 54  | 12:3094  | ·049256       | ·60631  | ·0902369  | ·6924591           | *7826947   | 54  |
| 55  | 11:9851  | ·051454       | ·61668  | ·0786416  | 7114191            | ·7900599   | 55  |
| 56  | 11:6570  | ·053803       | ·62718  | ·0665868  | 7308065            | ·7973922   | 56  |
| 57  | 11:3253  | ·056315       | ·63778  | ·0540498  | 7506241            | ·S046709   | 57  |
| 58  | 10:9907  | ·059003       | ·64848  | ·0410253  | 7708741            | ·8118966   | 58  |
| 59  | 10:6533  | ·061885       | ·65928  | ·0274842  | 7915854            | ·S190699   | 59  |
| 60  | 10:3141  | ·064972       | ·67013  | ·0134313  | ·\$127262          | ·\$261591  | 60  |
| 61  | 9:9737   | ·068281       | ·68102  | ·998×563  | \$342999           | ·\$331599  | 61  |
| 62  | 9:6334   | ·071823       | ·69190  | ·9837796  | ·\$562635          | ·\$400433  | 62  |
| 63  | 9:2932   | ·075623       | ·70279  | ·9681653  | ·\$786539          | ·\$468256  | 63  |
| 64  | 8:9550   | ·079686       | ·71359  | ·9520656  | ·9013\$20          | ·\$534488  | 64  |
| 65  | 8·6188   | ·054043       | 72435   | 9354468   | ·9245015           | ·\$5994\$5 | 65  |
| 66  | 8·2864   | ·058697       | 73498   | •9183659  | ·9479089           | ·\$662755  | 66  |
| 67  | 7·9580   | ·093677       | 74548   | •9008039  | ·9716330           | ·\$724360  | 67  |
| 68  | 7·6348   | ·095996       | 75581   | •8827977  | ·9956176           | ·\$784126  | 68  |
| 69  | 7·3176   | ·104674       | 76596   | •8643687  | ·0198388           | ·\$842064  | 69  |
| 70  | 7:0070   | ·110732       | 77590   | ·\$155321 | ·0412731           | :8898058   | 70  |
| 71  | 6:7035   | ·117193       | 78560   | ·\$263016 | ·0689017           | :8952015   | 71  |
| 72  | 6:4078   | ·124077       | 79506   | ·\$067089 | ·0936912           | :9003999   | 72  |
| 73  | 6:1204   | ·131405       | 80425   | ·7867798  | ·1186119           | :9053911   | 73  |
| 74  | 5:8416   | ·139203       | 81317   | ·7665318  | ·1436484           | :9101813   | 74  |
| 75  | 5·5719   | 147489        | :82179  | ·7460033  | ·16\$7596          | :9147609   | 75  |
| 76  | 5·3115   | 156288        | :83012  | ·7252172  | ·1939257           | :9191409   | 76  |
| 77  | 5·0606   | 165622        | :83815  | ·7042020  | ·21911\$0          | :9233217   | 77  |
| 78  | 4·8192   | 175520        | :84587  | ·6829750  | ·2443266           | :9273036   | 78  |
| 79  | 4·5874   | 186006        | :85328  | ·6615666  | ·2695269           | :9310916   | 79  |
| 80  | 4·3653   | 197096        | ·\$6038 | ·6400141  | ·2946778           | ·9346903   | 80  |
| 81  | 4·1526   | 208830        | ·\$6719 | ·6183201  | ·3197929           | ·9381143   | 81  |
| 82  | 3·9493   | 221227        | ·\$7369 | ·5965201  | ·3148381           | ·9413574   | 82  |
| 83  | 3·7554   | 234300        | ·\$7989 | ·5746562  | ·3697723           | ·9444284   | 83  |
| 84  | 3·5703   | 248106        | ·\$8551 | ·5527047  | ·3946373           | ·9473406   | 84  |
| 85  | 3·3944   | ·262620       | *89144  | ·5307630  | 1193278            | 9500921    | \$5 |
| 86  | 3·2267   | ·277931       | *89680  | ·5087586  | 14439370           | 9526956    | \$6 |
| 87  | 3·0674   | ·294026       | *90190  | ·4867701  | 14683857           | 9551584    | \$7 |
| 88  | 2·9162   | ·310929       | *90673  | ·4648173  | 14926612           | 9574780    | \$8 |
| 89  | 2·7726   | ·328689       | *91132  | ·4428872  | 15167852           | 9596709    | \$9 |
| 90  | 2·6363   | ·347337       | ·91568  | ·4209948  | 5407511            | *9617437   | 90  |
| 91  | 2·5072   | ·366868       | ·91951  | ·3991890  | 5645098            | *9636981   | 91  |
| 92  | 2·3848   | ·387340       | ·92373  | ·3774520  | 5880923            | *9655450   | 92  |
| 93  | 2·2689   | ·408759       | ·92743  | ·3558154  | 6114673            | *9672811   | 93  |
| 94  | 2·1591   | ·431173       | ·93095  | ·3342728  | 6346515            | *9689264   | 94  |
| 95  | 2·0552   | ·454588       | •93427  | 3125541   | ·6576180           | ·9704724   | 95  |
| 96  | 1·9567   | ·479082       | •93742  | 2915242   | ·6804098           | ·9719342   | 96  |
| 97  | 1·8635   | ·504642       | •94040  | 2703294   | ·7029834           | ·9733126   | 97  |
| 98  | 1·7752   | ·531334       | •94322  | 2492473   | ·7253676           | ·9746130   | 98  |
| 99  | 1·6916   | ·559173       | •94590  | 2282977   | ·7475462           | ·9758452   | 99  |
| 100 | 1.6122   | .588288       | -94844  | 2074189   | 7695900            | ;9770099   | 100 |

English Life Table (No. 3).  $3\frac{1}{2}$  per-cent Interest.

|  | ā  |  |                                      |  | Logarithms                               | 5  |                      |
|--|--|--|--------------------------------------|--|--|--|----------------------|
| x  | $a + \left(5 - \frac{\mu + \delta}{12}\right)$ | P  | Ā                                    | ā  | P  | Ā  | x                    |
| $\frac{1}{2}$                              | 19·8310<br>20·8904                             | ·016025<br>·013467                       | ·31779<br>·28134                     | ·2973446<br>·3199467                     | ·2047980<br>·1292709                     | ·5021402<br>·4492315                         | 1 2                  |
| $\frac{3}{4}$                              | 21·3830<br>21·6431                             | ·012365<br>·011803                       | ·26140<br>·25545                     | ·3300686<br>·3353195                     | 0921941 $0719924$                        | 4222615 $4073059$                            | 3                    |
| 5<br>6                                     | 21·7808<br>21·8296                             | ·011511<br>·011408                       | ·25071<br>·24903                     | ·3380738<br>·3390457                     | ·0611131<br>·0572095                     | ·3991717<br>·3962517                         | 5                    |
| $\begin{array}{c} 7 \\ 8 \\ 9 \end{array}$ | 21·8180<br>21·7684<br>21·6824                  | ·011432<br>·011537<br>·011719            | ·24943<br>·25114<br>·25410           | ·3388149<br>·3378265<br>·3361073         | ·0581222<br>·0620929<br>·0688906         | 3969487<br>3999159<br>4050047                | 8 9                  |
| 10<br>11                                   | 21·5667<br>21·4273                             | ·011967<br>·012268                       | ·25808<br>·26287                     | ·3337837<br>·3309675                     | ·0779853<br>·0887738                     | 4117544<br>4197410                           | 10<br>11             |
| 12<br>13                                   | 21·2701<br>21·1002                             | 012613<br>012992                         | ·26828<br>·27413                     | 3277695<br>3242866                       | $\frac{1008184}{1136760}$                | · 1285883<br>· 4379566                       | $\frac{12}{13}$      |
| 14 $15$                                    | 20·9223<br>20·7403                             | ·013395<br>·013814                       | ·28025<br>·28651                     | 3206094<br>3168151                       | 1269427<br>1403195                       | ·4175156<br>·4571398                         | 1:<br>1:             |
| 16<br>17<br>18<br>19                       | 20·5576<br>20·3773<br>20·2015<br>20·0318       | ·014242<br>·014673<br>·015100<br>·015519 | ·29279<br>·29899<br>·30504<br>·31088 | 3129725<br>3091466<br>3053837<br>3017200 | 1535710<br>1665189<br>1789769<br>1908637 | 14665562<br>14756567<br>14843568<br>14925928 | 16<br>17<br>18<br>19 |
| 20<br>21                                   | 19:8695<br>19:7150                             | ·015927<br>·016322                       | ·31646<br>·32178                     | 2981870<br>2947968                       | 2021340<br>2127734                       | ·5003188<br>·5075590                         | 20                   |
| 22<br>23<br>24                             | 19·5574<br>19·3966<br>19·2322                  | ·016730<br>·017154<br>·017595            | ·32720<br>·33273<br>·33839           | ·2913111<br>·2877256<br>·2840290         | ·2234959<br>·2343654<br>·2453893         | 5148133<br>5220920<br>5294175                | 2:<br>2:<br>2:       |
| $\frac{25}{26}$                            | 19:0638<br>18:8912                             | ·018054<br>·018533                       | ·34418<br>·35012                     | ·2802094<br>·2762595                     | ·2565734<br>·2679457                     | ·5367856<br>·5442169                         | 20                   |
| 27<br>28<br>29                             | 18.7144<br>18.5330<br>18.3469                  | ·019033<br>·019556<br>·020104            | ·35620<br>·36244<br>·36884           | ·2721759<br>·2679457<br>·2635627         | ·2795072<br>·2912800<br>·3032825         | 5516939<br>5592361<br>5668380                | 2<br>2<br>2          |
| 30<br>31                                   | 18·1560<br>17·9601                             | ·020677<br>·021278                       | ·37541<br>·38215                     | 2590202 $2543087$                        | ·3154875<br>·3279308                     | ·5745058<br>·5822339                         | 3                    |
| 32<br>33<br>34                             | 17:7592<br>17:5529<br>17:3416                  | ·021908<br>·022569<br>·023264            | *38906<br>*39616<br>*40343           | ·2494234<br>·2443489<br>·2390892         | 3406027<br>3535123<br>3666844            | 5900166<br>5978706<br>6057682                | 3 3                  |
| 35<br>36                                   | 17·1250<br>16·9031                             | ·023993<br>·024759                       | ·41088<br>·41851                     | ·2336306<br>·2279664                     | ·3800846<br>·3937331                     | ·6137150<br>·6217058                         | 3                    |
| 37<br>38                                   | 16:6757<br>16:1432                             | .025566<br>.026414                       | 42633<br>43433                       | 2220841<br>2159864                       | 4076628<br>4218342                       | ·6297459<br>·6378198<br>·6459231             | 3                    |
| 39<br>40                                   | 16·2054<br>15·9623                             | ·027306<br>·028246                       | ·41251<br>·45087                     | ·2096597<br>·2030955                     | ·4362581<br>·4509570<br>·1650180         | 6540513                                      | 4                    |
| $\frac{41}{42}$ $\frac{43}{43}$            | 15:7140<br>15:4603<br>15:2015                  | ·029236<br>·030280<br>·031382            | 45942<br>46814<br>47705              | 1962867<br>1892180<br>1818865            | ·4659180<br>·4811559<br>·4966806         | 6622099<br>6703758<br>6785639                | 4:4:4:4:             |
| 44   | 14:9374<br>14:6684                             | ·032544<br>·033773                       | ·48613<br>·49539                     | ·1742751<br>·1663827                     | ·5124709<br>·5285696                     | ·6867524<br>·6949472                         | 4                    |
| 46<br>47                                   | 14·3940<br>14·1146                             | ·035072<br>·036447                       | ·50483                               | 1581815                                  | 5449605<br>5616618                       | ·7031452<br>·7113347                         | 4                    |
| $\frac{18}{49}$                            | 13.8298 $13.5402$                              | ·037907<br>·039453                       | ·52424<br>·53420                     | 0.1408159<br>0.1316251                   | ·5787194<br>·5960800                     | ·7195302<br>·7277039                         | 49                   |

English Life Table (No. 3).  $3\frac{1}{2}$  per-cent Interest.

|                            | ã  | _   |  |   | Logarith           | 18   |                      |
|----------------------------|--|---|--|---|--------------------|--|----------------------|
| x                          | $a + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | P   | Ā  | ã   | P                  | Ā  | x                    |
| 50                         | 13·2452  | ·041098   | ·54435   | ·1220586  | ·6138207           | ·7358782                                     | 50                   |
| 51                         | 12·9446  | ·042851   | ·55469   | ·1120887  | ·6319610           | ·7440503                                     | 51                   |
| 52                         | 12·6461  | ·044675   | ·56496   | ·1019566  | ·6500646           | ·7520177                                     | 52                   |
| 53                         | $12.3441 \\ 12.0381$                                   | ·046609   | ·57535   | ·0914594  | ·6684698           | ·7599321                                     | 53                   |
| 54                         |  | ·048668   | ·58587   | ·0805579  | ·6872435           | ·7678013                                     | 54                   |
| 55                         | 11·7275  | .050868   | ·59656   | 0692054   | .7064447           | 7756541                                      | 55                   |
| 56                         | 11·4133  | .053216   | ·60737   | 0574113   | .7260422           | 7834533                                      | 56                   |
| 57                         | 11·0951  | .055728   | ·61831   | 0451312   | .7460735           | 7912063                                      | 57                   |
| 58                         | 10·7735  | .058419   | ·62938   | 0323569   | .7665541           | 7989129                                      | 58                   |
| 59                         | 10·4489  | .061306   | ·64054   | 0190706   | .7875030           | 8065463                                      | 59                   |
| 60                         | 10·1218  | ·064396   | ·65180   | .0052578  | *8088589           | ·8141144                                     | 60                   |
| 61                         | 9·7935   | ·067707   | ·66309   | .9909379  | *8306336           | ·8215725                                     | 61                   |
| 62                         | 9·4647   | ·071254   | ·67440   | .9761069  | *8528092           | ·8289176                                     | 62                   |
| 63                         | 9·1357   | ·075059   | ·68572   | .9607418  | *8754028           | ·8361468                                     | 63                   |
| 64                         | 8·8082   | ·079130   | ·69699   | .9448872  | *8983412           | ·8432265                                     | 64                   |
| 65                         | 8:4824   | ·083489   | ·70s19   | 9285187   | ·9216293           | ·8501498                                     | 65                   |
| 66                         | 8:1599   | ·088149   | ·71929   | 9116848   | ·9452174           | ·8569040                                     | 66                   |
| 67                         | 7:8408   | ·093137   | ·73027   | 913604  | ·9691222           | ·8634835                                     | 67                   |
| 68                         | 7:5260   | ·098472   | ·74110   | 8765642   | ·9933128           | ·8698768                                     | 68                   |
| 69                         | 7:2173   | ·104155   | ·75172   | 8583748   | ·0176802           | ·8760561                                     | 69                   |
| 70                         | 6:9141   | ·110231   | ·76215   | ·8397357  | ·0423037           | ·8820405                                     | 70                   |
| 71                         | 6:6181   | ·116700   | ·77233   | ·8207333  | ·0670709           | ·8878029                                     | 71                   |
| 72                         | 6:3293   | ·123593   | ·78226   | ·8013557  | ·0919939           | ·8933511                                     | 72                   |
| 73                         | 6:0483   | ·130934   | ·79193   | ·7816333  | ·1170525           | ·8986868                                     | 73                   |
| 74                         | 5:7755   | ·138745   | ·80132   | ·7615896  | ·1422174           | ·9038060                                     | 74                   |
| 75                         | 5·5114   | ·147041   | ·81040   | 7412619   | 1674385            | 9086994                                      | 75                   |
| 76                         | 5·2562   | ·155850   | ·81918   | 7206719   | 1927068            | 9133793                                      | 76                   |
| 77                         | 5·0099   | ·165203   | ·82765   | 6998291   | 2180179            | 9178467                                      | 77                   |
| 78                         | 4·7730   | ·175110   | ·83580   | 6787914   | 2433109            | 9221024                                      | 78                   |
| 79                         | 4·5452   | ·185611   | ·84364   | 6575530   | 2686037            | 9261572                                      | 79                   |
| 80                         | 4·3268   | ·196716   | ·85115   | 6361668   | ·2938397           | 9300061                                      | 80                   |
| 81                         | 4·1175   | ·208464   | ·85835   | 6146336   | ·3190311           | 9336644                                      | 81                   |
| 82                         | 3·9175   | ·220863   | ·86523   | 5930090   | ·3441230           | 9371316                                      | 82                   |
| 83                         | 3·7263   | ·233961   | ·87181   | 5712778   | ·3691435           | 9404218                                      | 83                   |
| 84                         | 3·5441   | ·247758   | ·87808   | 5495060   | ·3940277           | 9435341                                      | 84                   |
| 85                         | 3:3702   | ·262317   | ·88406   | '5276557  | ·4188264           | 9464817                                      | 85                   |
| 86                         | 3:2048   | ·277630   | ·88975   | '5058000  | ·4434664           | 9492680                                      | 86                   |
| 87                         | 3:0476   | ·293726   | ·89516   | '4839580  | ·4679424           | 9519007                                      | 87                   |
| 88                         | 2:8981   | ·310652   | ·90030   | '4621134  | ·4922742           | 9543873                                      | 88                   |
| 89                         | 2:7561   | ·328431   | ·90519   | '4402950  | ·5164441           | 9567397                                      | 89                   |
| 90                         | 2·6214   | ·347074   | ·90982   | 4185333   | 5404221            | 9589555                                      | 90                   |
| 91                         | 2·4936   | ·366627   | ·91422   | 3968268   | 5642245            | 9610507                                      | 91                   |
| 92                         | 2·3725   | ·387094   | ·91838   | 3752062   | 5878165            | 9630224                                      | 92                   |
| 93                         | 2·2577   | ·408526   | ·92233   | 3536662   | 6112197            | 9648863                                      | 93                   |
| 94                         | 2·1489   | ·430955   | ·92608   | 3322162   | 6344320            | 9666485                                      | 94                   |
| 95<br>96<br>97<br>98<br>99 | 2·0459<br>1·9482<br>1·8558<br>1·7682<br>1·6851         | ·454382<br>·478893<br>·504451<br>·531145<br>·559035 | ·92962<br>·93298<br>·93616<br>·93917<br>·94203 | 3108844<br>2896335<br>2685312<br>2475314<br>2266257 | 6574211<br>6802385 | :9683055<br>:9698723<br>:9713501<br>:9727442 | 95<br>96<br>97<br>98 |
| 100                        | 1.6064   | .588110   | 91171  | 2266257   | 7474390            | 9740647                                      | 99<br>100            |

English Life Table (No. 3).  $3\frac{3}{4}$  per-cent Interest.

|         | ıī   |                | _         |                 | Logarithms | 3               |             |
|---------|--|----------------|-----------|-----------------|------------|-----------------|-------------|
| x       | $a + \left( \cdot 5 - \frac{\mu + \delta}{12} \right)$ | P              | $\Lambda$ | ιī              | P          | Ā               | x           |
| 1       | 18:9461  | .015967        | ·30252    | .2775198        | 2032233    | .4807541        | 1           |
| 2       | 19.9609  | 013284         | .26516    | $\cdot 3001801$ | 1233289    | 4235080         | 2           |
| 3       | 20.4360  | .012119        | 24767     | 3103959         | .0834668   | 3938734         | 3           |
| 4       | 20.6901  | 011518         | .23831    | 3157626         | 0613771    | $\cdot 3771423$ | 4           |
| 5       | 20.8280  | $\cdot 011198$ | 23324     | 3186476         | .0491405   | 3678030         | 5           |
| 6       | 20.8814  | 011075         | ·23127    | 3197596         | .0443437   | 3641193         | 6           |
| 7       | 20.8774  | 011085         | .23142    | 3196764         | 0447357    | .3644009        | 7           |
| 8       | 20.8372  | $\cdot 011177$ | ·23290    | 3188394         | .0483253   | .3671695        | 8           |
| 9       | 20.7626  | 011350         | .23565    | $\cdot 3172817$ | 0549959    | 3722674         | 9           |
| 10      | 20.6594  | ·011590        | .23944    | 3151177         | .0640834   | 3791967         | 10          |
| 11      | 20.5337  | .011886        | .24407    | 3124673         | 0750357    | 3875144         | 11          |
| 12      | 20.3909  | $\cdot 012228$ | 24933     | 3094364         | 0873554    | 3967745         | 12          |
| 13      | 20.2359  | .012603        | .25504    | .3061226        | 1004739    | 4066083         | 13          |
| 14      | 20.0730  | 013004         | .26103    | .3026123        | 1140770    | $\cdot 4166904$ | 14          |
| 15      | 19:9061  | $\cdot 013422$ | .26718    | 2989862         | 1278172    | .4268039        | 15          |
| 16      | 19.7386  | $\cdot 013848$ | 27334     | .2953163        | .1413871   | 4367032         | $1\epsilon$ |
| 17      | 19.5733  | .014276        | 27943     | 2916641         | 1546065    | 4462730         | 17          |
| 18      | 19.4120  | 014701         | 28537     | 2880703         | 1673469    | 4554083         | 18          |
| 19      | 19.2568  | $\cdot 015116$ | .29108    | $\cdot 2845841$ | 1794369    | 4640124         | 19          |
| 20      | 19:1085  | .015519        | .29654    | 2812266         | 1908637    | .4720833        | 20          |
| 21      | 18.9677  | .015907        | 30172     | $\cdot 2780147$ | 2015883    | .4796041        | 21          |
| 22      | 18.8240  | .016309        | 30701     | 2747119         | 2124273    | $\cdot 4871525$ | 22          |
| 23      | 18.6771  | 016727         | 31242     | $\cdot 2713094$ | 2234181    | $\cdot 4947388$ | 23          |
| $^{24}$ | 18:5267  | 017162         | ·31796    | 2677981         | 2345679    | 5023725         | 24          |
| 25      | 18.3726  | 017615         | ·32363    | 2641707         | 2458826    | .5100488        | 25          |
| 26      | 18.2143  | ·018088        | •32946    | 2604125         | 2573905    | .5178027        | 2e          |
| 27      | 18.0519  | 018582         | *33544    | 2565229         | 2690925    | 5256148         | 27          |
| 28      | 17.8850  | 019099         | •34158    | 2524889         | 2810106    | .5334924        | 28          |
| 29      | 17:7136  | 019640         | •34789    | 2483068         | 2931415    | 5414419         | 29          |
| 30      | 17:5374  | .020207        | .35438    | 2439652         | .3055018   | 5494692         | 30          |
| 31      | 17:3564  | .020802        | 36104     | 2394596         | 3181051    | .5575553        | 31          |
| 32      | 17:1704  | 021426         | •36789    | $\cdot 2347805$ | 3309411    | 5657180         | 32          |
| 33      | 16.9794  | 022081         | 37492     | $\cdot 2299223$ | 3440187    | 5739386         | 33          |
| 34      | 16.7832  | 022769         | .38214    | 2248748         | 3573440    | $\cdot 5822225$ | 34          |
| 35      | 16.5817  | 023493         | .38956    | 2196290         | 3709385    | .5905744        | 35          |
| 36      | 16.3751  | 024255         | ·39717    | $\cdot 2141840$ | 3848013    | 5989764         | 36          |
| 37      | 16.1631  | 025055         | 40497     | $\cdot 2085247$ | 3988944    | 6074229         | 37          |
| 38      | 15.9460  | 025897         | 41296     | 2026518         | 4132495    | 6159080         | 38          |
| 39      | 15.7234  | 026786         | ·42116    | 1965465         | 4279079    | 6244471         | 38          |
| 40      | 15.4955  | $\cdot 027721$ | 42955     | 1902056         | 4428089    | 6330137         | 40          |
| 41      | 15.2626  | 028705         | ·43812    | 1836285         | 4579576    | 6415931         | 41          |
| 42      | 15.0240  | $\cdot 029746$ | 44691     | 1767856         | 1734286    | 6502201         | 42          |
| 43      | 14.7805  | .030843        | .45587    | 1696891         | 4891566    | 6588410         | 45          |
| 44      | 14.5318  | ·032001        | •46503    | 1623194         | .5051635   | 6674810         | 44          |
| 45      | 14:2776  | $\cdot 033225$ | 47438     | $\cdot 1546552$ | 5214650    | 6761264         | 45          |
| 46      | 14:0183  | 034521         | •48393    | $\cdot 1466953$ | 5380834    | 6847825         | 46          |
| 47      | 13.7539  | 035892         | 49366     | 1384259         | •5549977   | 6934279         | 47          |
| 48      | 13.4841  | 037348         | .20360    | 1298219         | 5722674    | $\cdot 7020857$ | 48          |
| 49      | 13.2087  | .038893        | .51373    | 1208601         | 5898714    | $\cdot 7107349$ | 49          |

English Life Tuble (No. 3).  $3\frac{3}{4}$  per-cent Interest.

| 1                          | ιĪ   |   |  |  | LOGARITHMS   | ;  |                      |
|----------------------------|--|---|--|--|--|--|----------------------|
| x                          | $a + \left( \cdot 5 - \frac{\mu - \delta}{12} \right)$ | $\overline{\mathrm{P}}$                             | Ī.   | ιĨ   | Ρ̈́  | Ā  | x                    |
| 50                         | 12·92S2  | ·040536   | *52406   | ·1115380   | ·607\$409  | 7193810  | 50                   |
| 51                         | 12·6421  | ·042286   | *53459   | ·1018192   | ·6261966   | 7280208  | 51                   |
| 52                         | 12·357S  | ·044107   | *54506   | ·0919412   | ·6445075   | 7364443  | 52                   |
| 53                         | 12·0691  | ·046042   | *55569   | ·0816749   | ·6631542   | 7448326  | 53                   |
| 54                         | 11·7767  | ·048099   | *56645   | ·0710237   | ·6821360   | 7531616  | 53                   |
| 55                         | 11:4797  | *050297   | ·57739   | 0599306  | ·7015421   | 7614693  | 56 56 56             |
| 56                         | 11:1781  | *052647   | ·55549   | 0483680  | ·7213736   | 7697391  |                      |
| 57                         | 10:8726  | *055161   | ·59974   | 0363334  | ·7416321   | 7779630  |                      |
| 58                         | 10:5637  | *057850   | ·61111   | 0238161  | ·7623034   | 7861194  |                      |
| 59                         | 10:2508  | *060740   | ·62263   | 0107578  | ·7834748   | 7942300  |                      |
| 60<br>61<br>62<br>63<br>64 | 9:9361<br>9:6190<br>9:3014<br>8:9533<br>8:6656         | ·063×29<br>·067147<br>·070697<br>·074504<br>·0755×4 | ·63421<br>·64589<br>·65758<br>·66929<br>·68098 | 9972160<br>9531299<br>9655453<br>9534359<br>9377956      | \$050180<br>:\$270266<br>:\$494010<br>:\$721796<br>:\$953341 | \$022331<br>\$1015\6<br>\$1794\6<br>\$256143<br>\8331344         | 6:<br>6:<br>6:<br>6: |
| 65<br>66<br>67<br>68<br>69 | 8:3501<br>8:0364<br>7:7265<br>7:4203<br>7:1193         | ·0×2045<br>·0×7620<br>·092611<br>·097952<br>·103649 | ·69260<br>·70415<br>·71556<br>·72683<br>·73791 | ·9216917<br>·9050615<br>·8879828<br>·8704215<br>·8524373 | ·9157902<br>·9426032<br>·9666626<br>·9910133<br>·0155650     | *\$404\$25<br>*\$476652<br>*\$546461<br>*\$61432\$<br>*\$6\$0034 | 6<br>6<br>6<br>6     |
| 70                         | 6:8239   | ·109720   | ·74878   | ·\$340327  | *0403215   | 18743542   | 777777               |
| 71                         | 6:5348   | ·116213   | ·75943   | ·\$152323  | *0652547   | 18804877   |                      |
| 72                         | 6:2526   | ·123120   | ·76982   | ·7960606   | *0903286   | 1863802  |                      |
| 73                         | 5:9778   | ·130471   | ·77993   | ·7765414   | *1155140   | 18020556   |                      |
| 74                         | 5:7108   | ·138292   | ·78976   | ·7566970   | *1407971   | 18074951   |                      |
| 75                         | 5·4522   | ·146598   | ·79925   | 7365715  | ·1661280   | ·9026989   | 7777                 |
| 76                         | 5·2018   | ·155427   | ·80850   | 7161537  | ·1915264   | ·9076800   |                      |
| 77                         | 4·9603   | ·164786   | ·81739   | 6955079  | ·2169203   | ·9124293   |                      |
| 78                         | 4·7277   | ·174704   | ·82595   | 6746499  | ·2423029   | ·9169538   |                      |
| 79                         | 4·5039   | ·185215   | ·83419   | 6535557  | ·2676761   | ·9212650   |                      |
| 80                         | 4·2890   | ·196339   | ·\$4210  | 6323560  | ·2930066   | ·9253637   | 8 8 8                |
| 81                         | 4·0830   | ·208104   | ·\$4969  | 6109794  | ·3182804   | ·9292605   |                      |
| 82                         | 3·8860   | ·220520   | ·\$5694  | 5895028  | ·3434450   | ·9329504   |                      |
| 83                         | 3·6979   | ·233608   | ·\$6386  | 5679552  | ·3684877   | ·9364434   |                      |
| 84                         | 3·5180   | ·247439   | ·\$7049  | 5462958  | ·3934682   | ·9397635   |                      |
| \$5                        | 3:3466   | ·261997   | *87680   | 5246038  | ·4152963   | ·9429005   | 22222                |
| 86                         | 3:1833   | ·277325   | *88281   | 5028776  | ·4429891   | ·9458672   |                      |
| 87                         | 3:0279   | ·293448   | *88853   | 4811415  | ·4675311   | ·9486721   |                      |
| 88                         | 2:8802   | ·310385   | *89397   | 4594226  | ·4919007   | ·9513229   |                      |
| 89                         | 2:7398   | ·328177   | *89914   | 4377189  | ·5161081   | ·9538273   |                      |
| 90                         | 2·6066   | *346527   | ·90404   | ·4160744   | *5401129   | 9561876  | 9999                 |
| 91                         | 2·4801   | *366397   | ·90870   | ·3944692   | *5639519   | 9584205  |                      |
| 92                         | 2·3603   | *356862   | ·91311   | ·3729672   | *5875561   | 9605231  |                      |
| 93                         | 2·2465   | 408324  | ·91730   | ·3515064   | *6110049   | 9625114  |                      |
| 94                         | 2·1388   | *430737   | ·92126   | ·3301702   | *6342122   | 9643822  |                      |
| 95<br>96<br>97<br>98<br>99 | 2:0367<br>1:9398<br>1:5482<br>1:7612<br>1:6788         | ·454176<br>·478704<br>·504253<br>·530979<br>·558852 | ·92502<br>·92859<br>·93196<br>·93516<br>·93820 | ·3089271<br>·2577570<br>·2667490<br>·2458057<br>·2249990 | -6572242<br>-6800670<br>-7026485<br>-7250774<br>-7472968     | -9693973<br>-9708859   | 99999                |
| 100                        | 1:6007   | -587912   | .94107   | 2043000  | 7693123  | ,  | 10                   |

English Life Table (No. 3). 4 per-cent Interest.

|                | ιĪ   |                | _             | 1               | LOGARITHM       | ŝ               |     |
|----------------|--|----------------|---------------|-----------------|-----------------|-----------------|-----|
| x              | $a + \left(5 - \frac{\mu + \delta}{12}\right)$ | P              | Ā             | ιĩ              | $\vec{P}$       | À               | x   |
| 1              | 18:1297  | 015937         | .28894        | 2583906         | 2024066         | 4608077         | 1   |
| 2              | 19.1028  | $\cdot 013128$ | $\cdot 25078$ | 2810970         | 1181986         | -3992929        | 2   |
| 3              | 19.5613  | $\cdot 011901$ | 23279         | 2913978         | 0755835         | 3669643         | 3   |
| 4              | 19.8092  | .011261        | .22307        | 2968670         | 0515770         | 3484412         | 4   |
| 5              | 19.9466  | $\cdot 010913$ | ·21768        | ·2998689        | 0379442         | .3378185        | 5   |
| 6              | 20.0037  | .010770        | .21544        | 3011103         | 0322157         | :3333263        | - 6 |
| 7              | 20.0061  | 010764         | 21535         | 3011625         | 0319737         | 3331449         | 7   |
| 8              | 19.9742  | .010844        | ·21660        | 3004694         | 0351895         | 3356585         | 8   |
| 9              | 19.9093  | .011007        | •21915        | 2990560         | 0416690         | 3407415         | - 9 |
| 10             | 19·S173  | 011240         | .22275        | 2970145         | .0507663        | .3478177        | 10  |
| 11             | 19:7037  | .011531        | ·22721        | 2945478         | 0618670         | 3564274         | 11  |
| 12             | 19.5736  | .011869        | .23231        | 2916707         | 0744141         | 3660679         | 12  |
| 13             | 19.4318  | 012241         | -23787        | 2885131         | .0878169        | .3763397        | 13  |
| 14             | 19.2824  | ·012640        | •24373        | 2851611         | .1017471        | 3869090         | 14  |
| 15             | 19.1291  | 013056         | 24975         | .2816946        | .1158101        | .3975055        | 15  |
| 16             | 18.9751  | 013480         | .25579        | 2781841         | 1296899         | 4078836         | 16  |
| 17             | 18.8230  | .013906        | $\cdot 26175$ | .2746888        | 1432022         | ·4178867        | 17  |
| 18             | 18.6750  | 014327         | -26756        | .2712606        | .1561553        | .4274212        | 18  |
| 19             | 18.5326  | $\cdot 014738$ | 27314         | .2679363        | ·1684386        | 4363853         | 19  |
| 20             | 18:3969  | 015136         | .27846        | 2617446         | .1800111        | .4447628        | 20  |
| 21             | 18.2683  | .015519        | .28351        | 2616981         | 1908637         | 4525684         | 21  |
| 22             | 18.1371  | .015915        | -28865        | 2585679         | 2018066         | 4603716         | 22  |
| 23             | 18.0027  | 016326         | -29392        | .2553376        | 2128798         | 4682291         | 23  |
| 24             | 17.8650  | 016755         | $\cdot 29932$ | .2520030        | .2241444        | 4761357         | 24  |
| 25             | 17:7236  | .017201        | .30487        | 2485519         | .2355537        | .4841147        | 25  |
| $\frac{1}{26}$ | 17.5783  | .017668        | •31057        | 2449769         | 2471874         | +4921595        | 26  |
| 27             | 17:4288  | .018156        | .31643        | $\cdot 2412676$ | 2590202         | .5002777        | 27  |
| 28             | 17.2752  | 018666         | ·32246        | 2374231         | 2710513         | ·5084758        | 28  |
| 29             | 17.1171  | 019201         | ·32866        | 2334302         | 2833238         | .5167469        | 29  |
| 30             | 16.9544  | .019761        | .33504        | 2292824         | .2958089        | 5250967         | 30  |
| 31             | 16-7869  | .020350        | ·34161        | .2249706        | .3085644        | 5335306         | 31  |
| 32             | 16.6146  | .020967        | .34836        | ·2204899        | $\cdot 3215363$ | $\cdot 5420283$ | 32  |
| 33             | 16.4372  | 021617         | 35532         | $\cdot 2158279$ | 3347954         | ·5506197        | 33  |
| 34             | 16.2548  | ·022300        | 36248         | 2109817         | 3483049         | 5592840         | 34  |
| 35             | 16.0674  | .023017        | .36983        | 2059456         | 3620487         | .5680021        | 35  |
| 36             | 15.8747  | 023772         | ·37738        | 2007055         | 3760657         | .5767789        | 36  |
| 37             | 15.6768  | 024568         | *38515        | 1952576         | 3903698         | $\cdot 5856299$ | 37  |
| 38             | 15.4737  | $\cdot 025405$ | ·39311        | 1895942         | 4049192         | ·5945141        | 38  |
| 39             | 15.2654  | .026287        | ·40128        | $\cdot 1837082$ | .4197410        | 6034475         | 39  |
| 40             | 15.0519  | .027216        | ·40966        | .1775913        | .4348243        | 6124236         | 40  |
| 41             | 14.8329  | .028197        | .41825        | $\cdot 1712262$ | 4502029         | $\cdot 6214359$ | 41  |
| 42             | 14.6088  | 029232         | 42704         | .1646146        | 4658585         | .6304686        | 42  |
| 43             | 14.3793  | 030324         | 43604         | 1577378         | 4817865         | .6395263        | 43  |
| 44             | 14.1446  | $\cdot 031478$ | 14524         | $\cdot 1505907$ | 4980071         | 6485942         | 44  |
| 45             | 13.9047  | 032698         | 45465         | 1431616         | .5145212        | .6576772        | 45  |
| 46             | 13.6593  | .033990        | $\cdot 46428$ | 1354284         | 5313512         | .6667800        | 46  |
| 47             | 13.4088  | 035357         | .47410        | $\cdot 1273899$ | 5484754         | 6758700         | 47  |
| 48             | 13.1527  | .036809        | .48414        | 1190148         | 5659540         | 6849710         | 48  |
| 49             | 12.8913  | .038351        | •49440        | $\cdot 1102967$ | 5837767         | 6940785         | 49  |

English Life Table (No. 3). 4 per-cent Interest.

|                            | ιĪ  |   |   |  | LOGARITHM  | s  |                            |
|----------------------------|---|---|---|--|--|--|----------------------------|
| x                          | $a + \left( \cdot 5 - \frac{\mu - \delta}{12} \right)$                        | P   | Ā   | ιĪ   | $\bar{\mathbf{P}}$                                       | Ā  | x                          |
| 50                         | 12:6244   | .039991   | :50186  | ·1012107   | ·6019623   | 7031710  | 50                         |
| 51                         | 12:3517   | .047140   | :51556  | ·0917267   | ·6205524   | •7122792   | 51                         |
| 52                         | 12:0803   | .043550   | :52620  | ·0820777   | ·6390779   | •7211508   | 52                         |
| 53                         | 11:8049   | .045490   | :5370J  | ·0720622   | ·6579159   | •7299743   | 58                         |
| 54                         | 11:5250   | .047547   | :54798  | ·0616409   | ·6771231   | •7387647   | 54                         |
| 55                         | 11:2406   | ·049743   | ·55914  | ·0507894   | ·6967320   | 7475206  | 55                         |
| 56                         | 10:9516   | ·052090   | ·57017  | ·0394776   | ·7167544   | 7562328  | 56                         |
| 57                         | 10:6582   | ·054604   | ·58198  | ·0276\39   | ·7372245   | 7649081  | 57                         |
| 58                         | 10:3609   | ·057296   | ·59364  | ·0153974   | ·7581243   | 7735232  | 58                         |
| 59                         | 10:0600   | ·060183   | ·60544  | ·00259\0   | ·7794738   | 7820711  | 59                         |
| 60                         | 9°7563  | ·063277   | ·61735  | ·9892851   | :8012459   | ·7905315   | 60                         |
| 61                         | 9°4504  | ·066595   | ·62935  | ·9754502   | :8234416   | ·7988922   | 61                         |
| 62                         | 9°1430  | ·070153   | ·64141  | ·9610887   | :8460462   | ·8071357   | 62                         |
| 63                         | 8°8351  | ·073964   | ·65345  | ·9462115   | :8690204   | ·8152323   | 63                         |
| 64                         | 8°5274  | ·075048   | ·66555  | ·9308166   | :8923615   | ·8231807   | 64                         |
| 65<br>66<br>67<br>68<br>69 | 8:2206<br>7:9166<br>7:6149<br>7:3172<br>7:0238                                | ·082425<br>·087097<br>·092101<br>·097444<br>·103152 | ·67758<br>·68951<br>·70134<br>·71302<br>·72452      | ·9149035<br>·8985387<br>·8816642<br>·8613449<br>·8465721 | ·9160590<br>·9400032<br>·9642643<br>·9887551<br>·0134776 | *8309606<br>*8385406<br>*8459286<br>*8531017<br>*8600504 | 65<br>66<br>67<br>68       |
| 70                         | 6·7357  | ·109242   | ·735×2  | ·\$2\\$827   | ·03\$3\$96   | ·8667716   | 70                         |
| 71                         | 6·4534  | ·115736   | ·746×9  | ·8097\\$6  | ·06346\$6  | ·8732566   | 71                         |
| 72                         | 6·1777  | ·122652   | ·75771  | ·790\\$26\$  | ·05\$6747  | ·8795030   | 72                         |
| 73                         | 5·9088  | ·130018   | ·76×25  | ·7714993   | ·1140036   | ·8855026   | 78                         |
| 74                         | 5·6476  | ·137846   | ·77×50  | ·751\\$639   | ·1393941   | ·8912586   | 74                         |
| 75                         | 5:3940  | ·146170   | ·78844  | ·7319109   | 1648582  | *\$967686  | 75                         |
| 76                         | 5:1486  | ·155007   | ·79807  | ·7116892   | 1903512  | *9020410   | 70                         |
| 77                         | 4:9116  | ·164378   | ·80736  | ·6912230   | 2158437  | *9070672   | 77                         |
| 78                         | 4:6831  | ·174314   | ·81633  | ·6705334   | 2413323  | *9118658   | 78                         |
| 79                         | 4:4632  | ·184834   | ·82495  | ·6496463   | 2667819  | *9164276   | 79                         |
| 80<br>81<br>82<br>83<br>84 | 4·2519<br>4·0492<br>3·8552<br>3·6696<br>3·4923                                | ·195969<br>·207742<br>·220170<br>·233290<br>·247123 | ·\$3324<br>·\$4119<br>·\$4880<br>·\$5608<br>·\$6303 | .6255830<br>.6073692<br>.5860469<br>.5646187             | ·2921574<br>·3175243<br>·3427581<br>·3678961<br>·3929132 | ·9207701<br>·9248941<br>·9288054<br>·9325141<br>·9360259 | 80<br>81<br>82<br>83<br>84 |
| 85                         | 3·3231  | ·261704   | ·86967  | ·5215434   | ·4178103   | 9393545  | \$5                        |
| 86                         | 3·1621  | ·277025   | ·87598  | ·4999756   | ·4425190   | 9424942  | \$6                        |
| 87                         | 3·0086  | ·293160   | ·88200  | ·4783645   | ·4671047   | 9454686  | \$7                        |
| 88                         | 2·8625  | ·310124   | ·88773  | ·4567455   | ·1915354   | 9482809  | \$8                        |
| 89                         | 2·7238  | ·327193   | ·89317  | ·4351752   | ·5148040   | 9509341  | \$0                        |
| 90                         | 2·5920  | *3465\\2  | ·89834  | ·4136350   | 5398060  | 9534407  | 90                         |
| 91                         | 2·4670  | *366129   | ·90324  | ·3921691   | 5636341  | 9558032  | 91                         |
| 92                         | 2·3482  | *3\\6637  | ·90790  | ·3707351   | 5873034  | 9580380  | 92                         |
| 93                         | 2·2356  | *40\\80\\7  | ·91232  | ·3493941   | 6107528  | 9601472  | 93                         |
| 94                         | 2·1287  | *130519   | ·91651  | ·3281145   | 6340226  | 9621372  | 94                         |
| 95                         | $\begin{array}{c} 2.0275 \\ 1.9314 \\ 1.8406 \\ 1.7543 \\ 1.6725 \end{array}$ | 453998  | 92048   | *3069609   | 6570510  | 9640144  | 95                         |
| 96                         |   | 478539  | 92425   | *2\5\5722  | 6799173  | 9657895  | 96                         |
| 97                         |   | 504080  | 92781   | *2649594   | 7021995  | 9674590  | 97                         |
| 98                         |   | 530810  | 93120   | *2441039   | 7219391  | 9690430  | 98                         |
| <b>9</b> 9                 |   | 558685  | 93440   | *2233661   | 7471670  | 9705328  | 99                         |
| 00                         | 1.5949  | ·5877×0   | 93715   | 2233661  | 7692148  | 9705528  | 100                        |

#### Discussion.

The President said the only practical observation he would venture to make upon the subject at that time was that since the system of industrial assurance had now successfully emerged from the experimental stage and was firmly based upon settled principles, the just claim of industrial policyholders to a distinctive share (in a reversionary form on some roughly practicable plan, and after a certain duration of assurance reasonably indicative of permanent membership) in the large surpluses which they created would require

equitable consideration and admission.

Mr. G. H. RYAN thought the paper would be welcomed by the meeting, because it afforded them an opportunity of discussing a practical branch of their subject rather out of the beaten track, and also because it served to introduce a new contributor, who had, as it were, an hereditary claim on their regard. Industrial assurance was a subject of which none of the members should be ignorant; but except for the information in Mr. Rea's paper, and that read by Mr. Schooling some little time ago, it was difficult to see how the great majority of members could ever get any close acquaintance with the principles of It was not so long ago, certainly within their President's memory, that industrial assurance was the Cinderella of insurance, and was looked down upon and scarcely recognized by the Things, however, were now quite altered, and it was viewed in truer perspective as the complement of the ordinary system of life assurance, providing for the lower classes those facilities for thrift which the ordinary business endeavoured to provide for the upper and middle classes. Industrial business, as Mr. Rea had said, had now attained very great proportions; indeed he doubted whether Mr. Rea had at all exaggerated the position it had made for itself when he described it as a matter of national importance. Perhaps, of all the information Mr. Rea had set forth in his paper, the new tables he had produced would be the most valuable. Experts who had to deal with the valuation of industrial companies, as well as students generally, would find some of those tables of very great value to them, especially the table which gave the continuous units of value by the English Life Table No. 3. The others, while not of so much practical value in the mechanical work of valuation, were suggestive in throwing new light upon the subject. He could not help thinking that a very instructive part of Mr. Rea's paper was that in which he endeavoured to show how the enormous mass of statistics of an industrial company are reduced to order for valuation and other purposes. It was extraordinary to observe the mechanical contrivances in an industrial company by which an apparently hopeless mass of figures and papers were reduced to order. The part of Mr. Rea's paper which dealt with valuation was perhaps the least satisfactory, because he had been sparing of the information which he might have placed at their disposal. He (Mr. Ryan) suggested that particulars of the reserves at successive valuations of the company, the amount of negative value excluded, and facts of that sort, taken in conjunction with the average duration of the contract, would have been information of extreme value. He hoped that Mr. Rea might find

time before his paper appeared in the proceedings, to extend that section of his work. He agreed with the author that the current opinion of a generation or so ago, that industrial assurance was akin to fire insurance, was unsound and untenable. To prove that, it only had to be considered that nowadays an immense mass of paid-up policies was on the books of all industrial companies. Moreover, industrial business no longer consisted so largely of whole-life assurances, and endowment assurances were increasingly popular. All those things went to emphasize the statement of the author that the valuation of an industrial company needed just as strict an overhauling actuarially as the valuation of an ordinary company. was glad that Mr. Rea had dealt so trenchantly with the monstrous allegation that industrial assurance was a prominent cause of infantile mortality, but there were other unfair assertions which were frequently heard in connection with industrial assurance in ill-informed quarters. One was that the working man was overcharged for his death benefit. Now, he submitted that if the cost of production was considered in relation to the price which the working man had to pay for it, it would be found that he paid no more for his death benefit than he did for his coals and his beer, or any other commodity which he was forced to purchase in small quantities. It might be a regrettable thing, perhaps, on ethical grounds, that the class least able to afford it should pay a higher rate for their luxuries and necessities than those better off; but political economy, which had to take account of the cost of distribution, offered no way out of the difficulty, and the assertion was undoubtedly unfair. Another statement which also needed examination was that industrial companies' profits were exorbitantly high. On that point, he would ask any impartial person to examine the percentage of profit in relation to the premiums collected, and consider whether that bore a higher or lower ratio than the average profit which a fire insurance company made upon its turnover. If not, it was surely unreasonable to urge that what was perfectly fair and proper in the one case was immoral in the other. Misrepresentations naturally clustered round a comparatively new class of business which had made such enormous strides as industrial assurance, but in time, they would be laid to rest as heresies of the past, and industrial assurance would be recognized as a branch of insurance worthy of respect, and quite deserving of the very great popularity which it had gained for itself.

MR. T. G. ACKLAND heartily echoed the encomiums of Mr. Ryan as to the value of the paper. Members were all, of course, familiar generally with the subject of industrial assurance, with its huge figures and its widespread area of operations: but looking through the thirty volumes of the Journal, he found that there was only one paper which related to the subject, and that not an essay, but a memorandum submitted by Mr. Sutton to a Select Committee of the House of Commons upon topics that were somewhat foreign to those included in Mr. Rea's paper. The members were, therefore, specially indebted to Mr. Rea for bringing forward so interesting a paper on a subject which was comparatively new to the Institute's discussions. He had not omitted to note that Mr. Schooling had recently submitted a valuable paper on the subject, but his paper related more particularly

to methods of valuation, and not so much to the general operations of industrial companies. The figures of the growth of industrial companies given by Mr. Rea succinctly showed that in effect the number of policies, the sums assured, and the annual premiums had all doubled within the last ten years. The results were sufficiently striking, and required, he thought, no further comment. special characteristics of industrial companies were well dwelt upon and illustrated, and the Secession Table deserved some scrutiny. might be expected, the rate of secession was very considerable; but so far as could be judged from the table, that rate would appear to be a function both of the age and of the duration. He thought it had been found in offices doing a somewhat similar business, that the rate of secession usually tended to be a function solely of the duration and was practically constant at different ages of entry. In connection with the temporary increasing insurances on the lives of infants referred to by Mr. Rea, Mr. Ackland considered it was worthy of note that under the English Life Table No. 3, which was usually adopted for the valuation of those assurances, the rate mortality  $(q_x)$  steadily diminished from birth and reached a minimum at age 13, from which point it as steadily increased throughout life. That would appear to point to the fact that the policy reserves in respect of those infantile assurances during their earlier years practically became a negligible quantity. The comparison of claims was very interesting, as showing an apparent deviation from what Mr. Rea considered might have been expected in the case of an ordinary life office, the average duration of those selected claims being about the same as the average duration of the whole of the policies in force in the company. At the same time, he confessed that, to his mind, the result was not altogether conclusive, and he thought it would be more satisfactory if they were exhibited in some relation to the ages With regard to the diagram at the end of Mr. Rea's of the lives. paper, the general characteristics of the curve with regard to the relative distribution of the business at ages passed through, showed, as might be expected, a very marked difference between industrial and ordinary life business, the one curve being of a mountainous character, while the other descended steadily from infancy to old age. Mr. Rea referred to the fact of certain disturbances arising in the curve at quinquennial ages. The difficulty he (Mr. Ackland) found in this respect was that he was somewhat at a loss to understand how one could expect to find any accentuations in the curve at ages passed through. If the diagram showed the distribution of the business at ages of entry, he could well understand that at every fifth age there might be accentuations from the cause indicated by Mr. Rea, namely, the mis-statements of age by "rejection of the odd units." He had been unable to see, however, how Mr. Rea could expect to find in the table as it stood any such accentuations. The author's description of his methods of scheduling, which Mr. Ryan had justly referred to as excellent, gave very full information as to the methods followed. By the courtesy of Mr. Rea, he had, some months ago, the opportunity of surveying in some detail the methods of scheduling the new business, the secessions, and the deaths, and of registering the facts, year by year and age by age, for the annual valuation. The

methods appeared to him to be admirably adapted to the desired ends, and the conclusion he arrived at was that the schedules gave precisely in the form that was desired the results ready for a valuation. only very small criticism he would make upon the schedules would be in respect of Register "C", where it would appear to be somewhat unusual to start the sheet with "durations." The heading of the sheet was the present age, and he should have thought the first column down the side would naturally have been the age at entry, which would have been constant throughout the different sheets, whereas the durations necessarily varied in successive sheets. Mr. Rea made some suggestions as to the methods of valuation, but if he had expanded his remarks, that section of his paper would perhaps have had more value. It did not seem to him (Mr. Ackland) that the plan of throwing off a fixed percentage of the office premium was, in a case of that sort, the most satisfactory; but it appeared to him that the plan, suggested by Dr. Sprague and others, of taking a net premium at age (x+t) with due allowance for initial expenditure, and with regard also to obligations under the policy for surrendervalue and otherwise, was very much preferable to the method of throwing off an arbitrary percentage and then eliminating negative values. In one portion of his paper Mr. Rea referred to the surrender-values and paid-up policies being usually in proportion to the premiums paid. He himself, however, was under the impression that free policies were most usually in proportion to the reserve, and that where they were granted in a fixed proportion to the premium paid, such as 60 per-cent, it was quite inevitable that all sorts of arbitrary and inequitable results would ensue, in respect of granting too much or too little under the free policy. Mr. Rea's schedules were much simplified because he was able to assume that the sum assured and the premium were always in a constant ratio at the same age at entry, so that a penny a week always insured the same sum at the same age. In the case of a large concern, whose figures had come under his own notice, this simplicity was absent; and the difficulty had been very much accentuated by the fact that from changes in prospectuses or transfers, or arbitrary increase or diminutions of the sum assured, the penny a week insured all manner of sums, varying in one case, at a particular age, from £5 to £12. That, of course, necessitated the recording of the sums assured as well as the premiums in the schedules; whereas Mr. Rea's method and the methods indicated by Mr. Schooling, of setting out the premium only, showed at once the fixed proportion in which the sum assured would come With regard to the valuation of joint lives and endowment assurances, the tables published by Mr. Rea must be treated with a great deal of respect, because they bore the honoured name of Mr. Woolhouse as their computor. But from the present-day point of view, he did not consider that it was satisfactory to value joint-life assurances by a single equivalent age. It appeared to him to be open to the grave objection that every year the equivalent age assumed had to be re-computed, and from many points of view the methods indicated by Mr. Schooling, based on Makeham's hypothesis of taking assumed equal ages corresponding to the two joint unequal ages, appeared to be more satisfactory. He did not quite see why Mr. Rea

should make the distinctions he had made as between the valuation of a small or a large number of cases. If there were only a few they might be valued by the true joint ages, and if there were a great number he should certainly value them by Makeham's hypothesis. The method was strictly accurate. It was irrespective of the rate of interest, and he understood from those who had to practically do the work that one could put upon the eard by inspection without the use of any table the equivalent equal ages corresponding to any ordinary case where the ages of husband and wife did not materially differ. With regard to endowment assurances, he also held that the method of valuing by a single equivalent age was unsatisfactory, and he should certainly adopt some such plan as that suggested by Mr. Schooling of setting out the years of maturity, and taking an equivalent average age in each such year, with, of course, the valuable modification recently introduced by Mr Lidstone. The factors under the English Life Table, published by Mr. Rea, were undoubtedly a valuable contribution. It was a constant bother to one who had to use the English Life Table to find that the computor of those tables had not given all that was required for practical

valuation purposes.

Mr. A. H. Bailey said it was more than thirty years ago that his attention was first directed to the subject of industrial assurance, before the passing of the Act of 1870, when there were no data available for the purpose of investigating the financial condition of a wellknown industrial assurance company to which it was proposed to transfer another company. Amalgamation was very much the fashion in those days. It was then that he discovered the material difference between industrial assurance and ordinary assurance. greatest source of that difference was that so large a proportion of the industrial assurances was on the lives of children, an experience which the ordinary office had nothing of. The assurance on the life of the child was the first thing. The mothers cared very much about having what they called "a decent funeral" for their children, and the fathers were always willing to pay the penny a week. And so what is called industrial assurance began with the children, went on to the mothers, and ended with the fathers, and it was very noticeable that the number of women assuring to the number of men was somewhat in the same proportion as the census returns of the country showed in the general population; whereas in ordinary assurances about 10 per-cent only were on the lives of females. The singular thing was that the average premium was under 2d, per week, and the average amount insured about £10. By far the commonest premium was the penny a week, and it was those pennies which in particular instances had produced such enormous premium incomes. He came to see that the ordinary ideas of valuation would not do, and came to the conclusion, which the author spoke rather unfavourably of, that the contracts were of the nature of fire insurance or ordinary term risks. In those early days the weekly premiums paid the weekly claims and weekly expenses, and if there was anything over it was profit. companies then had to deal with the lives of children, and the rate of mortality of children, unlike that of adults, diminished from year to year. How absurd, then, it was to say "How wickedly the

"companies are behaving! What profits they are making on those "people who pay for a little while and then drop their payments!" The thing which excited his indignation more than anything else was the shameful way in which some people, who should have known better, said that men and women insured the lives of their children in order to poison them and get £10. It was utterly untrue, and was, he believed, repeated occasionally even now. He had got into some trouble for saying that the £10 insured was for the provision of funerals and nothing else. It was ridiculous to talk about thrift, for out of the £10, after providing for funeral expenses, there was not much provision for anything. He was a little bit tired of hearing about that in ordinary insurances. Those who, like himself. had had great experience in the matter, knew that a great number of assurances was effected on the lives of gentlemen whose expenditure exceeded their income. There was a notable case of a well-known character, who was in the Bankruptcy Court, and who had figured in the papers during the last few days, who had insurance for more than £100,000 effected on his life. That could hardly be looked upon as an indication of thrift! Those present were not wholly unacquainted with the document known as a Notice of Assignment, and knew perfectly well how large a proportion of claims was paid to other than the relations of the person who died. He should like to make a remark on what had been said about the enormous profits yielded by industrial assurances and the dividing of them amongst the assured. If the rate of profit was taken, it would be found that that rate was very small instead of very large, and in one particular case it might be imagined what it would be to distribute profits amongst one-third of the population of Great Britain and Ireland-13 millions of people! The expense would swamp everything. It was quite impracticable to make a distribution of profits.

Mr. R. Todhunter said the question occurred to him whether the method of prospective valuation was entirely satisfactory as applied to industrial business. Most people would recognize that if the premiums valued were the net premiums according to the table of mortality and the rate of interest employed in the valuation, then the system was entirely satisfactory, not because it necessarily gave a larger reserve than the method of valuing gross premiums less a percentage, nor because the difference between the gross premium and the net had any special validity in itself as a reserve, but because it did ensure continuity from valuation to valuation in the process of estimating the reserve, and also because it gave the result that would be obtained by the retrospective method—that was, by accumulating the net premiums paid less provision for mortality. But when one came to the other method, to taking the value of the sum assured, and deducting the value of the premiums less a percentage, it was very doubtful whether a satisfactory result was obtained. The resulting reserve then had no necessary relation to the premiums paid, and its adequacy or inadequacy depended entirely upon the sufficiency or otherwise of the margin of loading. He had in mind a case in which a valuation, from which all negative values had been excluded, brought out a net liability equal to  $1\frac{1}{2}$  per-cent of the total estimated liabilities under the sums assured,

and of which the surplus amounted to 1-40th of 1 per-cent of the total estimated liabilities. That raised at once a presumption that the percentage of reserved loading was determined after the valuation had been made and not before, and if so, the valuation was practically It was presumed that the principles upon which the liabilities of the company were valued, as stated in answer to the questions in the Sixth Schedule, were determined by à priori considerations before the valuation was made, and it was supposed that the surplus was the quantity brought out as the result of those assumptions. It seemed to him improbable, when a surplus was found equal to 1-40th of 1 per-cent of the total estimated liabilities, that that could be the process employed. He did not say necessarily that the valuation was a bad valuation for that reason; it might be that the expense reserve was more than sufficient; but, nevertheless, it was putting the world on its little end to represent the process as one by which a surplus was discovered by means of a certain method of valuation. Of course the case to which he had referred might be an exceptional one, but he doubted whether there were more than two companies whose net liabilities under their industrial business amounted to 10 per-cent of their total estimated liabilities. sure that the addition of 5 per-cent to the margin of loading in the case of almost any industrial company would be sufficient to turn a substantial surplus into a large deficiency. In the particular case he had under consideration an addition of 1 per-cent in the margin would be sufficient to double the liability. In his view he preferred the method Mr. Ackland had briefly supported. If it was the practice of any one company to stick to one percentage from valuation to valuation, instead of making a valuation by throwing off now 40 per-cent, now 85, and now 45, and in one case 50, one could see some method in the process, but when applied as it was at present, he thought it made it difficult for the management of the company or for the shareholders or anyone interested to understand whether the business was really worked at a profit or at a loss. A point which practically rendered that gross premium method of valuation inappropriate was the modern system of guaranteeing something which was not ordinarily supposed to be on the face of the policy, that was the guaranteeing of a free policy. Taking a policy effected on the life of an adult aged 30, assuring £6 for a penny a week, it would be found that it was now usual under the stress of competition to allow a free policy for about 15s. after 5 years. Under the English Table No. 3, the cash value would be about Ss. 6d. or 9s.; so that it seemed unsafe, or would be unsafe if the secessions were not so numerous as they were, to neglect or attach only a small value to that liability during the first 5 years. With regard to endowment assurances, they were of recent date, but they were increasing enormously in number and the liability under them would be very much heavier than the liability likely to arise under the whole life policy. The insurance was an investment rather than a provision for funeral expenses, and the policies were kept up much more persistently than whole life assurances. He hardly saw how the substitution of an addition to the age for the endowment term would meet the case, because it would not provide for the accruing liability

at the end of the endowment period. The tabulation of the sums assured under the year in which they fell due, and a full provision for those falling due within the earlier years, seemed to be necessary.

Mr. James Sorley thought that, in their criticisms, they were scarcely showing due sympathy with the distinctive position of industrial offices. For them a uniform pure premium valuation was not even an ideal to be aimed at, for it produced negative values during the period of childhood, and a large proportion of industrial assurances was on the lives of children. The system of valuation which valued the future premiums under deduction of a uniform percentage and excluded negative values seemed to him much more business-like than the one which assumed a pure premium applicable to an age n years older than the true age at entry. Under the latter, the premium valued for old ages at entry might be even larger than the premium actually payable: while under the former, assuming the percentage to be adequate, the company would be kept solvent even

in the extreme case of none but old lives insuring with it.

Mr. H. W. Manly thought that, now industrial assurance formed a very large portion of the business of insurance in this country, it would be considered more thoroughly, and that a great many of the points, which at the present moment seemed doubtful. would be threshed out. He should like to pay a very great tribute of praise to the organization which industrial companies had set up with a view of classifying the business, of tracing it from stage to stage, and their facility for forming schedules for valuation. He had been surprised particularly with one company with which he had been connected, to find with what rapidity, with what accuracy, they could produce schedules of statistics which he had asked for, and which certainly in their own offices would have taken a very long time and a great deal of trouble. It seemed marvellous to him how this was done. It was, of course, done by an intelligent system of organization, distribution of labour, and the collection and classification of facts with a view to the desired end, as had been shown by Mr. Rea in his paper; and he thought many of them might even take a lesson from some of the large industrial companies. He must confess he did not agree with Mr. Sorley. He did not think that in industrial premiums many cases would be found where five years added to the age would produce a net premium larger than the premium charged. He had never found it himself, and if such a thing did happen, he should be inclined to think that that company was charging too little for the benefits which it was giving. As had already been said, a very large portion of the early premium must necessarily go in expenses; and from the table which Mr. Rea had given, it would be seen how very little of the business remained on for five years. During that time the premiums were barely more than sufficient to provide for the claims and expenses, and until the business became permanent, that is, until it had been in force five years, he did not think there was any reason whatever for making a reserve, unless the company had the money to do it, which, he found, meant until it had an income of at least £250,000 a year. He put that income as somewhere about the lowest upon which it was possible for an industrial company to make any profits, because they had to set up a large organization, which

practically spread throughout the kingdom, was in itself expensive before any business was done, and was capable of transacting, with very little more cost, about treble the business with which it started. Until an industrial company had secured a premium income of £250,000 it could hardly expect to make any profit at all. He quite agreed with Mr. Bailey, having studied the matter for some years, that for the early years of the business it should be treated very much as fire business. If the premiums paid for the expenses and the claims, they could not do much else, but after five years it might be presumed that that business was going to be permanent, and from that date might be treated as new ordinary business, and a net premium reserve made as from that date. Endowment assurances had descended in the social scale, and the industrial classes were now largely taking them up at 1d., 2d., 4d. and 6d. a week. Unless the companies were very eareful with this class of business, he thought it was very possible that they might find themselves landed some day in difficulty. No doubt the people who took up industrial endowment insurances did so with the intention of keeping them up and of maintaining them, at any rate, as long as possible. Moreover, endowment assurances, especially for the shorter terms, would not bear the same rate of loading as the whole life premiums; and it might be possible, in the case of a comparatively short term endowment of 15 or 20 years that, by adding 5 years to the age, the net premium would be larger than the premium charged. regard to the paid-up policies which were granted as surrender values, he was distinctly of opinion that they should have the full net reserve in hand for them. In the valuation of an industrial company, they were bound to set aside sufficient to provide for those endowment assurances which would mature in the early years and for the paid-up policies, as well as for those policies which had practically become permanent and had been in existence over five When that was done, it should be seen that a proper reserve was made for expenses on the whole business, always excluding He complimented Mr. Rea upon the production negative values. of his paper, and felt sure that it was only an earnest of what he could do and would do for the Institute in the future.

Mr. A. R. Barrand agreed with Mr. Rea that "the infantile "business now transacted by our industrial companies differs in almost "every important feature from the business of ordinary life offices", but he believed that with regard to adult insurances it was rather a difference of degree than of kind. It had been remarked that industrial companies had been the Cinderellas of the life assurance world, but to some extent the positions were now becoming reversed, for at least one well-established company in Great Britain had paid the industrial companies the high compliment of starting to collect its premiums weekly by special arrangement with large firms. With regard to the criticism that the rate of expenditure was exceedingly high, he thought Mr. Rea had shown very conclusively that, in view of the large amount of work that had to be done, the rate was not extravagant, and when it was borne in mind that the agent was not merely an agent to collect premiums, but was practically medical referee as well, it would be admitted that he was not too highly paid.

He need hardly say anything about the profit from lapsing policies, except, perhaps, to point out that it appeared from the paper that valuing by a gross premium method, and reserving presumably sufficient to fully meet the expenses, negative values were brought out for about the first five years of the insurance, so that there was practically no reserve up to that time. After that time the free policies were granted, forming a very substantial proportion of the true net premium reserve held against the policy valued by a strict net premium method. With regard to the remarks as to the dangers of offering free policies based upon the amount of premiums paid, as far as his experience went, it was not customary to do so, but the free policies were based upon the actual reserve held against that policy at the time the assured became entitled to a free policy, and under those circumstances there seemed to be very little or no risk in offering such inducements. Mr. Rea spoke of a policyholder becoming entitled to a surrender-value policy, which was equal to about 30 per-cent of the premiums paid. He imagined Mr. Rea only referred to the paid-up policy and not to the policy having any surrender-value attached to it. It was not customary to offer any cash surrender-values, but the equivalent of a free policy based on the amount of the reserve held against it. As to the charges that industrial assurance encouraged murder among children, it was not necessary to refer to that except to point out that, taking into account the numbers assured, ordinary assurance would appear to be a much stronger incentive to murder than industrial assurance, and every argument brought against the latter on this ground could be brought with greater force against the former. There was another point in reference to medical examination being dispensed with in industrial assurance; as, doubtless, most of the members were aware, the agent's examination, although it might be effectual, yet was not altogether relied upon. The family history was taken into account, and there was a graduated system of benefits extending from a quarter of the sum assured for the first three months, the full amount not being payable until the policy had been in force twelve months, thus guarding against extremely bad lives being introduced in spite of the careful scrutiny of the agent. regard to the ages given not being correct, it was not surprising, because the assured had everything to gain and nothing to lose in giving a wrong age in industrial assurance. If he gave a wrong age the policy was not forfeited, but he got the amount which he would have got if he had given his right age. Some remarks had been made on Mr. Rea's table of single life equivalents for joint lives. It was difficult to see what advantage could be gained by that method under any circumstances over the method of two equivalent joint lives. Little or no trouble would be saved by adopting the former method, and very serious inconvenience was entailed by it in connection with carrying on the valuation from year to year. Mr. Rea suggested that he had followed Mr. Schooling's method of grouping endowment assurances, but he did not say whether he had also adopted that method of not only dividing it into years to run, but sub-dividing those into groups so as to get smaller limits of ages. The President had suggested that the time had at length arrived when the insurers under industrial assurance contracts should be allowed to

share in the profits. To some extent they were already doing so, as the companies were giving larger benefits than those which they originally guaranteed under their contracts. Some of the difficulties had arisen from the fact that the large profits, or what appeared large considered without regard to the actual total of premiums, had arisen in the following way. Industrial assurance business had been a very expensive thing to earry out, particularly in its early days, and companies had done everything in their power to reduce the expense ratio, and had been very successful, in many eases, in doing so. But the premiums they were charging were the premiums they originally had to charge when their expenses were larger. The tendency was, and would increase as time went on, for companies to allow their insurers a share in those profits which they were able to make by great economy in their business. A question had been raised as to the relative merits of the gross premium method and the net premium He was not going to advocate the gross method of valuation. premium method, because he should certainly follow the speakers who had advocated the net premium method. The point had been noted already, that if the net premium was valued under the English Life Table No. 3, negative values were obtained for the infantile policies. Speaking from memory, a policy taken out between ages 0 and 1 had a negative value up to age 33, but that was a somewhat exceptional Although he was not prepared to advocate the gross premium method, there was a justification for it in the case of industrial assurance companies. As already pointed out, the premiums had had to be loaded very heavily for expenses. Under careful management, those expenses had proved to some extent to be a decreasing quantity. Under those circumstances, with the premiums based on the English Life Table No. 3, loaded with 50 to 100 per-cent, it might have been necessary originally to reserve the whole of the loading, but if the expenses had been reduced this would no longer be necessary.

Mr. F. J. Vincent asked the author if he did not find that the average weekly premium was rather increasing as time went on, probably due, as Mr. Bailey had pointed out, both to the increase in adult industrial assurance and also to the increase in the number of endowment assurances amongst them. If the statistics were examined, he thought it would be found that the average premium was increasing, and that 2d was a little below the average on the whole. He thought that the ages in industrial assurance must be taken as being only approximate, and in no sense so exact as in ordinary life assurance, inasmuch as the statements of the proposers must be taken on trust to a very great extent, and verification would prove too troublesome and costly. In this respect very curious instances sometimes came to light. It was not unknown in one part of the country, he believed, for the policy to be marked in small figures in the corner in order to give the person who registered the death a hint that at the date of the policy the person insured was of that age, so that he might register the death in accordance with it and thus avoid any deduction in the sum assured when the claim eame to be settled. He instanced a fact that had come before him that very afternoon, when, in examining a file of claim papers, he found some correspondence in which the claimant elaborately explained that if somebody else had registered the death the age at death would have been recorded very differently to what was given on the Registrar's certificate. Having regard to the systems of valuation that had been mentioned that evening and the differences in the rates of expenditure between various offices, in criticizing valuations and making comparisons of one office against another, regard ought to be had to the expense ratio. It might be necessary for one office to deduct an average of 40 per-cent from its gross premium income for future expenses, whereas another office only required to reserve 35 per-cent for that purpose, and yet the financial position of both offices be equally sound: for even with net premium valuations the results were not always to be taken as absolutely true unless after comparison with the results obtained by the deduction of the average percentage of expenditure experienced during the quinquennium under review.

The PRESIDENT, in expressing the cordial recognition of the meeting to Mr. Rea for the service he had thus rendered to his profession and the Institute, took the opportunity of adding that the members assuredly looked forward to a happy continuance of contributions on Mr. Rea's part to their professional literature.

Mr. C. H. E. Rea, in reply, thanked the President and members for the very kindly way in which they had received his paper, the first contribution he had had the honour of presenting to the Institute. He had prepared the paper under circumstances of some difficulty, and it only pretended to be an introduction to the subject. The last speaker (Mr. Vincent) had questioned the average amount of premium, but he thought he was right in stating that the average was slightly under 2d. rather than over that amount at the present time: however, he agreed that it was tending to increase with the continued demand for endowment assurance. With regard to the surrendervalue on industrial policies, he thought it was the almost invariable rule for industrial companies to decline to give a cash-value, and that the surrender took the form of a paid-up policy. When he mentioned the fact of its being something equivalent to 30 per-cent of the premiums paid, he did not wish it to be understood that the value was, or should be, calculated on that basis. This rate was only mentioned roughly to show the approximate value of the return. He was much indebted to Mr. Ackland for his remarks with regard to the diagram, because they brought to his notice that the expression in the first part of the paragraph was not quite clear. The reference made there to the irregularity in the curves intended to apply to the several diagrams referred to in the paragraph, and not to the drawing given in the paper where the ordinates are, as pointed out by Mr. Ackland, functions of the present age. In the several curves of the individual office referred to the entry age was laid down on the abscissa, and at the decennial ages referred to there was a decided bump, followed by a depression. He hardly followed Mr. Ackland in his reference to Schedule C. Each page of this register represented the classification at exact age at valuation date. The various durations were given in the first column down the side, which seemed to him to be the most convenient and comprehensive form for the records. He did not, throughout the paper, advocate

any particular basis of valuation. He only put forward a means of classifying, which could be adapted to almost any form of valuation, just as was thought best. Of course, the method of estimating the reserve on a net premium basis carried very important considerations, and it was very simple in those schedules to have a column for net premiums and to continue them in the classification. The formula given for the elimination of negative values of course only applied to gross premium valuations. With regard to the tables of equivalents, he stated distinctly that in the recent valuations he had made he had followed Mr. Schooling's methods with advantage. He thought there was no question that it was better to employ two equal joint lives than one single life equivalent. It was next to impossible to use a single life equivalent where it was intended to keep up a continuous method of classification. The single life tables were given as there were other distinct values attaching to them. He again thanked the members for their courtesy in listening to his paper.

## Sheppard Homans.

THE news of the sudden and somewhat tragic death of Sheppard Homans must have been much of a shock to his many friends among the members of the Institute. He was probably known to nearly all as the oldest and most prominent American actuary, since his name has appeared in the Journal for forty years past, either in commendatory references by the foremost men, or in connection with original contributions of great value, and he had also visited England very frequently, becoming personally acquainted with nearly all the actuaries of London and Edinburgh who have been prominent during the last forty years.

Mr. Homans was born in Baltimore, 12 April 1831. After graduating at St. Mary's College there, he entered Harvard University in 1849, devoting himself specially to mathematics and astronomy; in these studies he so excelled that, while still a student, he was appointed by the United States Government to conduct an astronomical expedition to determine the difference in longitude between Liverpool and Boston. This work occupied many months, during which he made two visits to England, and it was so well done that he was appointed an officer on the United States Coast Survey, and afterwards Astronomer on several expeditions across the then wild western plains. In 1855, Professor Gill, the first Actuary of the Mutual Life Insurance Company of New York, died, and Mr. Homans' friends

procured his appointment to the position without his knowledge. He was at that time on a western expedition, and, as he once remarked, "did not know anything whatever of life insurance—not even what a premium was."

His high mathematical ability and singular industry soon enabled him to master the science so fully, that his researches, "The Mortality Experience of the Mutual Life Insurance Company," (1859), earned the commendation of his seniors in the profession—see the Paper of Samuel Brown (J.I.A., vol. viii, p. 193)—so that when he visited England in 1861, he was warmly received by the Institute, among whose members his cordial manner made him many friends. During this visit he consulted English actuaries as to the best and most equitable mode of dividing surplus, having become convinced that the percentage method should be abandoned. The Jellicoe method was recommended and was adopted by the Mutual Life Insurance Company on Mr. Homans' return, but shortly afterwards further consideration led him to recommend a different and entirely novel method, which he urged so forcibly, that his company adopted it, namely, that which is known as the "contribution plan." It was applied to the company's quinquennial distribution in February 1863, when \$3,000,000 was divided. Despite his arduous labours at home, he immediately contributed an article on "The Equitable Distribution of Surplus" to the Journal of the Institute (Vol. xi, p. 121). This is thought by many to have been, directly or indirectly, the means of revolutionizing the methods of division everywhere.

Mr. Homans' next most important achievement was the production of a mortality table based partly on the experience of the Mutual Life Insurance Company of New York and partly on previous tables. This was adopted first by his own company and afterwards by the State of New York, by which it was entitled the American Experience Tables; it soon became—as it is still—the basis of the premium tables of all the New York companies, besides many others, and thus promises to prove a lasting memorial of him.

In 1870, he attended the Statistical Congress at the Hague, where he presented the experience of his company for its first twenty-two years, and called attention to the remarkable fact, that the rate of mortality amongst those insured under endowment assurance policies was much less than among holders of whole life policies, while among the latter, those who held limited

payment life policies had a lower death rate than those who took the ordinary continuous premium life policy; from which he deduced the general proposition that in any particular company, the rate of mortality among persons selecting high premium forms of policy would always be found to be less than among those insuring on low premium forms, which probably had never been so clearly demonstrated before, if, indeed, it had been ever pointed out.

In 1871, he resigned his appointment as Actuary of the Mutual Life of New York and became a consulting actuary. In 1875, he founded the Provident Savings Life Assurance Society, to embody his plans for insurance without large reserves, and his idea that business could be obtained by employing salaried solicitors or agents without paving commissions. Though at first he obtained considerable business from the large circle of persons friendly to himself or his shareholders, the system did not prove a permanent success, but Mr. Homans struggled bravely onward, and by giving up the salaried agency system, finally plucked victory from the very jaws of defeat. His health became broken under the long strain, and in 1886 he made a trip around the world. In 1890 and 1894 he visited England and the Continent, making long stays, and, on his return in 1895, resigned the presidency of his company and resumed independent actuarial work, but his attention was much engrossed by various outside ventures, so that he did little strictly professional work.

In 1889, he took part in the movement for the formation of the Actuarial Society of America and became its first President. He was a very active and zealous member, never missing any meeting unless abroad, and was most highly esteemed by all the members. His courtesy to the youngest or least prominent was as marked as his cordiality to his older associates, so that all feel they have lost a dear friend. Mr. Homans won the regard of all his subordinates by treating them, as one lately said to the writer, "more like an elder brother than a superior officer." His disposition to do them justice was strikingly illustrated in the last paragraph of the above-mentioned article on the distribution of surplus (Vol. xi., p. 127).

Mr. Homans was much opposed to the excessive Governmental supervision that obtains in the United States, and in February 1897 presented to the Institute a paper on that subject, which was cordially welcomed, not only as the production of an old friend,

but for its clear, graceful, and forcible treatment of that important subject.

His home was in a suburb of New York; he also had considerable real estate and a seaside house on Long Island. He was fond of cycling, and probably over-exerted himself. On the evening of the day he died, 8 January 1898, after riding a bicycle he ran to get on a trancar, the exertion was too great, his heart gave way, and he died a few rods from home, where his family were awaiting him.

His genial yet dignified cordiality will ever be remembered by those who knew him personally, and his scientific achievements will insure him a permanent place in the insurance Valhalla.

D. P. F.

### ACTUARIAL NOTE.

On Gompertz's Law of Mortality. By John Govan, F.F.A.

THE following is, I believe, a new proof, more direct than any that has yet been offered, of the propositions regarding Gompertz's and Makeham's hypothesis, treated by Messrs. Allen and Levine in the January number of the *Journal*.

The fundamental condition may be stated as either

$${}_{n}p_{x\,n}p_{y\,n}p_{z}\dots$$
 (N factors) =  $({}_{n}p_{w})^{\mathrm{M}}$   
 $p_{x+n}p_{y+n}p_{z+n}\dots$  (N factors) =  $(p_{w+n})^{\mathrm{M}}$  . . (A)

w being a function of xyz . . . . M, but not of n. Either of these equations is easily derivable from the other.

The two propositions are

- (1) If M is to be any positive integer at will, independent of N, the function called the Force of Mortality is necessarily of the exponential form.
- (2) If M=N, the differential coefficient of the Force of Mortality is necessarily of the exponential form.
- (1) Let  $\log p_x$  be denoted by f(x), and let xyz..., which, ex hyp., may have any value we please, be supposed all equal, and of the value zero. Then from  $(\Lambda)$  we have

$$Nf(n) = Mf(n+w)$$
 or 
$$f(n+w) = \frac{N}{M}f(n) . . . . . . . (B)$$

which means, that when the variable is increased by the constant w, the function is multiplied by the constant

 $\frac{N}{M}$ ; whence  $f(tw) = f(v) {N \choose M}^t$  or say, taking w as our unit of measurement,  $f(x) = Bc^x$ . Log  $p_x$  being of the exponential form, it follows that the Force of Mortality is of that form also. For  $\log p_x$  is  $\Delta \log l_x$  and  $\Delta_x = 1$ , when the unit of time is small, whilst the Force of Mortality is  $-\frac{\Delta \log l_x}{\Delta_x}$  under the same conditions.

(2) Reverting to  $(\Lambda)$ , and again employing f(x) as meaning  $\log p_x$ , we have

 $f(x+n) + f(y+n) + f(z+n) \dots$  (N terms) = Nf(w+n)Differentiate with respect to any one of the quantities  $xyz \dots$  say x, then

$$f'(x+n) = Nf'(w+n)\frac{dw}{dx}. \quad . \quad . \quad (C)$$

This is the same equation as (B), with the exception that the function in (C) is the derivative of the function in (B).

The following demonstration proceeds upon entirely different lines, but is almost equally direct (M=N).

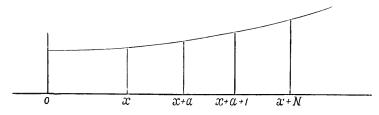
The quantities xyz.... as above said, may be anything we please. Taking them in a.p. with unity as their common difference, we have from (A), n being zero, and w=x+a,

$$p_{x}p_{x+1}p_{x+2}\dots p_{x+N-1} = (p_{x+a})^{N}$$

x has now become the independent variable, and a is a function of N, but not of x. That is to say, x may be anything we please, N may be anything we please independently of x, and the value of a, which lies between 0 and N-1, is regulated solely by that of N. From this we easily pass to

$$\log l_{x+N} - \log_x = N \log p_{x+a} \quad . \quad . \quad . \quad (D)$$

By way of illustration, let  $\log l_n$  be represented in the following figure by the area enclosed by the axis of y, the axis of x, the curve, and the ordinate at n, the said area being taken negatively.



The essential characteristic of our function is this. Take any perpendicular section of the area whatever, say that with base x, x+N. The said area will, of course, be equal to N times the area of a section having unity for its base, and situated at another part of the curve, say from x+a to x+a+1, the magnitude of a being determined solely by that of N. Thus, if we go to any other part of the curve whatever and erect two ordinates standing at the distance N from one another, say x', x'+N, we shall find that the area enclosed by these two ordinates is again N times the area enclosed by the ordinates x'+a, x'+a+1. Therefore, if we represent by  $\Delta$  the difference of the function for difference unity of the variable, we have, at any part of the curve whatever,

$$\Delta F(x) = N \delta F(x+a).$$

We can give to X any value we please, and we can likewise choose as our unit of measurement any value we please. If we take the latter small, this equation becomes

$$\mathbf{F}(x+\mathbf{N}) - \mathbf{F}(x) = \mathbf{N} \ dx \ \mathbf{F}'(x+a),$$

or since dx is unity by supposition,

$$F(x+N) - F(x) = NF'(x+a)$$
. (E)

Differentiate (E) first with respect to x and then with respect to N,

$$F'(x+N) - F'(x) = NF''(x+a)$$
 . . . (F)

$$F'(x+N) - F'(x+a) = NF''(x+a) \frac{da}{dN}$$
. (G)

but by (F), which is true for any values whatever of x and N, we get, writing x + a for x and N - a for N,

$$F'(x+Y) - F'(x+a) = (Y-a)F''(x+a')$$
. (H)

where the value of a' lies between a and N. Finally we have from (G) and (H),

$$\mathbf{F''}(x+a') = \frac{\mathbf{N}}{\mathbf{N}-a} \frac{da}{d\mathbf{N}} \mathbf{F''}(x+a)$$

This is the same equation as (C), because  $a \ a' \ N \ \frac{da}{dN}$  are all constants with respect to x.

Students of the Differential Calculus will recognize the similarity between equation (E) and that upon which Taylor's Theorem is founded. What distinguishes the function above

called F from continuous functions in general is that x does not

enter into the composition of a.

The foregoing deals only with Makeham's hypothesis. To deduce Gompertz's formula on the same lines, differentiate (A) in logarithms with respect to M, keeping the left-hand side of the equation constant. Then

$$0 = f(w+n) + Mf'(w+n) \frac{dw}{dM}$$

This means that  $\frac{f'(n)}{f(n)}$ , that is, the differential coefficient of  $\log \log p_n$ , is constant. Thus  $\log \log p_n$  is of the form kn + a, and  $\log p_n = \epsilon^a (\epsilon^k)^n$  or say BC<sup>n</sup>.

#### CORRESPONDENCE.

ON THE GROUPING OF ENDOWMENT ASSURANCES FOR VALUATION.

To the Editor of the Journal of the Institute of Actuaries.

Sehr geehrter Herr,—Zu meinem grossen Bedauern bin ich der englischen Sprache nicht so weit mächtig, dass ich im Stande wäre einen längeren Brief zu schreiben, darum bitte ich Sie um Entschuldigung, wenn ich deutsch schreibe.

Es ist mir immer ein besonderes Vergnügen das Journal des Institute of Actuaries zu lesen, so war es auch diesmal der Fall, wo die April Nummer zwei hochinteressante Abhandlungen über die Berechnüng der Prämien-Reserven gemischter Versicherungen

"Valuation of Endowment Assurances" brachte.

Da bei uns die gemischten Versicherungen schon seit Yahrzehnten eine sehr wichtige Rolle spielen, und da 70–75 % der neuen Geschäfte nach dieser Combination abgeschlossen werden, mussten wir eine einfache Methode der Prämien-Reserve-Bereehnung ersinnen, und da ich glaube, dass diese Methode für die englischen Fachcollegen von Interesse sein wird, erlaube ich mir, sie hier kurz zu erlaütern.

$$\begin{aligned} {}_{k}\mathbf{V}_{x\bar{n}} &= \mathbf{A}_{x+\kappa|\overline{n-\kappa}|} - \mathbf{P}_{x\bar{n}} \cdot \mathbf{a}_{x+\kappa|\overline{n-\kappa}|} & \left[ \mathbf{a}_{x\bar{n}} = \mathbf{1} + |_{n-1}a_{x} \right] \\ &= \mathbf{1} - d \cdot \mathbf{a}_{x+\kappa|\overline{n-\kappa}|} - \mathbf{P}_{x\bar{n}}|\mathbf{a}_{x+\kappa|\overline{n-\kappa}|} \\ &= \mathbf{1} - d \cdot \mathbf{a}_{x+\kappa} - \mathbf{P}_{x\bar{n}}|\mathbf{a}_{x+\kappa}| + \left[ \mathbf{P}_{x\bar{n}} + d \right] \frac{\mathbb{N}_{x+n}}{\mathbf{D}_{x+\kappa}} \\ &= \mathbf{A}_{x+\kappa} - \mathbf{P}_{x\bar{n}}|\mathbf{a}_{x+\kappa}| + \frac{1}{\mathbf{D}_{x+\kappa}} \mathbf{H}_{x\bar{n}}| \text{ wo } \mathbf{H}_{x\bar{n}}| = \frac{\mathbb{N}_{x+n}}{\mathbf{a}_{x\bar{n}}} \end{aligned}$$

Die Gruppirung der Versicherungen hat also nach dem Geburtsjahr zu erfolgen, damit für alle Versicherungen  $x+\kappa$  denselben Werth habe, in diesen Gruppen können auch die lebenslanglichen Ver-

sieherungen untergebracht werden. Das folgende Schema zeigt den Vorgang deutlich:—

Geburtsjahr 1800 + u.

| Policen- Versicherungs- Versicherung   |   | Јанкисне Ркаме   |   | Hilfszahl  |  |
|--|---|--|---|--|--|
| Nummer Summe Art   | P   | P.S  | н   | H.S  |  |
| $\begin{array}{c cccc} S_1 & P_{30} \\ S_2 & A_{32} \\ S_3 & P_{32;\frac{120}{32}} \\ S_4 & A_{33}; \overline{15} \\ & & & & \\ \end{array}$ | $egin{array}{c} P_{30} \\ 	heta \\ P_{32}; \overline{20} \\ 	heta \\ 	heta \\ 	heta \\ 	heta \end{array}$ | $egin{array}{cccc} P_{30}, S_1 & & & & \\ & P_{32;20}, S_3 & & & \\ & & & & \\ & & & & \\ & & & &$ | $egin{array}{c} 	heta \ 	het$ | $egin{array}{c} \\ H_{32}, \overline{20}, S_3 \\ K_{33}, \overline{15}, S_4 \end{array}$ |  |
| Σ(S)   |   | Σ(P.S)   |   | Σ(H.S)   |  |

\* 
$$\mathbf{K}_{xn} = d \mathbf{\Pi}_{x+n}$$
.

- Prämien-Reserve am Ende des Kalender-Jahres 
$$1800 + w = 1800 +$$

$$\begin{split} & \frac{1}{2} \Sigma(S), [A_{w-u} + A_{w-u+1}] - \frac{1}{2} \Sigma(P.S) [\mathbf{a}_{w-u} + \mathbf{a}_{w-u+1}] \\ & + \frac{1}{2} \Sigma(H.S) \left( \frac{1}{D_{w-u}} + \frac{1}{D_{w-u+1}} \right) + \frac{1}{2} \Sigma(P.S) \end{split}$$

Der bei uns herrschende Usus, als Eintritts-Alter (x) das Alter an dem dem Eintritts-Datum zunächst liegenden Geburtstag auzunehmen und die Dauer der Versicherung (n) in vollen Versicherungs-Jahren zu bestimmen, macht es in vielen Fällen nothwendig, das Geburtsjahr zu korrigiren u. zw. derart, dass man bei der Gruppirung als Geburtsjahr die Differenz zwischen Eintritts-Jahr und Eintritts-Alter annimmt.

Diese Methode hat den Vorzug, dass sie sehr einfach und—wenn man die Voraussetzung, dass alle Versieherungen in der Mitte des Jahres abgeschlossen werden, gelten lässt—genau ist, weshalb sie sich wohl bald in der Praxis der englischen Actuare einburgern dürfte.

Will man den Werth der Kapitalien und den Werth der Prämien separat bestimmen, so muss man einfach  $\mathbf{H}_{xn}$  in seine beiden Theile  $\mathbf{K}_{xn} = d \mathbb{M}_{x+n}$  und  $\mathbf{K}'_{xn} = \mathbf{P}_{xn} \mathbb{M}_{x+n}$  zerlegen, es ist dann pro Ende eines Versicherungs Jahres.

Werth der Kapitalien=
$$\Sigma(S)$$
.  $A_{x+k} + \Sigma(K_{xn}|.S) \frac{1}{D_{x+k}}$  und

Werth der Prämien = $\Sigma(P)$ .  $\mathbf{a}_{x+k} - \Sigma(K'_{xn}|.S) \frac{1}{D_{x+k}}$ , also

Prämien-Reserve = $\Sigma(S)$ .  $A_{x+k} - \Sigma(P)$ .  $\mathbf{a}_{x+k} + \frac{1}{D_{x+k}} [\Sigma(K_{xn}|.S) + \Sigma(K'_{xn}|.S)]$ 

Diese Methode kann auch auf alle zahlreichen anderen Versicherungs-Kombinationen, die bei uns üblich sind, angewendet werden, doch glaube ich, dass es überflüssig ist hierauf näher einzugehen.

In der angenehmen Hoffnung, durch diese Mittheilung meinen Herren Collegen in England einen klienen Dienst erwiesen zu haben, bitte ich Sie den Ausdruck meinen ganz besonderen Hochachtung zu genehmigen.

Ihr. ergebener,

#### JULIUS ALTENBURGER,

Vice-Präsident des Verbandes der oest. und ungarischen Actuare.

Triest, 27 April 1898.

#### (Translation.)

Honoured Sir.—To my great regret I am not sufficiently conversant with the English language to be in a position to write a long letter; I ask your indulgence, therefore, if I write in German.

It is always a special pleasure to me to read the Journal of the Institute of Actuaries, and this was the case again when the April number brought two highly interesting papers on the valuation of endowment assurances.

Seeing that among us endowment assurances have for some decades already played a very important part, and that 70 to 75 per-cent of the new business belongs to this class, it became necessary for us to devise a simple method of valuation, and as I believe that this method may be of interest to my English colleagues, I venture to explain it here briefly:—

$$\begin{aligned} {}_{k}\mathbf{V}_{x\overline{n}} &= \mathbf{A}_{x+k|\overline{n-k}|} - \mathbf{P}_{x\overline{n}|}.\mathbf{a}_{x+k|\overline{n-k}|} & \left[\mathbf{a}_{x\overline{n}|} = \mathbf{1} + \mathbf{1}_{n-1}a_{x}\right] \\ &= 1 - d\left[\mathbf{a}_{x+k|\overline{n-k}|} - \mathbf{P}_{x\overline{n}|}\mathbf{a}_{x+k|\overline{n-k}|}\right] \\ &= 1 - d\left[\mathbf{a}_{x+k|} - \mathbf{P}_{x\overline{n}|}\mathbf{a}_{x+k} + \left[\mathbf{P}_{x\overline{n}|} + d\right] \frac{\mathbf{Y}_{x+n}}{\mathbf{D}_{x+k}} \\ &= \mathbf{A}_{x+k} - \mathbf{P}_{x\overline{n}}\mathbf{a}_{x+k} + \frac{1}{\mathbf{D}_{x+k}} \mathbf{H}_{x\overline{n}|} \text{ where } \mathbf{H}_{x\overline{n}|} = \frac{\mathbf{N}_{x+n}}{\mathbf{a}_{x\overline{n}|}} \end{aligned}$$

Grouping the Assurances now according to the year of birth and so bringing together all those for which x+k has the same value, we can bring the whole-life policies also into the group. The following scheme will show the procedure clearly—

## Year of Birth 1800+u.

| Policy | Sum  | um Class of   |  | Рвемиим  | Auxili  | ARY No.  |
|--------|--|---|--|--|---|--|
| No.    | Assured  | Assurance   | Р  | P.S  | Н   | H.S  |
|        | S <sub>1</sub><br>S <sub>2</sub><br>S <sub>3</sub><br>S <sub>4</sub> | $\begin{array}{c} P_{30} \\ A_{32} \\ P_{32}, \overline{_{20}} \\ A_{33}, \overline{_{15}} \\ \vdots \end{array}$ | $egin{array}{c} \mathbf{P}_{30} \\ 	heta \\ \mathbf{P}_{32}, \overline{20} \\ 	heta  | $\begin{array}{c} P_{30}, S_1 \\ \\ P_{32;20}, S_3 \\ \\ \vdots \\ \vdots \end{array}$ | $egin{array}{c} 	heta \ 	het$ | $H_{32:\overline{20}}, S_3 \atop K_{33:\overline{15}}, S_4 \atop \vdots$ |
|        | $\Sigma(S)$  |   |  | $\Sigma(P.S)$  |   | Σ(H.S)   |

The reserve at the end of the calendar year 1800 + w will be

$$\begin{split} & \frac{1}{2} \Sigma(S) \cdot \left[ A_{w-u} + A_{w-u+1} \right] - \frac{1}{2} \Sigma(P.S) \left[ a_{w-u} + a_{w-u+1} \right] \\ & + \frac{1}{2} \Sigma(H.S) \left( \frac{1}{D_{w-u}} + \frac{1}{D_{w-u+1}} \right) + \frac{1}{2} \Sigma(P.S) \end{split}$$

The prevailing practice with us of taking the age at entry (x) as the nearest age at the date of entry and to determine the duration of the assurance (n) by integral policy years, makes it necessary, in many cases, to correct the year of birth in such a way as to take in the grouping for the year of birth the difference between the year of entry and the age at entry.

This method has the advantage that it is very simple, and (if we assume that all policies are effected on the average in the middle of the year) exact, on which account it may soon be adopted by English

Actuaries.

If one wishes to determine the values of the sums assured and of the premiums separately, one need simply express  $H_{x\overline{n}}$  in its two parts,  $K_{x\overline{n}} = d \, \mathbb{M}_{x+n}$  and  $K'_{x\overline{n}} = P_{x\overline{n}} \, \mathbb{M}_{x+n}$ . We then get, at the end of the Policy-year,

Value of Sum Assured 
$$= \Sigma(S) . A_{x+\kappa} + \Sigma(K_{x\overline{n}}|.S) \frac{1}{D_{x+\kappa}}$$
 and

Value of Premiums  $= \Sigma(P) . a_{x+\kappa} - \Sigma(K'_{x\overline{n}}|.S) \frac{1}{D_{x+\kappa}}$ , so that

Reserve  $= \Sigma(S) . A_{x+\kappa} - \Sigma(P) . a_{x+\kappa} + \frac{1}{D_{x+\kappa}} [\Sigma(K_{x\overline{n}}|.S) + \Sigma(K'_{x\overline{n}}|.S)]$ 

This method can also be applied to all the many other kinds of assurances which we grant, but I do not think it necessary to dwell on this further.

In the pleasant hope that by this communication I may be of some slight service to my colleagues in England, I beg you to accept the expression of my special regard.

Yours faithfully,

## JULIUS ALTENBURGER,

Vice-President of the Society of Austrian and Hungarian Actuaries.

Trieste, 27 April 1898.

[Many of our readers will probably remember the very similar formulas given by Mr. Wm. D. Whiting, and published in the Transactions of the Actuarial Society of America, No. 12, p. 427.—Ed. J.I.A.]

# THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1897, published in 1898.]

| INCOME                              | Ordinary<br>Companies | Industrial<br>Companies | TOTAL       |
|-------------------------------------|-----------------------|-------------------------|-------------|
| Balance at the beginning of the     | £                     | £                       | £           |
| Year                                | 204,379,825           | 14,329,635              | 218,709,460 |
| Adjustment: For Balance transferred | -140,741              | + 140,741               |             |
|                                     | 204,239,084           | 14,470,376              | 218,709,460 |
| Premiums                            | 19,604,748            | 7,151,109               | 26,755,857  |
| Consideration for Annuities         | 2,330,381             | 5,734                   | 2,336,115   |
| Interest and Dividends (less Tax)   | 7,964,761             | 480,264                 | 8,445,025   |
| lucrease in value of Investments .  | 196,515               | 1,363                   | 197,878     |
| Fines, Fees, &c                     | 11,307                | 1,324                   | 12,631      |
| Capital Paid-up                     | 3,240                 | 116,353                 | 119,593     |
| Customs Timber Measuring, &c        | $3,\!264$             | •••                     | 3,264       |
| Transfers from other Accounts .     | 805,152               | 42,320                  | 847,472     |
| Miscellaneous                       | 44,104                | •••                     | 44,104      |
|                                     | 235,202,556           | 22,268,843              | 257,471,399 |
| OUTGO                               | Ordinary<br>Companies | Industrial<br>Companies | TOTAL       |
|                                     | £                     | £                       | £           |
| Claims                              | 13,191,929            | 2,751,230               | 15,943,159  |
| Premiums                            | 1,201,353             | 579                     | 1,201,932   |
| Surrenders                          | 993,397               | 27,702                  | 1,021,099   |
| Annuities                           | 1,377,819             | 4,733                   | 1,382,552   |
| Commission                          | 1,115,114             | 1,880,662               | 2,995,776   |
| Expenses of Management              | 1,725,310             | 1,279,244               | 3,004,554   |
|                                     | 3,961                 | 971                     | 4,932       |
| Bad Debts                           | 57,625                | 3,298                   | 60,923      |
| Interest on Capital and Dividends   | 37,023                | 9,200                   | 00,828      |
| and Bonuses to Shareholders .       | 458,961               | 382,831                 | 841,792     |
| Transfers to other Accounts         | 839,279               | 500,075                 | 1,339,354   |
| Miscellaneous                       | 110,369               |                         | 110,369     |
| Balance* at the end of the Year .   | 214,127,439           | 15,437,518              | 229,564,957 |
|                                     | 235,202,556           | 22,268,843              | 257,471,399 |

<sup>\*</sup> This Balance includes the whole of the Life and Annuity Funds (£223,625,773), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

## Summary of the Balance Sheets (1897).

| LIABILITIES   | Ordinary<br>Companies    | Industrial<br>Companies | TOTAL                   |
|---|--------------------------|-------------------------|-------------------------|
| D'1 G'21('11' )   | £                        | £                       | £                       |
| Paid-up Capital (including sundry Shareholders' Balances) | 11 100 750               | 1,347,087               | 12,470,837              |
| Life and Annuity Funds                                    | 11,123,750 $209,221,502$ | 14,404,271              | 223,625,773             |
| Fire Funds of Companies trans-                            | 200,221,002              | 14,404,-/1              | ==0,0=0,770             |
| acting Life Business                                      | 10,561,024               |                         | 10,561,024              |
| Marine Funds of Companies trans-                          | 000.070                  |                         | es 0 25 c               |
| acting Life Business                                      | 680,256<br>4,128,099     | 500,000                 | 650,256<br>4,628,099    |
| Other Funds   | 938,501                  | 194,872                 | 1,133,373               |
| Profit and Loss Balances                                  | 4,079,460                |                         | 4,079,460               |
| Depreciation and Investment Ba-                           | 1,0,0,100                | ***                     | 2,0.0,100               |
| lances  | 1,114,584                | 17,877                  | 1,132,461               |
| Globe Annuitants (Liverpool and London)                   | 1,102,800                |                         | 1,102,800               |
| London)   | 3,447,232                | 34,266                  | 3,481,498               |
| Outstanding Accounts                                      | 614,668                  | 12,905                  | 627,573                 |
| Temporary Loans   | 306,471                  | 12,000                  | 306,471                 |
| remporary norms   |                          |                         | 1                       |
|   | 247,318,347              | 16,511,278              | 263,829,625             |
| ASSETS  | Ordinary                 | Industrial              | Total                   |
| ASSEIS  | Companies                | Companies               | TOTAL                   |
|   | £                        | £                       | £                       |
|   |                          |                         |                         |
| Mortgages   | 80,774,095               | 1,267,363               | 82,041,458              |
| Loans on Policies   | 10,971,187               | 41,799                  | 11,012,986              |
| ,, Rates  | 22,836,110               | 6,078,393               | 28,914,503              |
| British Government Securities .                           | 4,882,139                | 1,312,670               | 6,194,809               |
| Indian and Colonial Government                            | 1 = 0 = 0 0 = 1          | 201.002                 | 17 0, 0 070             |
| Securities  | 17,078,671<br>6,827,009  | 304,002<br>123,409      | 17,382,673              |
| Debentures  | 39,108,179               | 1,794,105               | 6,950,418<br>40,902,284 |
| Shares and Stocks   | 25,115,258               | 30,089                  | 25,145,347              |
| Companies' own Shares                                     | 642,990                  |                         | 642,990                 |
| Land and House Property and                               | 012,000                  | •••                     | 0 *=,550                |
| Ground Rents  | 17,719,836               | 4,298,355               | 22,018,191              |
| Life Interests and Reversions .                           | 5,468,110                | 358                     | 5,468,468               |
| Loans on Personal Security                                | 1,565,015                | 5,237                   | 1,570,252               |
| Agents' Balances and Outstanding                          |                          |                         |                         |
| Premiums  | 5,186,096                | 508,835                 | 5,694,931               |
| Outstanding Interest                                      | 2,244,085                | 156,552                 | 2,400,637               |
| Cash, Deposits, Stamps, &c                                | 6,726,925                | 179,985                 | 6,906,910               |
| Customs Timber Measuring Balances, &c.                    | 9 115                    |                         | 0 44*                   |
|   | 3,445                    | •••                     | 3,445                   |
|   |                          | 410,126                 | 579,323                 |
| Deficiencies, Preliminary Expenses, &c                    | 169,197                  | 110,120                 | 0,1,1                   |
|   | 169,197<br>              | 16,511,278              | 263,829,625             |

Increase (+) or Decrease (-) in the Chief Items of this Year's Summary as compared with the corresponding Items for the previous Year.

|   | Ordinary<br>Companies | Industrial<br>Companies |
|---|-----------------------|-------------------------|
| INCOME.   | £                     | £                       |
| Premiums  | + 947,400             | + 534,496               |
| Consideration for Annuities                           | -35,085               | - 380                   |
| Interest and Dividends (less Tax)                     | + 388,479             | + 43,961                |
| Net Result of Realization and Re-valuation            |                       |                         |
| of Investments  | - 21,677              | - 1,988                 |
| Outgo.  |                       |                         |
| Claims  | - 422,209             | - 22,871                |
| Annuities   | + 147,119             | + 582                   |
| Surrenders  | + 19,660              | + 3,349                 |
| Commission  | + 53,200              | + 54,191                |
| Expenses of Management                                | + 72,249              | + 186,186               |
| LIABILITIES. Paid-up Capital (including sundry Share- |                       |                         |
| holders' Balances)                                    | + 29,347              | + 119,928               |
| Life and Annuity Funds                                | +9,887,428            | + 849,526               |
| Assets.   |                       |                         |
| Mortgages (including Loans on Rates) .                | -1,458,145            | + 682,993               |
| Life Interests and Reversions                         | + 180,135             | - 4                     |
| Loans on Policies                                     | + 230,105             | + 3,342                 |
| British Government Securities                         | + 106,611             | + 325,445               |
| Indian and Colonial Government Securities             | + 328,422             | + 29,134                |
| Foreign Government Securities                         | +1,303,513            | + 118,303               |
| Debentures  | +5,613,253            | + 27,068                |
| Shares and Stocks                                     | + 4,663,060           | + 1,469                 |
| Companies' own Shares                                 | + 20,108              | •••                     |
| Land and House Property and Ground                    |                       | 000 177                 |
| Rents   | +1,191,093            | + 330,116               |
| Loans on Personal Security                            | + 81,214              | - 2,067                 |

#### NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 92, of which 81 have been classed as Ordinary, 7 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The Returns of the "National Mutual" and the "Scottish Accident" are included for the first time.

During the year three names have been removed from the official List of Companies, namely, the Mutual, National Life, which Companies have amalgamated as the National Mutual; and the Positive, whose business has been transferred. And eight names have been added, namely, the Australian Mutual Provident Society; British Homes Assurance Corporation, Limited; Employers' Indemnity Company, Limited; Mutual Life Association of Australasia, Limited; National Mutual Life Association of Australasia, Limited; New Era Assurance Corporation, Limited; Perseverance Home and Life Assurance Company, Limited; and the Sickness Accident and Life Association, Limited; in which cases the Board of Trade have issued their warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, as shown by the last Returns of the Companies.

ORDINARY BUSINESS.

|  |                                       | OF                     | DINAR                   | Y BUSIN  | ESS.                     |   |                                 |                                 |
|--|---------------------------------------|------------------------|-------------------------|--|--------------------------|---|---------------------------------|---------------------------------|
|  | WITH PROFITS                          |                        | WITHOUT PROFITS         |  | TOTAL                    |   | Re-assur-<br>ances              | Net                             |
|  | No.                                   | Amount                 | No. Amount              | No.  | Amount                   | Amount  | Amount                          |                                 |
| Assurances. Whole Term of Life                   | 753,750                               | £<br>366,100,054       | 137,692                 | £<br>67,722,544  | \$91,442                 | £<br>133,822,598  | £<br>23,957,594                 | £<br>409,865,004                |
| Limited number of<br>Premiums                    | 38,385                                | 23,250,728             | 6,828                   | 3,212,114  | 45,213                   | 26,462,842  | 1,080,798                       | 25,382,044                      |
| Endowments Endowment Assur-                      | 792,135<br>2,098                      | 389,350,782<br>430,814 |                         | 70,934,658<br>3,578,941  |                          | 160,285,440<br>4,309,755  | 25,038,392<br>3,000             | 435,247,048<br>4,306,755        |
| ances Joint Lives                                | 513,807<br>14,475<br>1,016<br>21<br>5 | 3,005,119              | 2,643<br>1,133<br>3,650 | 13,554,181<br>1,152,819<br>1,397,858<br>5,556,005<br>4,562,064 | 17,118<br>2,149<br>3,671 | 104,369,939<br>4,157,938<br>2.203,526<br>5,583,218<br>4,577,012 | 395,670<br>367,103<br>1,471,555 | 1.836,423<br>4,111,363          |
| Miscellaneous                                    | 987                                   |                        |                         | 7,742,627  |                          | 8,688,477   | 1,664,793                       |                                 |
| Annuities.                                       | 1,324,544                             | 485,396,152            | 218,452                 | 108,779,158  | 1,543,026                | 594,175,305   | 31,530,337                      | 562,344,965                     |
| Immediate Deferred                               |                                       |                        |                         |  | 26,111<br>7,240          | 1,327,668<br>231,888  |                                 |                                 |
|  |                                       |                        |                         |  | 33,351                   | 1,559,556   | 48,254                          | 1,511,302                       |
| INDUSTRI   | AL BU                                 | SINESS—(               | Sickness                | and Friend   | ly Society               | Contracts   | not include                     | ed).                            |
|  | WITH PROFITS                          |                        | WITHOUT PROFITS         |  | TOTAL                    |   | Re-assur-<br>ances              |                                 |
|  | No.                                   | Amount                 | No.                     | Amount   | No.                      | Amount  | Amount                          |                                 |
| Assurances. Whole Term of Life Limited number of |                                       |                        |                         |  | 15,302,75                | £<br>8 143,892,67   | £ 1,000                         | £<br>143,891,678                |
| Premiums   |                                       | •••                    |                         |  | 70                       | 3 12,53   | 34 500                          | 12,034                          |
| Endowments<br>Endowment Assur-                   |                                       |                        |                         |  | 15,303,46<br>102,16      | 1 143,905,21<br>2 1,720,43                                      |                                 | 143,903.712<br>1,720,437        |
| ances Joint Lives                                | •••                                   |                        |                         | <br>   |                          | 8 4,484,90<br>4 1,91  | 0 <b>S</b><br>10 400            | 1,962,952<br>4,484,968<br>1,510 |
| Miseellaneons                                    | ***                                   |                        |                         |  | _                        |   | 00                              | 300                             |
| ANNUITIES.                                       |                                       |                        |                         |  |                          | 1152,075,80   | _                               | 152,073.819                     |
| Immediate<br>Deferred                            |                                       |                        |                         |  |                          | 3 3,18<br>2 1:  | 55<br>23                        | 3,185<br>123                    |

The above figures are based on Returns deposited, for the most part, during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the ase of one Company, namely, the Northern, the amount of business at a more recent date has been neluded. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not eparately show the extent of business in the United Kingdom.

The Relation of the Actuarial Profession to the State. By John Nicoll, F.F.A. Being the Essay to which a Prize, presented by Mr. J. Chisholm, was awarded.

[Read in abstract before the Institute, 28 February 1898.]

EVEN within comparatively recent years it was not uncommon to hear the question asked, "What is an Actuary"?; and, before dealing with our subject proper, it would therefore seem necessary, or very desirable at least, to trace to some extent the origin and development, so to speak, of the Actuary, and the rise and progress of the Actuarial Profession.

From our Latin Dictionary we find that the word Actuarius means a Notary or Clerk (probably a "ready writer", seeing that the adjective form of the word means light or nimble); and, in his Address to the Institute of Actuaries, on 28 November 1881, Mr. Bailey went very fully into the derivation of the word Actuary. He gives the meanings of it (1) from Johnson's Dictionary as (a) "The registrar who "compiles the minutes of the proceedings of a court; (b) a "term of the civil law"; and (2) from Webster's Dictionary, as (a) "A registrar or clerk—a term of the civil law, and "used originally in courts of civil jurisdiction, but in Europe "used for a clerk or registrar generally; (b) The manager of "a joint stock company, particularly of an insurance com-"pany." He then goes on to say—"Hence it seems clear "enough that the original conception of an Actuary was that of "an officer of a court of law. An officer with this designation " has always been, and still is, attached to the Lower House of "Convocation of the Church of England; and it is worthy of " notice that the Chief Officer of the first Life Assurance Society "established in this country, the now extinct Amicable, was " called Registrar. The first mention of an Actuary in connection "with a life office is to be found in the Deed of Settlement of "the Equitable Society, which bears date 7 September 1762. "His duties are described as follows:-That he shall, in a fair " and clerk-like hand, method, and manner, write and keep all " such books as the future occasions of the said Society shall " require.' Thus it seems to have come to pass that the " keeper of the registers of the risks incurred came to be em-" ployed to determine the proper rates of premium for these

"risks: the registrar had to be a scientific computer, and became "the Actuary in the modern sense of the word."

In examining the origin of the word Actuary, we must not, of course, forget that actuarial principles were applied by many learned and scientific men long before the title came to be used in the sense in which we now generally think of it. We shall therefore consider next the beginnings of the employment of such principles, and endeavour to trace the steps in their discovery and improvement.

One writer on life assurance thinks that "if we put aside "names, and consider simply things, life assurance is seen to be "coeval with the human race, at least, as soon as civil govern-"ment had its birth." The same writer considers that Joseph, the Hebrew, as Prime Minister of Egypt in the year 1702 B.C., acted on Actuarial principles in fixing that the Egyptian landholders should give one-fifth of the produce of their land in perpetuity to the King in consideration of the latter's making them an annual allowance during the remaining three years of Leaving, however, such cases—which are, it must be admitted, somewhat faneiful-out of account, there is no doubt that what we know as guilds, the precursors of friendly societies, have existed from very remote times. They were, however, really assessment clubs, against sickness chiefly, and only required contributions from their members of sufficient amount each year to meet the claims and expenses of that particular year. little knowledge of any special principles was, therefore, required in their establishment and management.

What looks, however, like a genuine attempt to arrive at the values of annuities is found under the Roman law, and is referred to by various writers. The following account is taken from the Insurance Cyclopedia:—"Most of the best-informed writers "agree that the first judicial occasion for valuing annuities on lives arose in consequence of the Falcidian law, which, in "B.C. 40 was adopted in the Roman Empire; and which declared that a testator should not give more than three-fourths of his property in legacies, and thus one-fourth was required to be "secured to his legal heirs. It became necessary, in the execution of this law, to value all such legacies as were charged upon the succession for limited terms or as annuities for life. "It is possible that several means of accomplishing this were attempted. We learn from Emilius Macer (230 A.D.), that "the method which had been in common use at that period was

"as follows:—From the earliest age until 30, to compute 30 years; from age 30 so many years are computed as are wanting until 60; therefore, never more than 30 years are computed. It was, no doubt, the employment of this defective method which caused the great jurist, the Prætorian Præfect, Ulpianus, to give his attention to the subject. It is impossible to ascertain what materials he called to his aid in preparing his estimate of the value of annuities. The values he gives are—birth to 20 = 30 years; 20 to 25 = 28 years, &c.; 55 to 60 = 7 years; 60 and upwards 60 and upwards 60 years. Interest was not apparently taken into account, and, in that connection, it may be mentioned that in England, in earlier times, interest for the use of money was forbidden."

How little progress was made for 1,300 years in the theory of life annuities, we gather from a statement in an address by Mr. T. E. Young to the Institute on 30 November 1896. He says—" From the mystic number seven, derived from Chaldean "usage, septennial periods of life were assumed to be naturally "expressive. In an Act of Parliament passed in 1540, it was "accepted on that basis that a single life was equivalent to a lease of seven years, two lives to a lease of 14 years, and three to a "lease of 21 years, thus creating the stereotyped restriction of "leases to 7, 14, and 21 years. Even after the publication of "Halley's researches in 1693, Acts were passed in 1694 and "1703, in which the ancient numerical relationship (somewhat "modified), was adopted in the assessment of pecuniary values."

Just as the early guilds were assessment societies, so the first annuity societies were Tontines, and, no more in the case of the one than of the other, was any special knowledge or training required for their foundation and management. A certain number of individuals clubbed together to form a fund, the interest of which was to be divisible among the survivors from year to year; and, in the later Tontines, it was provided also that the capital sum should be divided among those who survived a fixed term of years.

In 1692 the English Government first sought to raise moncy by means of life annuities, and, in that case also, it was in the form of a Tontine. The measure was known as the "Million Act", because the object in view was to raise one million sterling for the purpose of carrying on war with France. There was no account taken of the ages of the subscribers, who were all therefore admitted on equal terms. A table was appended to the Act, showing how many subscribers out of 10,000 of all ages will die each year. A copy of this Table is in the British Museum, but there is no means of knowing how it was constructed, or by whom. It was not until the English Tontine of 1789 was issued that the amounts of the annuities were regulated by the ages of the annuitants.

Previously to the year 1671, there is no doubt that the method adopted of calculating annuities was "a purely conjectural one." In that year the celebrated John de Witt, Prime Minister of the States of Holland, submitted to his Government a Report and Treatise in regard to the terms on which life annuities might be granted; and, looking to that Report, it is evident that we must accord to that most distinguished man the honour of being the originator of true actuarial principles, and of first applying them to the calculation of life annuities.

Dr. Halley, in England, followed shortly after De Witt. In 1693 he read a paper before the Royal Society of London on "An Estimate of the Degrees of the Mortality of Mankind, &c.", giving therein the first Life Table (The Breslau Table), properly constructed from real observations. Following so comparatively shortly after De Witt, and considering that the latter's Treatise had been suppressed by his Government, on account apparently of his rates of annuities being too low to induce purchasers, seeing that the Government's credit was rather shaky at the time—it seems only fair to consider Dr. Halley as an entirely independent worker, and to accord to him, equally with De Witt, the distinction of conceiving and employing those true scientific principles which have been so marvellously extended and built up since their day. With these two great men began the dawning of the morning which has ever since been increasing towards the perfect day.

Mr. Young has pointed out the very important fact that De Witt and Halley "established our work upon a basis which "subsequent experience and study have failed fundamentally to "improve." This is a matter in regard to which we cannot be too grateful to these illustrious men, as, had the foundation been imperfect, the building-up of our science must, at the least, have been greatly hindered thereby.

The readiest means of following up the development of actuarial science, in this country specially, will perhaps be to enumerate, as far as possible in chronological order,

the various treatises on the subject that have been published since the days of De Witt and Halley.

In 1707, a small edition of Smart's Interest Tables was published, in which is given what may be called a hypothetical Table of Mortality for London. A larger edition of the same work was published in 1726, and to it some remarks on annuities upon lives are appended.

De Moivre improved on what Dr. Halley had begun, and the

first edition of his Annuities on Lives was printed in 1724.

Mr. Thomas Simpson, in 1742, published his treatise on the Doctrine of Annuities and Reversions. The Tables in that work are based on the Mortality Table for London contained in Mr. Smart's work referred to above. Simpson's Select Exercises for Young Proficients in the Mathematics, containing a supplement relating to annuities, appeared in 1752.

In 1753, Mr. James Dodson completed and issued his Mathematical Repository. It may be mentioned that Mr. Dodson was great-grandfather of the celebrated Professor De Morgan, who writes a memoir of him; and that, owing to his being refused admission to the Amicable Society, on account of his being over 45 years of age when he made application, he determined to found a new assurance society (The Equitable) upon the principles laid down by Dr. Halley, namely, that the rate of premium should increase with the age of the assured. Mr. Dodson, however, died before the Equitable Society was started.

In 1769, Dr. Richard Price wrote some observations addressed in a letter to Dr. Franklin on the Expectations of Lives; the Increase of Mankind; and the Population of Loudon; which were published in the Philosophical Transactions of that year. About the same time some gentlemen in the profession of the law referred to Dr. Price for his opinion of a plan by which they proposed to form themselves into a society for providing annuities for their widows. This, Dr. Price found, upon examination, to be very defective; and he represented its insufficiency so strongly to the promoters, that they determined to lay it wholly aside. A great number of other societies had been forming about the same time, with the same views, and founded upon principles equally defective. In order to expose the fallacious principles on which such annuity societies were founded, and to prevent, or at least limit as far as possible, the evils which he saw must result to the public from them, Dr. Price published, in 1769, his Treatise on Reversionary Payments. After the publication of this book, the rage for establishing new societies ceased, and many of those already in existence were led to revise their rates. In 1770, Dr. Price communicated to the Royal Society some Observations on the proper method of calculating the values of Contingent Reversions in which he corrected an error into which De Moivre had fallen. As illustrating the fear and trembling with which the early Actuaries pursued their investigations, and as a sign of Dr. Price's natural diffidence, his biographer mentions that, "from "the high opinion he entertained of the accuracy of De Moivre, he conceived the error to be his own rather than that of so eminent a mathematician, and, in consequence, puzzled himself so much in the correction of it, that the colour of his hair, which was naturally black, became changed in different parts of his head into spots of perfect white"!

In the Journal of the Institute, Vol. vii., Mr. Frederick Hendriks reprints a rather curious and interesting work, the Case Book of John Rowe, containing actuarial questions put to that Actuary between the years 1775 and 1790, with their solutions. We find also that, in 1764, reference was made to Mr. Rowe by the "Law Society for the benefit of Widows" as to whether the premiums proposed to be charged by that Society were likely to prove sufficient. Apparently this was a different Society from that which made application to Dr. Price some years later. In the present case, Mr. Rowe calculated a table of rates which were much higher than those proposed to be charged, and the result was that this "Law Society" also was dissolved. Mr. Hendriks remarks that, though the name of Rowe neither has nor merits the same degree of distinction as the names of De Moivre, Simpson, or Price, "nevertheless a respectable share of honourable mention may, with much propriety, attach to it." Rowe states that he had intended writing a treatise on annuities, but he gave up the idea when Dr. Price's work appeared.

In 1779, Mr. Morgan, Dr. Price's nephew, published his Doctrine of Annuities and Assurances. Dr. Price had given the officers of the Equitable Society valuable assistance in its early days, and, on his recommendation, Mr. Morgan was appointed first Assistant Actuary, and in 1775 Actuary, of that Society.

In 1783, Mr. Baron Maseres' Principles of the Doctrine of Life Annuities was published. The tables in that Work are constructed on the basis of M. De Parcieux's observations.

In 1808, Mr. Baily published a treatise on the Doctrine of

Interest and Annuities, and his Doctrine of Life Annuities and Assurances, published in 1813, is stated by himself to be a continuation and amplification of the earlier treatise.

After enumerating most of the above-referred-to works of English authors who have written on the subject of life annuities and assurances, Baily remarks, "they are few in number; and "the whole of their productions, taken collectively, by no means "contain a complete view of the science." As to foreign writers on the subject, he remarks—"their productions are more "numerous than ours, but their enquiries are not so extensive, so "that, if the works of the English authors by no means contain "a complete view of the science, much less do the works of the "foreign writers."

In 1815, Mr. Joshua Milne published his treatise on Annuities and Assurances on Lires, introducing for the first time in it his famous Carlisle Table, which was to supersede the Northampton Table, and form the basis of calculation during the following 50 years.

Mr. Griffith Davies' treatise on Annuities was partly printed in 1825. The complete treatise was published after his death by his executors in 1856.

In 1843, the celebrated work by David Jones on Annuities and Reversionary Payments—which, until recent years, formed the text-book for actuarial students—was published under the superintendence of the Society for the Diffusion of Useful Knowledge.

It would be interesting to give an account also of various special investigations undertaken by Actuaries up to this date, but we have only time to refer to some of these very shortly.

In 1829, Mr. Griffith Davies investigated and reported upon the Bombay Military Fund, and he subsequently wrote other 19 reports at various times on different Indian funds. Similar investigations were also made by Messrs. Woolhouse, Neison, Finlaison, and Brown and Hardy. Mr. Finlaison also carried through an enormous amount of work for the Home Government, but we must refrain from describing the results of these and other investigations for the present.

Mr. T. E. Young states that "Mr. Griffith Davies, in 1825, "first practically solved the problem of the market value of a life "interest, which has resulted in such commercial and financial "advantage to the community."

Earliest Recognition by Government of the Recognition by Government of the Actuarial profession appears to have occurred in the there was a clause requiring tables to be approved by two persons known to be professional Actuaries.

Then, in 1821, Mr. John Finlaison was appointed to the newly-created office of Actuary to the Commissioners of the National

Debt.

The Act of 1819 was repealed by the Act of 1829, which did not require an Actuarial Certification of Tables.

The Act of 1846, however, enacted that the Registrar of Friendly Societies should not certify the rules of any society until its tables had been certified by an Actuary.

By the Act of 1850, the enactments of 1829 and 1846 were modified, and the employment of an Actuary is made optional, but the Registrar shall not grant a certificate to a society transacting annuities, unless such tables are certified by an Actuary.

The law, as it at present stands, requires Tables of Annuities to be certified by the Actuary of the National Debt, or by some Actuary approved by the Treasury, before a friendly society

granting annuities becomes entitled to registry.

"By an Act of 1852, to abolish the office of Master in Ordinary of the High Court of Chancery, and to make provision for the more speedy and efficient despatch of the business of said court, authority is given to any judge of the court to obtain the assistance of Actuaries or other scientific persons the better to enable such court or judge to determine any matter at issue in any case or proceeding, and to act upon the certificate of such persons." Since this Act was passed, the services of Actuaries have been frequently sought for by the courts, and with good results.

Foundation of the We now come to an event which marks the beginning Institute of Actuaries.

of a very important epoch in the history of the Actuarial profession, namely, the founding of the Institute of Actuaries, the progress of which is hereafter the history of the profession. As the Bank of England was founded by a Scotsman, so there is no doubt that the Institute of Actuaries, which has done such immense service in the cause of actuarial science over the whole world during these fifty years, owed its origination to a Scotch Actuary. It is a fact of which Scotch Actuaries may be justly proud. In order to complete this part of our subject, we shall give here a short connected account of the foundation of the

Institute, deduced from various presidential addresses and other sources.

Mr. Wm. Thos. Thomson, of Edinburgh, in the early part of the year 1848, suggested to Mr. Robert Tucker, a London Actuary, the advisability of forming in London an association of Managers and Actuaries of Assurance Companies such as had existed in Edinburgh for some years previously. Mr. Tucker took up the suggestion, and very kindly introduced Mr. Thomson to the leading London Actuaries of that day, with a view of having the matter considered by them. As the result of these introductions, a preliminary meeting was held on 15 April 1848 of the Actuaries and chief officials of the London Assurance Offices, to consider whether it would be desirable to form in London a business organization such as had been established for 15 years previously in Edinburgh, under the title of the Managers' Association. At this meeting, a committee of ten, including some of the then senior Actuaries, was formed, to which the consideration of the matter was referred. Their report, presented to a subsequent meeting convened by circular, was to the effect that it was not expedient to lay down rules for the formation of a society of any description, but that occasional meetings might be held with advantage if this should from experience be found to establish uniformity in dealing with points of practice. There were, however, among the officials at this meeting some (Mr. Peter Hardy and Mr. Jellicoe among the number) who took a wider, and, shall we say wiser and loftier, view of the matter, and advocated the formation of a college or society of Actuaries on the model of other learned professions. Though they did not afterwards succeed as they had hoped, in getting a Bill passed giving legislative powers for regulating the profession, yet we must remember nevertheless that the Institute still owes a very great deal to those early founders for the high ideal they sought to set before themselves in the formation of the society they had in view. was apparent to Messrs. Hardy, Jellicoe and others, that the decision of the preliminary meeting of officials was not the opinion of the body of the Insurance profession; and, accordingly, another meeting, this time of representatives of all the London offices, was held on 10 June 1848 at which it was decided by an overwhelming majority that the report of the first committee be not approved, and a resolution was moved by Mr. Hardy and carried: "That this meeting considers it desirable to establish a scientific "and practical association amongst the Actuaries, Secretaries, and

"Managers of the Life Assurance Societies of Great Britain." Another committee of fifteen, chosen by ballot, was appointed to devise the best means of carrying this resolution into effect; and their efforts resulted in the formation of the Institute of Actuaries.

After the meeting of 10 June 1848, when the report of the Committee of ten was set aside by the General Meeting, the minority, who were in favour of the report, appear to have agreed to form themselves into a private Association, which they called the Actuaries' Club, and restricted to the Actuaries of life offices, who must not become members of the Institute of Actuaries. Referring to this Club, Mr. Newbatt, in his presidential address of 1890, speaks of "The small but notable band of "capable Actuaries whose sense of duty—mistaken, as we have come "to think it—or dim perception of the drift and needs of their "time, kept them aloof from the early struggles, and strangers "to all but the latest achievements of the Institute."

The first meeting of the Institute was held on 14 October 1848, and the first president was Mr. John Finlaison, Actuary of the National Debt, who warmly approved of the objects of the Institute.

It is not possible here to follow the Institute through all the vicissitudes of its earlier and later years, but we must refer to one incident that occurred a few years after its foundation. In the year 1855, difficulties began to arise between the members of the Institute of Actuaries resident in Scotland, and their colleagues in London; and, as a result, almost all the Scotch members, numbering more than one-third of the whole body of the Institute resigned. The immediate consequence of the withdrawal of the Scotch members was the establishment, in 1856, of the Faculty of Actuaries in Scotland. The Faculty of Actuaries was incorporated by Royal Charter in September 1868, apparently without encountering any of the difficulties that attended the grant of a Charter to the Institute of Actuaries sixteen years later.

A very pleasing matter should be referred to here, as showing how real and disinterested a concern the early founders of the Institute had in its welfare—an interest which has continued to be shown by their successors in office. The Assurance Magazine now called the Journal of the Institute of Actuaries, had not, of course, the circulation at the beginning that it has now, and it was probably carried on for some years at an actual loss. Mr. Jellicoe, however, discerned how great a power it was likely to

become, and he accordingly not only edited it, but had the earlier numbers published at his own risk.

In 1853, Mr. Jellicoe gave evidence before the Select Committee on Assurance Associations. He read three resolutions which had been passed shortly before at Special General Meetings of the Institute, one of which was "That no legislation on Life "Assurance Associations could be permanently effective that did "not exact tests as to the responsibility and acquirements of "persons allowed to practice as Actuaries." Mr. Jellicoe also laid before the Committee a copy of the Bill the Institute had at its foundation proposed to endeavour to get passed for regulating the profession of an Actuary, but which had been allowed to drop. Referring to Mr. Jellicoe's evidence specially, the Committee report as follows :- "Your Committee will conclude their report "by calling attention to a part of the evidence which advocates "the formation of an incorporated society of Actuaries with a "view to the advancement of that important science, and also "with a view to issue diplomas or certificates to persons qualified "to practice as Actuaries. If an effort should be made to induce "Parliament to grant such an incorporation, the Committee are " of opinion that it will be worthy of consideration; but further "investigation would be needful before such measures should be "adopted, as considerable difference of opinion prevails on the "subject among Actuaries themselves."

The editor of the Journal was greatly at a loss to understand what grounds the Committee had for stating that there was difference of opinion among Actuaries as to measures being adopted for incorporation, as there seemed nothing in the evidence given before them to warrant the statement. The account of the difficulties encountered on a subsequent occasion in obtaining a Charter for the Institute will, however, show in what quarter the "difference of opinion" existed.

We might narrate many circumstances of importance and interest in connection with the history of the Institute of Actuaries, but we must content ourselves with giving one other extract in regard to a matter which should certainly not be left unnoticed.

The question of the incorporation of the Institute was again brought before, and carefully considered by, the Council of the Institute at their meetings in the early part of 1882, and it was agreed to present a recommendation to the Annual General Meeting in favour of such incorporation. Accordingly, at the

Annual General Meeting, held on 3 June 1882, such a recommendation was made by the Council and unanimously approved, and the Council was requested to take the necessary steps towards obtaining a Charter of Incorporation. The Council accordingly considered and settled the Draft Charter, and accompanying Petition, which were presented to Her Majesty's Most Honorable Privy Council on 15 September 1882. The application of the Institute was referred by the Privy Council Office to the Board of Trade in the usual way; and it then transpired that, some time before the presentation of the Institute's Petition to the Privy Council office, an objection had been lodged by the Actuaries' Club with the Board of Trade. Thus, nearly 30 years after the Select Committee of the House of Commons had reported that, in reference to the incorporation of the Institute of Actuaries, "considerable difference of opinion prevailed on the subject among Actuaries themselves", it became evident that this difference of opinion had not ceased to exist.

The opposition of the Actuaries' Club led to an unfavourable report, in the first instance, from the Board of Trade; but, on its being represented to the Privy Council Office (evidently by Mr. Sutton) that, under the Friendly Societies Act, the Treasury had important duties imposed upon it in connection with Actuarial matters, the Institute's application was apparently referred to the Treasury to report upon. From various causes this second reference proved abortive. Further negotiations were, however, ultimately successful, and the Institute became incorporated by Royal Charter on 29 July 1884. Under and by virtue of the Charter, the members of the Actuaries' Club became Fellows of the Institute.

From what has preceded, it would seem as if there had not been much in the way of aid or protection accorded by the State to the actuarial profession in the performance of its duties. Our Free Trade Government has, however, been rightly—as it seems to us—very chary at all times of seeming to favour any particular society, or set of individuals, more especially if that favouring was at all likely to be at the expense of other members of the community. It is really very doubtful whether the policy of non-interference is not, in most circumstances, the best for a Government to pursue; and, as regards the Institute of Actuaries, it is very questionable if it would have been so vigorous, or so surely founded as it is at the present day if it had depended, at its inception, on assistance or support in any form from the

State. It is at least to the credit of our Government that they have not at any time willingly put obstacles in the way of the progress of societies or associations for legitimate purposes, even when they may not have seen their way to assist them by their countenance or aid; and, in this respect, they compare favorably with the Governments of many other countries.

A statement, made by Mr. Emory McClintock, at a meeting of the Institute held on 8 June 1895, furnishes a picture of the position of the Actuary under a Government exercising direct State Supervision of Assurance Companies, and, through them, to a great extent of Actuaries also; and, with a quotation of that statement, we shall close this introductory part of our subject. Mr. McClintock remarked as follows:-"The position of an "Actuary in the United States, in his judgment-and he spoke "for no others,-had been damaged by the system of official "valuations. The different States made yearly official valuations " of the various Companies on a uniform standard. The result of "that system had been what probably no one would have expected. "The Officer of the State whose business it was to make these " annual valuations went to a published set of tables, put down "the figures which he found there, added them up, and that was "the valuation. The Actuary might, and usually did, make his "own valuation for his own Company, very often on a different " standard; but, in the eye of the public, the valuation was made "by a State Official, and not by the Actuary of the Company. "He would leave it to those present to judge what the position of "Actuaries in this country would be if they were not "held responsible for their own valuations. That particular "element of professional authority, which came from making "the valuations, was largely lost in his country, for the simple "reason, that a person was accustomed to refer to the Superin-"tendents of the Insurance Department of his own State, or some "other State, to find out what the total reserve liability was. "These statements were published widely, and were in the hands " of a great many agents and a large number of people who were "not agents, and they knew as much, or thought they did, about "the value of a policy as any Actuary. The public seemed to "think that they must have as the surrender-value, the "amount of reserve on their policies, and they had to a "certain extent, an excuse owing to the system of Government " valuations."

It has been stated that "we may take it as established, "that no plan of Life Assurance, as we now under-assurance." stand it, had been contemplated by any Company or "Society, or had been considered by any legislature in Europe "prior to the year 1760." We should naturally, therefore, begin only from that year to look for any dealings of the Actuarial Profession with the State. There are, however, one or two matters of interest which date prior to that year, and to which we shall refer very shortly in the first place.

In one of the earliest volumes of the Journal of the Institute, there is given the preamble of what is believed to be the first English Statute on Insurance. That Statute is dated 1601, but it refers only—as all assurance in these days did—to Marine Insurance.

Again, we may recall the fact that in 1706 a Royal Charter was granted to the Amicable Society. At that time, however, and for nearly one hundred years after, that Office was simply a Mortuary Tontine, and in no proper sense of the word an Assurance Office in the present-day meaning of the term.

1720, the first Parliamentary Committee on First Parliamentary Committee

Insurance was appointed to enquire into and examine on Insurance, the several subscriptions for fishering insurances. the several subscriptions for fisheries, insurances, annuities for lives, and all other projects carried on by subscription in and about the cities of London and Westminster, and to inquire into all undertakings for purchasing joint Stocks or obsolete Charters. As a result of the Committee's Report, the Government brought in a Bill (known as the "Bubble Act") entitled "An Act for better securing certain powers and privi-" leges intended to be granted by His Majesty by two Charters for " assurance of ships and merchandize at sea, and for lending money "upon bottomry; and for restraining several extravagant and "unwarrantable practices therein mentioned." By this Act, Charters were granted to two London Assurance Companies to undertake Marine Insurance on condition, among other things, that they each paid into the Exchequer £300,000 for discharging the debts of the Civil List. In 1721, however, the Companies had only paid up about one-third of the required contribution, and new Charters were then granted to them limiting the amounts to one-half of the original sum, and giving them powers at the same time to undertake Life Assurance risks.

" calculations."

The first actual dealing between Assurance Offices, as properly constituted, and the Crown, was no doubt the presentation by the Equitable, in 1757, of a Petition for the grant of a Charter. The Charter was, however, refused by the advice of the Law Officers of the Crown for the following three reasons; (1) Because it appears altogether uncertain whether the project will or can succeed in the manner in which it is proposed; (2) The success of the scheme must depend upon the truth of certain calculations taken upon tables of life and death, whereby the chance of mortality is attempted to be reduced to a certain standard,—this is a mere speculation, never yet tried in practice, and consequently subject, like all other experiments, to various chances in the execution; (3) Parliament, in dealing formerly with other Insurance Societies, have sufficiently declared their opinion that such Charters ought not to be granted without some benefit accruing to the public therefrom. The Law Officers further remarked, "if the Petitioners are so sure of success as "they say they are, there is an easy method of making the "experiment by entering into a voluntary partnership, of which "there are several instances now subsisting in this business of "insuring; and if, upon such a trial, these calculations are found " to stand the test of practical experiment, the Petitioners will then "apply with a much better grace for a Charter than they can at "present, whilst the scheme is built only upon speculative

If we consider all the circumstances fairly, we shall probably admit that no blame attached to the Law Officers of the Crown in advising against the grant at this time of a Charter to the Equitable; and the promoters themselves seem to have taken this view of the decision, when they resolved to act upon the hint to form a voluntary partnership in the first instance. Of course, in that ease, the promoters were not freed from individual liability as they would have been if a Charter had been granted, but it was only right that, in an undertaking so novel in many respects, the originators should be prepared to incur more than ordinary risk. The fact of their doing so, tended no doubt, to give the public greater confidence in the scheme.

In 1773, we find the first reference to Old Age Mr. Dowdeswell's Pensions. In that year Mr. William Dowdeswell to the plan brought a Bill into the House of Commons, agreeably to the plan of Mr. Baron Maseres, to enable the purchase of deferred annuities to be guaranteed by the poor rates of the parish to

which the purchaser belonged. Notwithstanding that the Bill passed the House of Commons by a majority of two to one, it was rejected by the House of Lords. A Bill of a similar nature, with tables computed by Dr. Price, was introduced in the year 1789; but, though it too passed the House of Commons, it was thrown out by the House of Lords.

In the interval between the bringing in of the two Bills just referred to, a Committee of the House of Commons was appointed, in 1777, to take into consideration the laws then in being against usury and the present practice of purchasing annuities on the life of the grantor. It was suspected that annuities were frequently granted with the object merely of evading the usury laws. This Committee examined many leading solicitors and also the principal officers of the Amicable and Equitable, and thereafter presented to the House their Report embodying certain proposals as to the regulation of the grant of life annuities which formed the basis of the Act 17, Geo. III, c. 26. "An Act for registering the Grants of Life Annuities, and for the better protection of infants against such Grants."

What appears to be the first instance on record of 1786. Chancellor of Exchequer con-sults Dr. Price. Government applying to a private Actuary for advice, occurred two or three years after the conclusion of the American War, when Mr. Pitt determined upon a new and more regular arrangement of the taxes, and upon forming a plan which should reduce the debt. Amidst a great variety of plans which were proposed to the Minister for this purpose, he at length decided in favour of one which, previous to its being carried into effect, he submitted to the judgment of Dr. Price by a letter dated 8 January 1786. Instead of going much into the plan sent him, Dr. Price proposed three other plans, one of which he specially recommended. This plan, however, required the addition of £800,000 a year to the taxes, and Mr. Pitt therefore adopted one of the less efficient plans requiring £1,000,000 per annum aided by the falling in of Temporary and Life Mr. Pitt, in proposing the measure to Parliament, made no reference to the assistance he had received from Dr. Price in connection with it, and, in fact, took the whole credit of the scheme to himself.

From first to last a great deal of legislation has been enacted for the encouragement and relief of friendly societies.

The first Act for that purpose was passed in 1793. It is known by the name of its author, Mr. George Rose, and was

the beginning of that legislative interference or regulation of these societies which has been proceeding ever since then. We shall sce, as we go on, that legislation in regard to friendly societies has been, as was to be expected, much more paternal in character than in the case of the ordinary life assurance companies, and to some extent, perhaps on that very account, has not been attended with the success that has met the working of the Life Assurance Companies Acts, especially that of 1870. We shall, however, deal further with this point hereafter.

In 1807 we first hear of Industrial Assurance. 1807. The Poor's was proposed to establish an office to be called "The Assurance Office. Poor's Assurance Office." The management was to be under Commissioners appointed by the Crown, and the scheme was to be for the benefit of those "who subsisted wholly or principally by the wages of their labour." Under Canon Blackley's scheme, of a much later date, it may be mentioned that only those "earning wages" were to participate in the sickness A Bill was introduced to Parliament "for establishing " a Fund and Assurance Office for investing the savings of "the poor." It was sent to a Committee, and was afterwards thrown out by Parliament. As under the Savings Bank scheme of later years, the Post Office was to be utilized for working the proposed Poor's Assurance Office, but such a scheme at that time

3 per-cent Stock into

was evidently premature.

In his Doctrine of Interest and Annuities, published 1808. Mr. Baily recommends Conversion of in 1808, Mr. Baily suggested to the Government the advisability of converting the 3 per-cent stocks into life annuities; and he gives, in his subsequent work of 1813, a brief account of the plan adopted by the Government in carrying out his suggestion. Unfortunately, there was really only one English Mortality Table available at that Northampton Table used for time, namely the Northampton Table, and it had been shown, in the case of the Equitable Life Assurance Society, to be very profitable for the granting of assurances on lives. It never seems to have occurred to the Treasury that, on that very account, such a table must be very unprofitable for the granting of annuities; and they accordingly entrusted to Mr. Morgan, of the Equitable Office, the work of preparing, from the same Northampton Table, tables for the newly-instituted Government Life Annuities. These annuity-rates so calculated, continued to be used for the following 20 years; and, as every one knows, they caused a very great loss to the country, estimated as amounting,

during the first eleven years they were in use, to nearly two millions sterling. There was, of course, no Actuary attached to the National Debt Office at this time, though Mr. John Finlaison had probably begun about that date to qualify himself for that position. We know at any rate that it was in 1812 that he first directed his attention to the principles of life assurance in connection with a Widows' and Orphans' Fund, which he proposed should be established for all the employees of the civil departments of the Royal Navy. He worked very diligently at that undertaking for seven years, and he had the satisfaction of seeing the fund established by Order in Council, 17 September 1819. Mr. Finlaison entered the Government service in 1805, and passed through various departments, being finally appointed Actuary at the National Debt Office in the year 1821. He retired from that appointment in 1851, having thus served his country for the long period of 46 years in all.

1818. Clause of Bill as to Employment of Actuaries in Friendly Societies. It is curious to find that, so early as 1818, the following clause was inserted in a Bill laid before Parliament for the regulation of friendly societies, namely, "That the Lord High Treasurer or Com-

namely, "That the Lord High Treasurer or Com"missioners of H. M. Treasury shall, within three months after
"passing of this Act, nominate and appoint five or more persons
"skilled in arithmetical calculations to be a Committee for the
"purposes of this Act; and any Society or Institution sub"mitting tables or rules to such Committee for approbation shall
"pay to such Committee such reasonable fee or fees as the said
"Lord High Commissioner or Commissioners of H. M. Treasury
"shall appoint." The clause was omitted from the Bill, because
it occasioned alarm among certain friendly societies, chiefly in
the North of England, that the Government contemplated
interference with the funds of friendly societies. The spirit of
the clause was, however, re-introduced in subsequent friendly
societies' Acts, and, indeed, in an Act passed the following year.

The Act to which we refer was passed in 1819 for Benefit Societies Act. regulating benefit societies, and it contained a clause enacting that the tables and rules of all such societies should be approved by two persons at least, known to be professional Actuaries, or persons skilled in calculation as fit and proper according to the most correct calculation of which the nature of the case would admit. This seems to be the first recognition by Government of the profession of an Actuary, though who were to be entitled to style themselves Actuaries the Government were

very slow indeed to determine, in any distinct and authoritative manner. The Act of 1819 also provided that, in the event of the dissolution of a society, the certificate of two Actuaries should be required that, according to the most correct calculation of which the case will admit, the claims of all persons interested in the society are, by the proposed scheme of dissolution, fairly dealt with and secured. Mr. Finlaison states that it occurred to him, when this Act was passed, that it was necessary there should be a college of some kind to determine who were and were not Actuaries.

In this same year (1819), Mr. Finlaison was asked to 1819. Mr. Finlaison's report upon the annuity system of the Government, Report on Government and in this report, he first directed the attention of the Annuity System. Chancellor of the Exchequer to the enormous loss eaused by granting annuities at the rates then eurrent. would appear, however, that it was really Mr. Gompertz who first observed the unsafeness of the Northampton Table for granting annuities, and that he called Mr. Finlaison's attention to the Mr. Gompertz was about this time appointed Actuary of a London Assurance Company, whose directors proposed entering into competition with the Government for annuity business, but Mr. Gompertz strenuously opposed the proposal. not, however, till nine years after the Mr. Finlaison's report that the grant of annuities by Government at the erroneous rates was suspended.

In 1821, Mr. John Finlaison was, by Act of Parliament, appointed as the first Actuary for the Commissioners of the National Debt. By this step Parliament again recognized the Actuarial profession, without, however, doing anything to settle the question as to who were to be considered Actuaries.

The state of the law in regard to assurance companies had up to this time been very unsatisfactory. "Indeed, the only means by which they could obtain legal "recognition in conducting their business was to obtain a Special "Act of Parliament. They might have sought incorporation, but "this was a 'lottery.' Matters were rendered even worse "when, in 1824, the Standing Orders of the House of Lords in "relation to Private Bills were amended, and it was required "that any Company, other than those for public purposes—"railways, &c.—requiring to be incorporated, must satisfy "a Select Committee that three-fourths of its intended capital

"was paid up and deposited in the Bank of England. This "effectually put a stop to the incorporation of new insurance "companies." The fact that many good life assurance offices were founded really without any capital at all is sufficient proof that a large paid-up capital is quite unnecessary in the case of such institutions, and that they should at any rate have been exempted from the foregoing enactment.

In 1825, an Act was passed authorising the repeal of certain

elauses of the "Bubble Aet" of 1720.

On 5 July of the same year (1825), the report of a Select Committee on Friendly Societies was issued. It refers to the provision in the 1819 Act requiring the tables of payments in friendly societies to be approved by at least two persons known to be professional Actuaries, or persons skilled in calculation. The Committee remark that this provision is in principle of the highest utility, and has already, they believe, been productive of much good, but who, they ask, are professional Actuaries, or persons skilled in ealeulations? and, in what way are the Justices to satisfy themselves that the persons by whom the tables are signed really answer to the description of skilful calculators. It was found that the men who generally certified such tables were "petty " schoolmasters and accountants, whose opinion upon the proba-"bility of sickness and the duration of life is not to be " depended on."

The Committee were decidedly of opinion that, as regards the sanction of Actuaries, some alteration must be made in the law. There are various methods, they think, in which the object might be attained. (1) By the Parliamentary enactment of Tables applieable to all the purposes of friendly societies; but, even if the House were satisfied that the data exist for constructing such Tables with sufficient certainty, it may be remarked that this suggestion puts entirely out of the question all differences of locality or occupation. (2) The Justices of every county might name two or more persons upon whose certificate only they will sanction Tables. There is no guarantee, however, that the Justices would be careful to name Actuaries of ability and experience; and, if this suggestion were acted on, the object might be more effectually accomplished by providing for the nomination by some authority, not local, but extending over the whole of England. It seems rather curious that, with such a strong and clear recommendation before

them from the Committee, Parliament did not attempt at this time to define in some way who were to be considered qualified to undertake the duties of Actuaries. The suggestion that Government should nominate certain persons seems a very good one, and was practically adopted in "The Friendly Societies Act, 1875."

The enquiry by this Committee turned very much in the direction of the best data to be employed in the construction of Tables of Contributions for friendly societies. Their report states—"It is remarkable that, until within a very few years, no " data were collected whereon a calculation of the average "occurrence of sickness at the several ages of man could be "formed with tolerable accuracy; but it is fortunate that "a calculation, made many years ago upon a hypothetical " assumption, has been found to lead to practical results not very "different from those of the more recent enquiries. "hypothesis of Dr. Price was that in societies consisting of "persons under 32 years of age, a 48th part of the members "will be always in a state of incapacitation from illness or "accidents; that, from the age of 32 to 42, this proportion "increases to one quarter more than a 48th part; from 43 to 51, " to one half more; from 52 to 58, to three-quarters more; and " from 58 to 64, to double. The first branch of this proposition, " referring to an age under 32, is stated to have been founded in " some degree upon experience; the others proceed upon the " assumption that after that age, as life approaches to its close, sick-" ness becomes more frequent in the same ratio as life becomes less "valuable. In strictness, perhaps, some allowance ought to "have been made in respect of accidents, the liability to which "does not increase as life advances: but the correction would be " very inconsiderable."

Mr. Finlaison, in his evidence before the Committee, said that he thought his Government Annuity Tables would be "on the safe side," for the calculation of friendly society benefits; but he indicated another source from which he could construct tables suitable for such purposes, namely, from the mortality prevailing among the out-pensioners of Chelsea and Greenwich establishments during the seven years 1814-1822, numbering 75,000 men, and from which he had constructed a Mortality table.

Mr. Finlaison at first stated that he conceived it totally impossible to obtain authentic materials sufficient to reduce the subject of sickness to any certain law; but he was afterwards

led to alter this opinion, and to state that he was strongly inclined to think that the recurrence of sickness is constant to a much greater degree than was hitherto supposed.

Further on in his evidence, Mr. Finlaison suggested that the object of friendly societies should be limited entirely to provision against sickness, and that they should arrange for deferred annuities through Government. In most cases where the failure of friendly societies has taken place, he believed it had been brought about by inattention to the value of Annuities in old age.

In answer to further questions, Mr. Finlaison stated that he thought materials showing the probable fruitfulness of marriages might be obtained from the registers of the Royal Regiment of Artillery. He had commenced, but had not carried through, an observation on the decrements of life prevailing among the immediate descendants of peers from the materials given in Sir Egerton Brydges' edition of Collins' Peerage.

Mr. Joshua Milne was also examined before the Committee. He stated that he knew of no data on which a calculation as to sickness could be made which could be depended on. He knew of Dr. Price's tables, but was satisfied that very little dependence could be placed upon them on account of the method of their construction.

Mr. Morgan stated under examination that in his opinion a table founded on the Experience of the Peerage and Baronetage would be suitable for the grant of endowments to children.

It is interesting and instructive to notice what was the opinion of this, among the earliest, of Select Committees on the subject of State Supervision and Control. They remark, "Your Committee take this opportunity of observing that it is, in their "opinion, only in consideration of advantages conferred by the "law that any restrictive interference can be justified with volun-"tary associations established for lawful and innocent purposes." No doubt this expression of opinion has formed the key-note to much of the assurance legislation since that date.

On 29 June 1827, just two years after the date of the report, we have been considering, a report from another Select Committee on Friendly Societies was issued. This Committee directed its attention almost entirely to the sufficiency of the contributions for Old Age benefits, or deferred annuities, and to sums payable at death. They state that they have nothing to add to the report of 1825 as regards contributions for sickness allowances and endowments to children.

The Committee recommended the adoption of the Carlisle Table in calculating deferred amuities. As regards Mr. Finlaison's Government Tables, they thought the objection that they were based on *select* lives applies, in a considerable degree, to them as regards their use for friendly society purposes. For death benefits, the Committee considered that tables founded on the Northampton Table might be made use of.

Among the witnesses examined by the Committee were Messrs. Joshua Milne, Francis Baily, Charles Babbage, Benjamin Gompertz, John Naylor, Griffith Davies, J. D. Bayley, Wm. Morgan, and John Finlaison.

Mr. Babbage stated in his evidence, "I strongly object to "using tables giving a greater mortality than is expected to take "place, a course which has sometimes been defended on the ground "of safety. Safety is much more certainly secured by judging as "nearly as possible the true risk, and adding an additional sum "for security. If tables not representing the mortality of the "class for whom they are designed are employed, every step in "the reasonings which are deduced from them is liable to "increased error; and, if the calculations are at all complicated, "the errors so introduced may not improbably act on the "opposite side to that which they were introduced to favour. My "view in all cases is-let us get as nearly as we can the law of " Mortality of the class for which we want to calculate, and add "to the prices computed from it some proportional part sufficient "to insure the safety of the establishment which uses them." In connection with these remarks of Mr. Babbage, it is to be borne in mind that in the early part of this century the premium rates of all offices were based on the Northampton table, generally with three per-cent interest, and without any direct addition for expenses and contingencies.

The 1827 Committee seemed sceptical in respect to the considerable difference in value brought out in Mr. Finlaison's tables between male and female life, as it went much beyond what they had received from any other quarter. They accordingly desired to hear from Mr. Finlaison any further observations on the subject that he might think useful. Such incredulity must have been somewhat trying to Mr. Finlaison, but we know that this was not the only occasion on which his efforts for the public good did not meet with the reception they deserved. We are told that, from the first moment of his undertaking the subject of life assurance until the production of his report of 1829, he continued constantly

impressing upon the members of the Government the enormous loss which the country sustained by persisting in using the erroneous Annuity tables; but, so far from meeting with the encouragement and commendation which might have been expected, he received nothing but neglect and contempt. One great function of the Institute is to try all new advances and discoveries in our science, and the weight of its approval must unhesitatingly secure the acceptance by the public of new views, and further discoveries by its members, and effectually kill the suspicion and neglect which frequently fell upon the opinions of Actuaries in earlier and less happy days.

At the request of the Committee of 1827, Messrs. Finlaison and Davies computed two tables: (1) showing the single and monthly payments to be made by males and females to insure a weekly payment of 10s. bed-lying pay and 5s. working pay, and 5s. per week superannuation allowance from age 70; (2) Single

and monthly premiums to secure £10 at death.

In computing these tables, Mr. Davies took the Carlisle Mortality, and Mr. Finlaison the mean of his male and female Government Annuity Tables. As regards sickness, both took the mean between the Highland Society's Tables and the rates of sickness for the Army deduced from the Adjutant General's official returns. The results of their calculations agreed very closely indeed.

All the Actuaries examined before this Committee, with the exception of Mr. Morgan, condemned the use of the Northampton table for deferred annuities.

The Committee submitted certain tables of rates for superannuation allowances to Messrs. Baily, Gompertz, Milne, Davies, Naylor and Finlaison separately, and obtained opinions from these Actuaries as to their sufficiency. This might have formed a valuable precedent in regard to testing the sufficiency of other subsequent Government measures; and, in that case, as we shall presently see, much loss might have been saved to the country subsequently.

In the beginning of the year 1828, a Committee on Public Income and Expenditure was appointed. Their report was very short, and is as follows:—"The Select "Committee appointed to enquire into the state of the public "income and expenditure of the United Kingdom, and to consider and report to the House what further regulations and "checks it may be proper in their opinion to adopt for establishing

"an effectual control upon all charges incurred in the receipt, "custody, and application of the public money; and what further measures can be adopted for reducing any part of the public expenditure without detriment to the public service, have proceeded to an examination of the matters referred to them, and agreed to the following report: The Committee having, in the course of their enquiries, discovered that the conditions under which the Commissioners of the Sinking Fund are required, by "the Act for enabling the Commissioners for the reduction of the National Debt, to grant Life Annuities, are extremely disadvantageous to the public, feel it to be their duty to call the "immediate attention of the House to the expediency of repealing "this Act."

Repeal of Acts for granting Life Annuities. In accordance with this report, a Bill was immediately passed by Parliament "To repeal so much of several "Acts as empowered the Commissioners for the reduction of the "National Debt to grant Life Annuities."

A new Act as to granting annuities was passed in A new Act as to granting annually annual to tables in connection with it were annually Tables. 1829, and the tables in connection with it were those of Mr. John Finlaison, known as the Government Annuity Tables, 1829. The tables were not appended to the Act, but were published separately, and were liable to be changed as occasion arose. It was fortunate that this provision had been inserted in the Act, for the amounts of annuities allowed at the higher ages were apparently much too high. instance, for every £100 of purchase money, a male of the advanced age of 90 could obtain an annuity of no less than £62. The manager of one of the Scotch Assurance Companies is understood to have at once called the attention of the Government to the ruinous rates at the older ages, but the reply he received was not over courteous, and he was in effect told that he had better mind his own business. This suggestion he proceeded to carry out to the full; for, finding it useless to argue with Government, he adopted another method of convincing them of their error. To this end he advised his own and other assurance offices, and also many of his private friends, to select robust males of advanced age and to buy annuities on their lives. Other speculators also no doubt discovered the weak points about the Government Tables, with the result that annuities on no less than 675 old men were purchased before these rates for the advanced ages were removed from the table. One of the speculators who had bought several annuities, was much distressed because one

of the lives he had selected died shortly after the annuity was purchased. This however was, on the whole, a somewhat rare experience, and the Government in the end were the losers by the deal to the extent of something like one million sterling.

In 1829 a Select Committee on Life Annuities was Committee on Appointed. The origin of this Committee was not the loss on annuities just referred to, but a Petition by Mr. Cadogan Williams praying that opportunity might be given to the lower classes of society of buying deferred annuities for small amounts from Government.

Mr. Williams himself does not appear to have formulated any plan or means of carrying the object of his Petition into effect.

Mr. Samuel Higham, Comptroller of the National Debt Office, proposed to the Committee that the Savings Banks should be utilised for the purpose in view—that a depositor should be allowed to transfer each year, the whole or part of his deposit to an Annuity Account, and therewith the Trustees of the Savings Bank should purchase an annuity from the Commissioners of the National Debt. Mr. Higham in his evidence states, however, that he doubts whether the poorer classes will avail themselves of the opportunities to purchase deferred annuities, even if they be given them; as, ever since the year 1817, the Commissioners of the National Debt have only granted about £14,000 per annum deferred annuities. The smallest annuity they are empowered to grant is, he says, about £20.

Mr. John Finlaison was examined, and he submitted various Tables of Annuities—the rate of interest being in all cases, £3. 8s.  $5\frac{1}{4}d$ . per annum payable half-yearly, being the rate allowed by the Trustees of Savings Banks to their depositors. The mortality was that of the Government Annuity Table, 1829. (The Trustees of Savings Banks were at this time allowed by Government interest at the rate of Twopence halfpenny per £100 per diem, and they were authorised to give their depositors Twopence farthing per £100 per diem, i.e., £3. 8s.  $5\frac{1}{4}d$ . per annum—the other farthing retained being required to meet expenses, &c. On this account, Mr. Finlaison made no addition in his tables for expenses).

Mr. Finlaison advised the Committee that, if a person effected a deferred annuity by annual premiums, and after a number of payments, failed to meet his premium, he should be entitled to a deferred annuity for a reduced amount to begin at the same age, or even at an age further deferred, so as to increase the amount of the annuity, if only a very few premiums had been paid. He would not advise any return in money to be made in such cases on account of the question of health; and he thought the smallest paid-up deferred annuity that should be granted should be £1 per annum. Instead of calculating deferred annuities at the rate of the day, Mr. Finlaison thought that an average rate might be more properly taken. He went over all the purchases made by the Commissioners for the Reduction of the National Debt during the 40 years 1786-1827, and he found that the variable rate of interest resulting from the investment of money in the 3 per-cents at their fluctuating prices from day to day was, in the long run, the very same, to the minutest fraction, as the constant rate of  $4\frac{1}{2}$  per-cent. therefore thought 4 per-cent might be relied on for many years to come, and that it would be quite safe to calculate deferred annuities at that higher rate, or even at  $4\frac{1}{4}$  per-cent. The difference between the rate assumed and that realized would, he thought, meet all expenses. A drawback to calculating Savings Bank Annuities at an average rate of interest was pointed out to Mr. Finlaison, namely, that as Government Life Annuities are not calculated at an average rate of interest, but according to the fluctuating price of the day, a person buying an annuity would get a better bargain by going to the Savings Bank than to the Government. Mr. Finlaison admitted this, but thought the drawback would be quite well met by limiting the amounts of the Savings Bank Annuities. Mr. Finlaison was further asked whether it might not be practicable, by a system of Bonuses, to arrive at the same result as by calculating at a higher average rate of interest; but he pointed out that such a system would entail a great deal of valuation and calculation which could not be done by Trustees of Savings Banks, and which could not be drawn up in tabular form. Moreover, he thought that the class for whose benefit these annuities were intended would prefer a smaller present annual contribution to a possible increase afterwards in the amount of the annuity. Being asked, "Could "you by calculation ascertain the exact value of a life annuity "at a rate of interest less by 7s.  $7\frac{3}{4}d$ . than  $4\frac{1}{4}$  per-cent"? Mr. Finlaison gave the following answer, which must surely have impressed the Committee—"Yes", he said, "I could compute it at a rate less by the millionth part of a penny"! This Committee, after going over the evidence, conclude their

Report as follows—"Your Committee cannot, at this advanced "period of the Session, contemplate the introduction of a Bill "founded upon the views which they have taken; but, should the "House think it expedient to re-appoint your Committee in the "next Session, they hope it may be in their power to frame such "a Measure as may obtain the approbation of Parliament."

In 1829, the various Laws in regard to Friendly Societies were consolidated; and, as the amount to be insured on any single life by a society registered under the Act was not restricted, Mutual Life Assurance Societies took advantage of it as a means of securing legal protection, and other benefits. We shall see later on that the ordinary Life Offices were not allowed to enjoy for long the privileges they had thus somewhat unfairly secured.

A Select Committee sat in the year 1833 on Parochial Registration, and Mr. Finlaison stated before it, that the diversities in mortality and siekness which existed in different parts of the country were utterly unknown.

In the same year, 1833, Mr. Finlaison commenced extensive computations in regard to the duration of slave and Creole life on the West Indian plantations in connection with the measures that were undertaken for emancipating the slaves. Two years later, he made the necessary calculations for bringing out the West India Loan of £15,000,000 to compensate the owners of the slaves.

During the years 1835, 1836, and 1837, Mr. Finlaison was busy with various measures emanating from the Ecclesiastical Commissioners, and among the subjects on which he reported, may be mentioned the means for improving Church property by the abolition of fines, the great question of Church leases, and the steps leading to the "Appropriation Clause", and the Church rate question.

The Act requiring the registering of Births, Deaths,
Births, dec.,
Registration Act for England.

Mr. Finlaison, as was to be expected, was consulted on many points connected with the establishment of the system.

In 1840, the Friendly Societies Acts were amended in the following particulars:—No policy assuring a sum of more than £200 to be exempt from stamp duty.

Societies issuing policies of larger amount were not to be allowed to invest funds with Savings Banks or with the Commissioners of the National Debt. Ordinary Mutual Life Offices were in these

respects excluded from the benefits which they had obtained under the Act over Proprietary Companies, and which it was, no doubt, never intended that they should have become entitled to.

Next year (1841), a Parliamentary Committee on 1841. Committee on Joint Stock Companies. Joint Stock Companies was appointed, chiefly with the view of enquiring into those "Bubble" Companies which had been started, ostensibly for the purpose of carrying on Assurance and Annuity business. The Committee issued its first Report in 1844. It states that the Committee believe that Assurance and Annuity Societies generally are deserving of the highest credit on account of the respectability of their management, and the soundness of their schemes; but that instances of abuse which have occurred justify and demand enquiry. The Committee accordingly took some evidence and issued a list of queries to the various companies. The Report then goes on to say, "Your Committee propose, as soon as the information to be " supplied by the answers to the queries can be properly collected, "to prosecute their enquiries in this direction with a view, on "the one hand, to suggest such practical facilities and encourage-"ments as it may be in the power of the Legislature to afford " to these Companies for the furtherance of the System of Life "Assurance, so important to the well-being of the Nation at "large; and, on the other hand, to prescribe such reasonable " preeautions as may be requisite to insure the faithful observances " on the part of such Companies of the extensive obligations into " which they enter with the public."

As a result of the foregoing Report, "The Joint Stock Joint Stock Companies Act, 1844", was passed. Its two leading features were (1) Provisional Registration, requiring certain information to be furnished regarding new Companies before they could even be brought before the public; and, (2) Complete Registration before a company could begin to do business. A copy of every balance-sheet was also required by the Act to be registered. Many hundreds of Insurance Companies were founded under this Act, some good, and some only fraudulent from the start. It has been said that far greater mischiefs followed the Act than had preceded it. The chief causes of failure of this Act appear to have been (1) That no specified form of accounts was required, and (2) That distinct and sufficient powers were not provided to compel compliance with its requirements. The result in many cases was that its provisions were evaded. From the failure of this Act, the lesson was no doubt afterwards learned, as regards Life Assurance Companies at any rate, that while they might be left free to conduct their business in the way they considered best, it was necessary, both in their own interest as well as in the interest of the public, to compel them all, without exception, to disclose full information as to the methods they had employed in conducting their business, as well as the results arising from the methods employed. In the case of companies, as of individuals, there must necessarily be a limit to freedom of action; and, as regards Assurance Institutions, that limit has, perhaps, been as happily fixed as it can be by the Act of 1870.

It may be mentioned here that there appeared in the year 1843, the first Mortality Table prepared from statistics collected by Government, namely, the English Life Table No. 1; and also the first collective Experience of Assured Lives, the 17 Offices' Experience Table.

In 1847, a Bill was introduced to Parliament having the following preamble: "Insurances on lives having proved very advantageous to the families of persons effecting the same, and for other purposes, it is desirable to encourage Insurance, and, with that view, to secure as much as possible that the claims on account of such policies shall not be defeated by objections on the part of Insurance Companies." This attempt to interfere with the rights of contract between the Companies and their policyholders was most properly very promptly rejected by the House of Commons. Experience has at least shown that such an Act was altogether uncalled for, seeing that it is very seldom indeed, that a claim is contested by a Life Assurance Office. In fact, in place of Assurance Offices being accustomed to dispute claims, they too frequently make concessions in regard to disputable points in order to avoid litigation.

A Select Committee of the House of Lords was floored floored appointed in 1848 to consider and report upon a Bill which had been introduced into Parliament bearing the following title:—"An Act for the more effectual protection "from Fraud and Misappropriation of the Funds of Certain "Charitable and Provident Associations, and for the relief of the "Members thereof from the provisions of two Acts, namely, 'An "Act for the more effectual suppression of Societies established for "seditious and treasonable purposes, and for better preventing "treasonable and seditious practices"; and 'An Act for the more "effectual prevention of Seditious Meetings and Assemblies."

The Committee in their Report made various suggestions and advised a modification of the Acts then in force.

Among the witnesses examined were Messrs. Neison, Ansell, Davies, John Finlaison, and A. G. Finlaison.

Mr. Ansell was examined as to what constituted sickness in a friendly society, and as to the applicability of tables of sickness compiled from general statistics to small societies composed of separate trades. In ordinary healthy trades he did not think there was sufficient difference in the rate of sickness to necessitate a separate table for each trade. From his practical experience he had found that small local friendly societies had succeeded best.

Mr. Griffith Davies was asked if there is more real siekness in female than in male life, and he replied, he had no means of determining that. From his observations on the point, Mr. Ansell was of opinion that the siekness among females, taking age for age, was throughout the greater part of life more than twice as great as among the males.

Mr. John Finlaison did not think that it had by any means been proved that the duration of life is longer in country districts than in town districts. He thought there were countervailing causes in an agricultural district, and in time the balance is pretty equal.

All the Actuaries examined agreed entirely as to the want of proper data for mortality and siekness, and in urging that Government should take steps to collect such data, and submit them to a Committee of Actuaries to be dealt with.

Mr. John Finlaison attended, in his capacity of Government Actuary, as a witness before the Committee of 1848 on Feargns O'Connor's Land Scheme; and, in the same year, he sent in to the Treasury the second of two Reports on the Act of the previous Session for lending money to Irish landlords, with tables necessary to give effect to it.

Still another friendly society measure comes before us in the shape of "A Bill to make better provision "for eertifying the Tables of Contributions and "Payments of Friendly Societies." It was referred to a Select Committee of the House of Commons in June 1849; and the Committee having read the Bill, thought it advisable to apply to the House for leave to take evidence on the subject. The actuarial witnesses were Messrs. Neison, John Finlaison, Alex. Finlaison, Ansell, and David Jones.

Mr. Neison stated that, up to the year 1824, there were no

data existing of much value to friendly societies, and absolutely nothing in regard to the element of sickness. About the year 1820, the Highland Society of Scotland published an Analysis of Returns obtained from 70 or 80 societies. A year or two afterwards, the Society for the Diffusion of Useful Knowledge issued schedules to societies in England requiring returns on the rate of sickness, and these were analyzed by Mr. Ansell. By the advice of a Committee of the Statistical Society, London, Mr. Neison, about the year 1840, offered prizes for the best returns from Scotch societies, and obtained in that way a considerable number of most valuable returns. These, along with the returns made to Government by friendly societies, were analyzed by Mr. Neison.

Mr. Neison was asked how many Actuaries it would take to make a quinquennial valuation of the 14,000 enrolled friendly societies during a period of three months, and he replied 20 if they had assistants. He did not think it would be wise to have the work done in a Government Office.

Mr. John Finlaison stated that he constructed, for the Committee of 1825, what he believes was the first Table of Sickness that was ever computed. He thought the ultimate insolvency of friendly societies had been too much dwelt upon, and that the good they really accomplished, while they existed, was rather overlooked. He was of opinion that friendly societies should not be too much interfered with. Mr. Finlaison considered that the whole body of Actuaries then existing was insufficient to value all the enrolled friendly societies quinquennially. Being asked: "Do you not think it highly desirable, in a national point of "view, that friendly societies should be enlightened as to their "real condition"?—he replied: "I know of no means of accommission it, but by laying them under such stringent conditions "as would probably break up these societies."

Mr. Ansell proposed the establishment of a Board in London to regulate and counsel friendly societies rather than to enforce particular rules. He considered that the provision of the Act of 1846, which requires a certificate from a competent Actuary before the Registrar certifies any society, is not a desirable provision. The tendency of the existing law is to prevent societies from being enrolled at all. They are now exposed, not only to the chance of an unsound table, but also to the chance of fraud among themselves, without any legal remedy against it.

Mr. David Jones, on the other hand, from his own experience,

considered it very desirable that the opinion of an Actuary should be obtained upon tables before they are laid before the Registrar.

There seems no question that Actuaries find, in most cases, that friendly societies they are called on to value are insolvent, and that they do not generally adopt any suggestions made for their improvement. Mr. R. P. Hardy stated before a subsequent Committee, "speaking from my own experience, I cannot recall a "case (save that of the Hearts of Oak Society) where my "recommendations for adjustment were accepted by a friendly "society."

As a result of the evidence tendered to them, this Committee recommended that the Bill referred to them should be withdrawn, and that a new Bill, which they had drafted, should be introduced.

Many matters of interest and importance have to be referred to as having occurred in the year 1852.

select Committee A Select Committee was appointed to enquire into the Property Tax. working of the Income and Property Tax; and one branch of the enquiry was specially to examine into the principle of the assessment of terminable and variable annuities.

The Committee examined Messrs. Hill Williams, Erskine Scott, Brown, Jellicoe, Edmonds, Peter Hardy, F. G. P. Neison, and Dr. Farr. All these Actuaries agreed that the method of levying the Tax under the Act then in existence was very unjust, owing to the fact that all incomes were taxed alike, being considered as entirely of the nature of profits, whereas in many cases, such as annuities, a large proportion of the yearly return consisted of capital. Various methods by which the Tax might be more equitably assessed were suggested by the Actuarial witnesses. Many assurance offices it appeared had been in the habit of investing in terminable annuities, but it was stated that the way in which the Tax was imposed was making them withdraw from that form of investment.

One of the non-actuarial witnesses, Mr. Warburton, M.P., submitted an algebraical proof as to the levying of the Tax on annuities to the Committee in the course of his evidence. The Chairman of the Committee wrote Dr. Farr, stating that he did not clearly comprehend this proof, and asking him to see the Actuaries and get a statement from them showing the fallacies in Mr. Warburton's evidence. Messrs. Hardy, Neison, Brown, Williams, and Jellicoe, accordingly prepared the statement required, and signed it as Members of the Institute of Actuaries. Dr. Farr also dealt with the matter separately.

The Report to the Annual General Meeting of the Institute, held on 3 July 1852, remarks on the foregoing Committee, and the evidence submitted to it as follows—"A Committee of the "House of Commons has been engaged during the Session in "the consideration of the subject of the Property and Income "Tax, with a view to determine the true principles of direct "taxation. It is gratifying to the Council to observe, from the "remarks of the Chancellor of the Exchequer, that the evidence "given before the Committee by Members of the Institute has "tended to throw much light on that intricate question. The "Council are glad to perceive one of the most important "functions of the Institute so soon called into exercise."

There occurred about this date what was known as the "Insurance Controversy." It arose from a letter which Mr. Christie, of Edinburgh, addressed on behalf of the older insurance offices, to the President of the Board of Trade, requiring "A thorough scrutiny and investigation into the "affairs and responsibility of every life and annuity institution "in the United Kingdom, with a view to such enactments as "shall protect extensive public interests from the alarming "prospective evils of fraud and ignorance." We need not now enter into the war that was waged over the matter.

In this same year, 1852, a Parliamentary Committee was appointed, nominally to consider the law relating to friendly societies, but, in fact, to solve the difficulty in regard to "nomination" policies still being issued by several offices. The greater part of the Report of the Committee is taken up with the consideration of evidence in connection with a Petition from 83 assurance companies complaining that certain five offices established under the 10 Geo. 4, C. 56 before the year 1840 " claim to have the power to make their policies, by a clause in "the policy, payable to widow or widower, or child of a member "to any amount, and that such policies are exempt from legacy " and probate duty; that these societies have ceased to possess "the character for which Parliament granted privileges; that "they have an unfair advantage from being constituted under the " Friendly Societies Act-which is equivalent to a charter-being "in reality trading life assurance companies; and they ask to "have such societies confined in their transactions to the limits "prescribed by the Act, namely £100, or compelled to be " reconstituted in the same manner as other life companies."

The other alternative the Committee inclined to recommend to

Parliament was to allow the privilege of exemption from stamps and from probate and legacy duty to the policies of all life assurance offices, as well as friendly societies, to the amount of £500. (Parliament did not, however, assent to this recommendation.) In return for these privileges, the Committee advised that such societies or assurance companies as might avail themselves of them should be required to make an annual return of their accounts to be laid before Parliament. This would also, the Committee thought, be a means of checking the formation of delusive and ill-founded schemes for establishing life assurance companies.

Mr. Tidd Pratt, Registrar of Friendly Societies, suggested, in his evidence, that the returns that had been made to him by nearly 3,000 societies should be laid before the Institute of Actuaries, with a view to having tables of sickness deduced from them. He also mentioned that, owing to the want of proper data, he found in many cases the calculations of Actuaries differ much from one another, and this causes the labouring classes not

to place much confidence in such calculations.

Mr. Jellicoe stated that the Highland Society of Scotland, finding that the very elaborate tables put forth by them for the general use of friendly societies in Scotland were not to be depended on, have withdrawn them, and have requested the Institute to look into the matter with a view to supplying other tables, or to suggest such other means of regulating the societies as may be best calculated for their welfare. He also stated that the members of the Institute were in favour of the Petition presented by the 83 Offices. Mr. Jellicoe was in favour of removing altogether from friendly societies the power of making life insurances or granting annuities. He did not believe there was stability in a small friendly society to make a contract of that nature with them safe. Mr. Jellicoe suggested the consolidation of the laws relating to friendly societies; and the deposit by new assurance companies with Government of a sum of £10,000, in order to check the formation of merely mushroom undertakings.

We find the following allusion, by the Council of the Institute, to the subject of the suggestion by Mr. Tidd Pratt in his evidence before the foregoing Committee: "In connection with the col-"lection of the experience of friendly societies obtained from the "quinquennial returns to 31 December 1850, received by the "Registrar of Friendly Societies in England with a view to the "deduction of tables therefrom, the Council of the Institute of

"Actuaries was consulted by the Registrar of Friendly Societies "as to the necessity of further information; also if they would "consent to superintend the preparation of such data and the "computation of the tables; and generally, they were requested "to advise as to the manner in which the subject should be "treated. The Council, in reply, expressed their opinion that "such a valuable opportunity should not be lost for adding to "our knowledge, and offered to undertake the public trust of "analyzing the returns, and forming from them tables of the "money values of allowances in sickness and sums at death. "The Lords of the Treasury, on considering the report of the "Registrar, thought they would be justified in incurring the "expense for so great a public object; but, on the whole, con-"sidered it would be better that the task should be undertaken "by the Government officials in the department for friendly "societies, and under the Commissioners for the Reduction of the "National Debt." The tables were accordingly deduced by Mr. A. G. Finlaison, and were presented, in accordance with an order of the House of Commons, and ordered to be printed 16 August 1853.

Mr. A. G. Finlaison also submitted to Government tables of annuitant life which he had deduced from various Tontines from 1773 to 1850, but they were not adopted, owing apparently to his own suggestion that a further period of years might restore the value of female life to the higher rate indicated by Mr. John Finlaison's tables.

A great many actuarial witnesses were examined before

the Select Committee on Assurance Associations, 1853. They were all generally asked very similar questions in regard to certain subjects, such as the working of the 1844 Act, and as to its improvement; what general form of Accounts and Returns, if any, could be obtained; whether a deposit with Government should be required from Assurance Offices; and whether it was desirable or necessary that Assurance Companies should have a Paid-up or Subscribed Capital, and if so, of what amount. They were also asked as to rates of expenditure, especially in new offices, and as to the proper data for valuations, and whether a uniform method of valuation could be devised for all the offices. Several Actuaries, but notably Mr. Jellicoe, were

Mr. John Finlaison stated that, by the desire of the late King,

asked particularly as to the qualifications of an Actuary, and

if it was desirable or possible to test these in any way.

he had given counsel in the formation and conduct of an Insurance office, the object of which was to enable officers in the army and navy to insure their lives against tropical risks at known and definite rates. He computed tables from very extensive data, showing the proper premium for Europeans serving in the Tropies, and he thought these tables of his were the first of the kind that were ever computed,—that was about 17 years ago. Mr. Finlaison's opinion was that "an Actuary is not well used "by his Company unless he has a voice potential in the acceptmance of lives." He also advocated a "Confidential System of Investigation" by Government in determining the solvency of life offices.

Mr. Charles Ansell thought it would be quite desirable that, after setting forth the liabilities of an Institution in regard to their policies of assurance, there should be some strict declaration (a declaration before a Magistrate would be best) as to the condition of their assets. He stated the following objections to requiring all offices to value by the same Mortality Table and rate of interest: (1) The method of valuation is already prescribed by the Deeds of Settlement of many offices; (2) A great deal of the work is done by tables specially prepared at great expense and trouble, and these would all be useless; (3) Various systems of Division of Profits are regulated by the methods of He suggested a separate registry for Assurance Companies, with an officer at the head of it thoroughly acquainted with assurance business, without, however, any power to interfere with the discretion and management of the office. Such officer might be empowered to report any failure in compliance with the requirements of the Act.

Mr. William Thomas Thomson advocated a separate Act being applied to the registration of assurance associations and extending to the United Kingdom. He was not in favour of publishing an annual balance-sheet by offices; but thought it should be recorded for the information of the Government solely. Government might have an officer (an Auditor) who should have power to refer to the general books of companies, and advise the Board of Trade, if he thought it necessary, as to the position of any of these institutions. He also thought that each assurance office should be required to appoint a professional Auditor of its own. Mr. Thomson handed in forms of balance-sheets which he suggested should be required from assurance offices. He also suggested that a deposit of £10,000 should be required from

new companies, and that that sum should be retained by the Government so long as the company continued in existence. He was satisfied, from an examination of their balance-sheets, that there were a number of companies in existence which it would be better to wind up.

Mr. T. R. Edmonds stated that he believed one-third of the existing English offices were not in a condition to meet their liabilities. Nevertheless, he thought there should be no Government interference with offices, except upon a requisition by a certain number of policyholders.

Mr. Samuel Brown mentioned that the office with which he was connected was founded on mutual principles without any capital, but that the promoters each guaranted £500 if it were required at any time within the first five years.

Mr. Jellicoe's evidence related chiefly to the status and qualifications of an Actuary, and has been already referred to in this respect. He was strongly of opinion that the person on whom the responsibility rests, as regards the management of an office, should always be an Actuary. He would be quite content that any tests as to qualification as an Actuary should be applied to new men only, and that those already in positions should be held to be qualified. He really placed more reliance upon the efficiency of the Actuary than on almost anything else.

Mr. Jellicoe also handed in a paper entitled a "Statement of views entertained by the undersigned Managers of Life Assurance Offices in Scotland with reference to the contemplated "amendment of the law regarding Life Assurance Associations" now under consideration of a Select Committee of the House of "Commons." The preface to the "Statement" points out that "the evils for which a remedy is sought consist not so much in "the adoption of insufficient rates, or the fraudulent misappropriation of the premiums received; they have rather arisen from "the recklessness with which new offices have been instituted, and "the unwarrantable expense incurred in forcing them into notice," and in seeking to procure business." The recommendations in the statement were as follows:—

 At the establishment of an office, not only the Deed of Settlement, but a copy of the prospectus of rates (with a statement of the data on which these rates are calculated) certified by two Directors and the principal officers, should be lodged with the Registrar.

- II. There should be lodged annually, in place of the present balance-sheets, accounts to be prepared in forms appended to the Act, showing—
  - (1) The subscribed capital, amount paid-up, number of shares, and list of shareholders.
  - (2) The amount of the Life Assurance Fund.
  - (3) The total funds, and the mode of their investment.
  - (4) The debts and all outstanding claims.
  - (5) The number and amount of the existing policies, and the annual premiums thereon.
  - (6) The amount of bonuses already declared.
  - (7) An abstract of the whole receipts and disbursements of the year.
  - (8) Similar particulars as above in regard to the annuity or other business.
- III. At each periodical investigation a statement should be furnished of the assurances and premiums thereon classified according to age, with particulars of the table of mortality, rate of interest, and other data employed.
- IV. A deposit of £10,000 should be made with Government.
  - V. No dividends or bonuses should be allowed till after an investigation at the end of not less than five years.
- V1. It should be the duty of the Registrar to report to the Board of Trade any failure of compliance with the requirements of the Act.
- VII. A copy of the returns to the Registrar should be laid annually before Parliament.

The Managers were also strongly opposed to combination of guarantee of fidelity, rents, provision for sickness, and taking of loans with life assurance.

Mr. F. G. P. Neison was most decidedly averse to any interference on the part of the State with life offices. He was also questioned as to the desirability of some tests being applied to persons calling themselves Actuaries, but he did not like to commit himself as to what was practicable. He thought a test desirable, if it could be had.

Mr. Higham handed in a table, on the principle of a "Model Office", showing the liability of an office in terms of the annual

premiums. It cannot, however, be applied to an office of more than 12 or 15 years' standing, and the amount of each year's new business is assumed uniform. Mr. Higham would only require offices to furnish particulars of their policies for valuation; and he had no doubt some one would soon come forward and value all the offices from the particulars supplied.

Mr. Farren handed in a paper illustrating the probabilities of gain or loss in life offices, being the expansion of  $[(1-m)+m]^n$  where m = the mortality and n = number at risk.

Mr. W. S. D. Pateman, publisher of the Post Magazine, gave an account of various "Bubble Companies." He also produced a circular which had been issued by a new London Company to almost every name in the Edinburgh Directory. The circular stated that a friend had recommended the person to whom the circular was issued to act as an agent, but Mr. Pateman humorously remarks that the effect of competition appears to have tempted the secretary of the said company to overstep the limits of truth, as it was not likely that any friend would recommend a Director, Manager, or Secretary of a Scottish Company to become agent of a new London Company, notwithstanding that it was "incorporated by Act of Parliament with a guaranteed capital of \$50,000." The fact is, he says, that, in sending out the circulars, it had been forgotten to omit the gentlemen connected with the Edinburgh Assurance Offices.

Dr. Farr referred to the impulse given to life assurance by Mr. Pitt's Income Tax Bill of 1798, wherein life assurance premiums were exempted from Income Tax. He thought the Board of Trade might have power, in certain cases, to send down an Actuary to look over the accounts of companies, and ascertain their accuracy so far as the return itself extends. He thought that would have a very salutary effect in preventing fraud.

The conclusions of the Committee were as follows:-

- (1) The provisions of the 1844 Act are very imperfectly carried out, and many of them the Registrar has no power to enforce if not complied with.
- (2) The Act has not prevented deception and misrepresentation being practised on the public. Companies have begun with small capital, and after complete registration, greatly increased it, and the Registrar has no power to interfere.

- (3) The Act prescribes no form of accounts, and it has failed to afford either the information or the security it was intended to give.
- (4) The Committee arrived at the conclusion that, in many ways, the distinction between companies established since the passing of the Act of 1844, and completely registered under that Act, and others established prior to that date, and not so registered, has operated prejudiciously, and that it would be eminently to the advantage of the offices themselves, as well as of the public at large, if all insurance companies could be brought under one law, leaving each company to recommend itself to the public upon its own merits.

As a result of the evidence submitted to them, and following on the conclusions they had arrived at, the Committee recommended as follows:—

- (1) That life assurance companies should be dealt with in a separate Act by themselves.
- (2) That powers be given to the Registrar to enforce any regulations Parliament may enact.
- (3) That all companies, old and new, should be placed under one general system of registration.
- (4) That a deposit of £10,000 be required from new companies.
- (5) That no fixed form of accounts can be made applicable to all cases which would not be exposed to much evasion, or which would practically afford any real security; but, that it should be imperative upon each company to make a complete investigation into its affairs at least once in five years, and that, in intermediate years, a statement should be registered of (1) Receipts from premiums during the year. (2) Expenses. (3) The number and amount of new policies. (4) The total number and amount of policies current. (5) Total premiums on same. (6) Amount of capital and how invested. (7) The average rate of interest realized. (8) Amount of interest in arrear. (9) The Table of Mortality, and the rate of interest used in calculating the premiums.

In regard to the foregoing, it is remarked, in the Journal of the Institute of Actuaries, that "it cannot but be gratifying to "the Institute to observe that the recommendations made by the "Committee tally as nearly as possible with the resolutions" adopted at the Special General Meetings held on the 12 and "19 April 1853."

These were:—

- (1) That the Act of 1844 has created an invidious distinction between the offices established prior to 1844 and those established since, and that the Act in question ought to be forthwith repealed, and provision made for putting all existing offices on an equal footing.
- (2) The second resolution deprecated any undue restriction, but advocated the requiring from every new company a deposit with Government of at least £10,000 as some guarantee to the public of its ability to fulfil its engagements.
- (3) That no legislation on life assurance associations could be permanently effective that did not exact tests as to the respectability and acquirements of persons allowed to practise as Actuaries.

The Select Committee of 1853 dealt very fully with the question of State interference with assurance societies, and their statement of the advantages and disadvantages of State control is so full and so clear that we cannot refrain from giving it in extenso, even although we have already given somewhat fully a good deal of the evidence submitted to this important Committee. They say—"In approaching the question of what alterations, in "the present state of the law, your Committee are prepared to "recommend, they feel that it is one which is surrounded by "great difficulties. The two distinct principles in respect to "subjects of this nature-namely, that of interference by the "Government for the protection of the public, and that of no "interference whatever-have been very fully and ably discussed " by the witnesses examined by your Committee. On the one " hand, even admitting the general wisdom of the principle of " non-interference on the part of the Government in matters of "trade, it has been contended that the question of life insurance "differs so materially, in its general character, from ordinary "trading transactions that it may fairly be considered as an "exception to that rule. This exceptional treatment has been "justified and supported on the ground that the obligations "undertaken by such associations have reference to a very

" remote and uncertain period; that the object which persons " have in view in effecting insurances upon their lives is generally " of an important and solemn character, namely, the provision for "widows and orphans after the death of their natural protectors; "that, unlike any ordinary transaction of trade, a contract once " entered into cannot be discharged or abandoned, if doubts of the "stability of an office should arise, without a great sacrifice of " premiums paid in past years, and the necessity of effecting new " policies in other offices at increased rates of premium, owing to the "greater age of the assured; and that, in the present state of "uncertainty which arises from the imperfect knowledge as to "the real condition of assurance offices, persons are thus placed in " the anxious and unhappy dilemma of being compelled to persevere " in paying premiums from year to year with some suspicion and "doubt as to the ultimate advantage of doing so, or of incurring " the serious loss which, under the most favourable circumstances, "must attend the abandonment or sale of a policy. On these " considerations, as a special case, it has been contended by "different witnesses of great experience that interference on the " part of the Government is not only justifiable, but a matter of "high duty for the protection and information of the public.

"On the other hand, it has been contended with equal ability, "that insurance business forms no exception to ordinary trade in "these respects: that the Acts which have already been passed "with a view of controlling the operations of Insurance Societies "have been rather hurtful than beneficial for the very objects "they had in view; that an apparent compliance with the " provisions of an Act of Parliament and a certificate of complete " registration, while they have proved entirely ineffectual for their " professed objects and no real security to the public, have afforded "facilities, under the sanction of Parliamentary authority "embodied in regulations administered by a public department, " for the formation of companies and the perpetration of frauds "which could not otherwise have been accomplished. And it "has further been contended that it is impossible to make such " regulations consistent with the free development of private "enterprise, which, so far as the public is concerned, will not " prove more prejudicial by lulling private prudence and vigilance "than beneficial in respect to any increased security which they "can confer. Much difference of opinion has been expressed by "the different witnesses favourable to the former of the two "principles alluded to, as to the extent to which Government

"interference should take place. Some have contended, not only for an enactment which should more strictly regulate the circumstances under which Assurance Companies can be formed, but also for the appointment of Government Actuaries under a department of the State, who should have a direct supervision of the proceedings of all Insurance Companies, while the generality of witnesses favourable to some regulations have confined themselves to much narrower limits, and have contended only for such precautions as should test the bond fides of new Companies, and as should from time to time afford a fair amount of information with respect to the condition of such Companies, relying upon the good faith of Directors and Actuaries for the accuracy of the information furnished.

"Prior to the year 1844, the great concern of legislation seems to have been to seeure ample guarantee for the fulfilment of their obligations by Assurance Associations; but the Act of 1844 went upon the assumption apparently that enterprise is of primary, and capital only of secondary, importance in such Societies. The Committee of 1853 were in substance of opinion that the old state of matters should be reverted to in these respects."

It appears that the Council of the Institute in this same year (1853), addressed the Chancellor of the Exchequer in regard to the Succession Duty Bill then being introduced, because it contained in their opinion, several provisions of a character likely to affect injuriously Life Assurance and Reversionary Interest Societies, and because of some curious anomalies appearing in the Schedule annexed to the Bill. One of the objectionable clauses was in consequence at once withdrawn, and the Council were not without hope, that the other suggestions made by them would also be given effect to.

In the following year, 1854, another Committee had Friendly Societies Bill referred to them, and suggested various alterations. The evidence taken referred, however, principally to Burial Societies, and to the charge that had been made that payment of a sum of money on the death of a child was a temptation to the crime of child murder. This charge was proved to be quite unfounded. No actuarial witnesses were examined by this Committee.

As a result of the various Committees appointed in Friendly Societies Act. previous years to enquire into matters affecting Friendly Societies, a new Act of Parliament was passed in 1855. This Measure, while it repealed all previous Acts, also embodied

in itself their principal regulations. Owing to the number of Acts that had been passed up to this time, considerable difficulty was beginning to be experienced in determining the true state of the law on certain points, and hence the necessity even at this early date for consolidation.

At the Annual General Meeting of the Institute, held on 7 June 1856, it was narrated that "in the pending enquiry by "Parliament as to Civil Service Pensions, no evidence given before "the Committee will be found more important than that contri- "buted by the Members of the Profession examined on that "occasion." We have not, however, been able to see the Report to ascertain the nature and scope of the evidence referred to.

In the Journal of the Institute of Actuaries, vol. viii, Life Insurance Life Insurance Companies which was introduced into Parliament at the close of the Session 1857 by Mr. James Wilson and others, but was deferred on account of the lateness of the Session. Many of those concerned were, it is stated, satisfied with the Bill as a whole, and it was thought that "it would at least serve—if not "productive of much advantage—to prevent the adoption of a "Measure of a more objectionable character." Another Bill for the same purpose was introduced towards the close of the Session 1858 by Mr. Brinsley Sheridan and Colonel French, but, "although characterised by much ability, it could scarcely be looked upon as an improvement of its predecessor." It also was deferred owing to the lateness of the Session.

In the early volumes of the Journal of the Institute, there are several Editorial Notes lamenting the unrestricted increase in the number of Life Assurance Companies, and the great competition and expense arising in consequence to all in obtaining new business. In view, however, of the bringing forward of the two Bills just referred to, the Editor of the Journal seems to have become uneasy lest the oft-repeated references to restriction were really at length to be dealt with. In vol. viii of the Journal there is an Editorial Note on "The proposed Legislation for the Regulation of Insurance Companies", in which the following occurs-"The impression by which so many well-" informed persons seem to be actuated that they (Life Assurance "Companies) might be multiplied with advantage almost indefi-" nitely appears now to have become thoroughly eradicated; and, in " its place, a conviction seems to have arisen that the country cannot " support a very large number of these companies, and that the

"sooner the existing ones even are reduced in number the better." Be this as it may, we cannot but regard the rational and moderate "ideas now prevailing on this subject as a vast improvement on "the extravagant notions heretofore generally entertained with "regard to it; and, if reliance could be placed on the continuance "of this better disposition, we think our readers will agree that "the necessity for legislative interference has, in a great measure, "passed away, and that the Assurance Companies might now be "left to pursue their course, guided by their own discretion, and "untrammelled by further restrictions." The Note goes on to say, "that it is more than probable that all will not coincide in "this opinion, and that attempts will again be made to carry "some measure for regulating these important institutions."

In his Address to the Institute, in July 1860, the President, Mr. Jellicoe, said, inter alia, "As regards the question of taxation, "it is very desirable that further investigations should be made to determine whether it be true or not, that trade is paralyzed by the imposition of such duties as those of the customs and excise, and whether it be true or not, that a vast means of employment for the population is thereby destroyed, and needless poverty created and enforced. On the other hand, there remains abundant room for discussing what is the true measure of liability in a system of direct taxation, and for illustrating, with the aid of statistical records, the merits or demerits by which such a system may be found to be characterized."

Then, in the following year, Mr. Jellicoe remarked in regard to the foregoing, "The suggestions made at the last Annual "Meeting have received remarkable confirmation in the measures "of the Government with reference to the abolition of the duties "on the manufacture and importation of paper, and in the efforts "made by Mr. Hubbard to modify the system under which the "income tax is at present levied. I cannot but regard the "measures of the one, and the efforts of the other as essentially in the right direction; and, having no other desire than the growth and prevalence of right principles, I am happy to think "that a similar opinion is entertained by a large majority of "those whom I have the honour to address."

From the above and other passages, it will be seen that the members of the Institute, in its early days, gave a good deal of attention to matters relating to the State, even although they were not purely actuarial in character; and it is perhaps, to be

regretted, that seemingly less attention has been given of late by the Institute to similar subjects. The annual financial proposals of the Government would seem very suitable matters for consideration by the members of the Institute, but we do not remember to have seen any papers on such subjects which have been read at the Institute, and, it is indeed, very rarely, that even reference is ever made to them now-a-days at the meetings. Since a Charter was granted, the public are entitled, even more than in earlier days, to look to the Institute for guidance in regard to proposals of the Government in matters of finance and taxation, and it seems very desirable, therefore, that more attention should be given to such subjects in future by members of the Institute than has of late been bestowed upon them.

In the year 1862, there was passed the Companies Companies Act. Act, by means of which "Life Assurance Societies, "whether Mutual or Proprietary, could obtain a form of incor-"poration sufficient for all business purposes, and at the same "time could secure a limitation of the liability of their members " to such an amount (in proprietary companies at least) as should "be mutually agreed upon." We have already seen that some such measure as this was greatly needed, and the wonder is that the bringing forward of it was so long delayed. It seems to us unaccountable that Government should ever have withheld from such Institutions as Assurance Companies, possessing such large funds and doing so much good in the community, the ordinary legal recognition in conducting their business which was accorded to private traders; and, that it should have compelled them to go to the expense of obtaining Private Acts of Parliament in order to obtain such recognition. It is simply astonishing that Life Assurance Companies should have multiplied and prospered as they did, notwithstanding the unfair and restrictive legislation to which they were subjected up to this time.

A project, which was expected in certain quarters to Government the Saurance and Annuity business was initiated in 1864 in the shape of the Government the Post Office. Before introducing into Parliament the Bill for authorizing the scheme, it appears, from a paper by Mr. Adler in vol. xii of the Journal, that the Chancellor of the Exchequer had an interview with the Actuaries of several London Assurance

Offices, though he was opposed to a general enquiry, and the calling for "persons, papers and records" by the Select Committee to whom the Bill was referred. It seems that the opinion of the Scotch Actuaries was also asked upon the measure, through the Managers' Association, and that that body replied that, if the amounts of the Assurances were limited to £100, there was not likely to be any objection to the scheme on the ground of the Government entering into undue competition with the ordinary Life Assurance Offices. Waiving such objections as might be entertained on public grounds, the project they thought was well entitled to the favorable consideration of the Legislature. By the Government Bill the minimum amount of a life policy was fixed at £20, and the maximum at £100.

In a further communication to the Journal, Mr. Adler states that it is a source of satisfaction to him to find that most of the suggestions contained in his previous paper had been adopted in the Government Bill. The table of mortality, the rate of interest, the loading in the case of premiums payable oftener than once a year, and the whole mode in which the tables are presented are precisely those which, he says, he ventured to suggest in the previous year.

Instead of achieving the success it was expected to attain to at its origination, the scheme of Government Assurance has, as is well known, been pretty much of a failure. During the first ten years it was in operation, there were issued in all 4,478 life policies assuring only £343,797; and, of those, there were remaining in force, in 1874, 3,630 policies, assuring £284,069. In the same ten years there had been purchased 6,531 annuities, at a cost of £80,374.

With regard to "The Policies of Assurance Act 1867", Policies of Assurance Act. We find the following very interesting note in the Journal of the Institute, vol. xiv. "The Bill, as originally "brought in by Sir Colman O'Loghlen, was much shorter and "open to various objections. It was hardly discussed at all in "either House of Parliament, but nevertheless, underwent many "changes during its progress which was watched with great "interest out-of-doors, several meetings of Actuaries and "Managers of Life Offices having been held in London and "Edinburgh to consider the subject. The original Bill intro-"duced by Sir Colman O'Loghlen passed the House of Commons "with little alteration. In the House of Lords it was referred to a Select Committee which held one meeting only, and whose

" proceedings, judging from their Report, were of the most sum-"mary character. No evidence was taken, but the Committee "cancelled the original Bill altogether, substituting for it an "entirely new one, submitted to them by the Managers of the " Scotch Insurance Companies, and drawn by two Counsel who, "at the same time, gave an opinion altogether hostile to the "principle of making policies assignable at law. The conse-"quence was that the new Bill, as was remarked by "Mr. Russell Gurney in the House of Commons, while pro-"fessing to make policies legally assignable, contained words "which were intended to render it practically inoperative. "The Bill, thus amended, was sent back to the House of "Commons, where several amendments were proposed by "Sir Colman O'Loghlen, after consultation with a Committee "of Actuaries in London, and carried. These amendments "were mostly accepted by the House of Lords; but the very "important one, to omit the words in the first clause, 'and "'possessing, at the time of action brought, the right in equity " 'to receive and the right to give an effectual discharge to the "' Assurance Company liable under such policy for monies "'thereby assured or secured,' was rejected."

1869. In the month of August 1869, the whole Assurance world experienced a severe shock by the announcement of the failure of the Albert Life Assurance Companyalthough the evidence given before the Committee of 1853, showed that many offices were even then known to be unsound. That Office was founded in 1838, and between that time and the date of its collapse, it had absorbed by amalgamation no less than other 26 offices. At the death of the original Manager in 1868, it was considered advisable to have its affairs thoroughly looked into, and it then transpired that a net premium valuation in 1866 had revealed a deficiency in its funds of more than a quarter-of-amillion. For some time previous to the intimation of the failure, it had been suspected by outsiders that there was something seriously wrong with the state of the Company, and, when the whole truth came out, it was seen that there was no course open to it but to It was found on investigation, that almost the stop business. entire deficiency in the funds arose from the sums paid in connection with acquiring the business of the various companies which had amalgamated with the Albert. After various attempts had been made to deal with the affairs of the Company, a special

Act of Parliament was obtained under which Lord Cairns was appointed Arbitrator, with full powers to deal with the rights of all parties. Mr. A. H. Bailey was at the same time appointed Actuarial Adviser to Lord Cairns. In little more than four years the whole affairs of the Office were wound up at very moderate cost, and apparently with satisfaction to all parties.

Only a month or two after the collapse of the Albert Office, it was seen that another similar institution, the European Life Assurance Company, was in difficulties. The cause of its misfortunes was the same as in the case of the Albert, namely, indiscriminate and reckless amalgamation. In the end it had merged into itself no less than 46 offices. It was wound up in much the same way as the other Company.

Mr. Cave's Bills. The result of these disastrous failures was that Government was forced to take action, and to legislate in regard to the affairs of Life Assurance Companies. Mr. Stephen Cave was at that time Vice-President of the Board of Trade, and he brought in a Bill to amend the law relating to Life Assurance Companies. The Council of the Institute of Actuaries considered this such an important step on the part of Government, that they decided to summon a meeting of the representatives of all the Life Offices in the Kingdom to consider the Bill. At that meeting, resolutions were passed which were communicated to Mr. Cave, and the Council stated, that they had reason to believe that the Bill would be materially improved in consequence. What became of this Bill is not very clear, but, in the report of the General Meeting of the Institute held in July 1870, we find it stated that Mr. Cave has again introduced into the House of Commons a Bill to amend the law relating to Life Assurance Companies. This Bill, the Council state, they have watched with interest, and have made some suggestions for its improvement, which they have reason to believe will meet with proper consideration. Among many notices of amendment that were given, one by Mr. Bowring proposed to enact that the Actuaries responsible for the periodical investigations of Life Assurance Societies shall be members of the Institute. The Council state, that they think it right to say that, so far as they are aware, this notice was not given at the suggestion direct or indirect of any member of the Institute, but was entirely spontaneous on the part of Mr. Bowring; and, whatever the result may be, the Council say that they cannot but regard this notice as a gratifying proof of the progress which the

Institute has made in public estimation, and a recognition of its efforts to raise the status of the Actuarial Profession.

As showing the feeling of the Actuarial Profession in regard to the proposed legislation by Government at this time, we may quote the following Note by the Editor of the Journal of the Institute. It says, "On the publication of accounts, and the "legislation needed for assurance offices, we would make but "this remark, that never more than now has interference on the "part of the State been needed in order to enforce such a "measure of publicity as would enable persons of ordinary " intelligence to judge of the solvency of the companies to which "they have committed their interests. The state of things "existing with regard to Life Assurance Companies has no " parallel. The public have, by the nature of things, to repose "in them great and, in theory at least, perpetual confidence; "and they in turn too often treat the public with the most " studied and persistent secrecy. None but companies that fear "the light would do other than welcome an impartial parlia-" mentary action which, whilst leaving them unfettered in the " conduct of their business, would compel them to state its " results with such clearness and precision as would at least rob " dishonesty of its congenial darkness."

The real author of the Bill which became "The Life Assurance Companies Act. Assurance Companies Act 1870", is said to have been the late Mr. Pattison, a member of the Institute, who was Mr. Cave's adviser throughout. The fifth and sixth Schedules were entirely drawn by Mr. Pattison, and were not, it is believed, altered in their passage through Parliament.

At the Annual General Meeting of the Institute following the passing of "The Life Assurance Companies Act 1870", it was remarked that "short as has been the period during which the "Act has been in operation, its good results are already "discernible in the internal reforms it is effecting, and in the "improved accounts now published by the offices."

(To be continued.)

X

## JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

The Relation of the Actuarial Profession to the State. By John Nicoll, F.I.A. Being the Essay to which a Prize, presented by Mr. J. Chisholm, was awarded.

[Read in abstract before the Institute, 28 February 1898.]

(Continued from page 208.)

AGAIN, we have the following reference to the same Act in Vol. xvi of the Journal, "That there have been great evils in "the state of Life Assurance business is admitted on all hands. " and the Act seems well adapted to cure these evils. It will "no longer be possible for a company that is hopelessly "insolvent to claim the public confidence and obtain large " amounts of new business, thus deferring the evil day, but only " with the result of largely increasing the magnitude of the loss " caused by its failure, and the number of persons injured thereby: "and it will be no longer possible for the managers of an un-" successful company to dispose of its business to an insolvent "company, receiving themselves a handsome bonus for the "transaction. Nor is it matter for regret that the mystery with " which many flourishing companies have enveloped their affairs "will now be dissipated. In future, when an Actuary is asked " his opinion of the solveney of an office, he will have materials "at command from which to form his own estimate of the " solvency and stability of the office, instead of having to say that " he knows nothing of it for certain."

Thus one of the greatest and most beneficial of Life Assurance enactments was consummated by Government without outward fuss, and with the cordial approval and support of the members of the assurance profession. We need not enter into its provisions here.

Mr. Sprague's paper on "Legislation as to Life Insurance and Life Insurance Companies", was read in November 1870, immediately after the passing of the 1870 Act. Mr. Sprague remarks in it that some persons consider that the Act stops short of what it should be, compared with similar legislation in the United States of America. He goes on to say that the object aimed at by American legislation may be briefly described as an attempt to secure the solvency of all Life Insurance Companies doing business in the States, and he proceeds to give his reasons for believing—

Firstly—That no legislative enactments whatever can succeed in absolutely securing the solvency of Life Insurance Companies;

Secondly—That, even if this could be done, it would be highly undesirable that it should be attempted.

He states that it is easy to make Companies publish the fullest particulars regarding their policies, but, when all this has been done, the object aimed at is far from being attained. The insolvency of a Life Office may arise from one or more of several causes: (1) Excessive Expenditure; (2) Declaring Excessive Bonuses; (3) Loss on Investments; and (4) Excessive Mortality.

The American System may prevent insolvency from the first two causes, but, if money has been lost from either of the other two, and the Company come to be wound up, the contracts will fall to be reduced. To prevent loss from No. (3) the Government will require, as they do in the State of New York, to superintend the investments of the Companies, or to lay down rules for investment of the Funds in certain securities. Even then loss may arise. No amount of supervision may prevent loss from No. (4).

If then Government desires to secure to persons insuring their lives certainty that their policies will be paid on becoming claims they must either (1) open a Government Insurance Office in Competition to existing companies, or (2) buy up the existing companies, as they did the telegraphs, and make a Government Monopoly of it. There seems no objection to the first, and it is

not likely the offices would require to fear the Government competition, as they could insure cheaper and give better returns than Government. There are many objections to the second proposition on the grounds of political economy, and Mr. Sprague adds some others. (1) A life declined by one company can very often get assured by another, but, if Government alone assured, there would be no chance for such cases. (2) The rates would doubtless be increased all round, and thus assurers would have to pay for Government security. (3) The Government would not likely grant loans on reversions, and, as the competition would be lessened, sellers would get much lower prices for them than at present.

In the event of a Government of a country attempting, with any real prospect of ultimate success, to secure the solvency of the Life Offices in it, they would require to make strict and minute regulations. These regulations would extend to the magnitude of the premiums to be charged, the manner of conducting the business, the securities in which the funds shall be invested, and the amount of the reserve to be made for the liabilities. In fact, we may say the offices would be bound hand and foot. This would prevent any possible improvements, and would, in fact, amount to prohibition of any radical alteration in the way of conducting the business.

Mr. Sprague also refers to the difficulties in the way of saying when a Life Insurance Company is insolvent.

Looking at the matter from whatever point we may, there is no doubt that the whole duty of the Government, as regards the supervision of Life Assurance Companies, should be strictly limited to simply insisting on the offices publishing the fullest and most minute details of their working, but leaving them at the same time to conduct their business altogether in the way they consider best. When this is done, public opinion will surely become—as it has even already to a great extent become—the best and the surest controlling power.

About this same time, Mr. Sprague read another valuable paper, suggested apparently by the 1870 Act, "On the Liquidation and Reconstruction of an Insolvent Life Insurance Company." In it he points out that the 1870 Act has laid down no standard by which the solvency or insolvency of a Company is to be decided, and if the question were brought before the Court, no doubt some Actuaries would say a Company was solvent, while others would say it was not. Mr. Sprague thought at this time that a

standard of solvency might be fixed in this country, and a public officer appointed, whose duty it should be to make valuations of all the Life Offices by the legal data, and who should publish an Annual Report showing how much each office exceeds or falls below the legal standard. He should, however, he thought, have no penal powers, his function being restricted to informing the public. In finding the legal standard of solvency, Mr. Sprague thought a modification of the hypothetical method of valuation would be most suitable.

In his Presidential Address to the Institute on 24 November 1884, Mr. Sprague, however, stated that he did not think it necessary that any legal standard of solvency should be enacted; but any question as to the solvency or insolvency of a Life Office might be safely left as at present to the decision of the Court, after hearing the evidence of Actuaries for and against the Company.

On 29 October 1870 another Royal Commission on Friendly Societies. Friendly Societies was appointed, the outcome of which was the Act of 1875, which was perhaps of as much importance to Friendly Societies as the Act of 1870 was to the Ordinary Life Assurance Companies. The Commissioners were "to enquire into the existing state of the Law relating to Friendly "Societies, and the Organization or general condition of "Societies established under such Acts respectively; and upon the "office and duties of the Registrar of Friendly Societies; with "power to suggest any improvements to be made in the law with "respect to matters aforesaid."

Of the Eight Commissioners, one, Mr. Wm. Pollard Pattison, was a Fellow of the Institute of Actuaries. On the recommendation of the Commission, four paid assistant Commissioners were appointed to collect information, and to digest and submit it to the Commission. The Commission sat at intervals during five years, and it issued a mass of literature consisting of eleven volumes in all.

Immediately before this Commission was appointed, we learn from the Journal that Mr. Lowe, who was then Chancellor of the Exchequer, asked Mr. Pattison to call upon him. Mr. Pattison found that the right hon. gentleman was at once for abolishing the office of Registrar, and forming the Societies on one principle, the Government being no more responsible than for Joint Stock Companies; but, when it was pointed out to him that a number of societies were unsound, through ignorantly charging inadequate

premiums, he at once replied, "Then this is a subject for a Royal Commission."

The responsibility of one chapter of the Commissioners' Reports "On Rates of Premium and Statistics of Sickness, and "on the rate of Mortality prevailing among the classes comprised "in Friendly Societies", was, it is said, attached entirely to Mr. Pattison by the Chancellor of the Exchequer.

Among the witnesses examined by the Commission were Messrs. Ratcliffe, Bailey, A. G. Finlaison, Neison, and Sprague.

Mr. Ratcliffe did not think any improvement was required in the form of the annual and quinquennial returns required by Government. In proof of this, he mentioned that the Sickness Tables deduced from the Experience of his own Society were compiled from duplicates of the Government returns. He referred to the fact that Mr. Finlaison's Tables do not distinguish between sickness at full, half, or quarter pay, and he was of opinion therefore, that, for existing Societies, Mr. Finlaison's prices are overstated.

Mr. Bailey thought it would be quite competent to use one Table of Sickness for various Trades, and that a plumber, for instance, might be rated up two or three years to meet extra risk. Mr. Bailey was also examined as to the forms of books that friendly societies might be required to keep, and as to the advisability of Government publishing a Table that might be recommended as giving prudent rates of contribution. He thought, however, that Government could not recommend such Tables without incurring some moral responsibility if societies should fail through the inadequacy of the contributions charged. The Tables of Sickness in existence were, he considered, very imperfect, and that Mr. Finlaisons' Tables were the best of their kind, but, he added, "bad are the best."

Mr. A. G. Finlaison was asked as to his experience of friendly societies generally. He stated he had had to do only with the larger societies, and that, as a rule, they were in a good condition. In making up his tables, he adopted the returns supplied to him by the Registrar as being correct; and it was quite the case that his tables only dealt with sickness generally, and did not distinguish between sickness at full, half, or quarter pay. The secessions in friendly societies were, he mentioned, three times as great as the mortality.

Mr. Neison thought it very desirable that friendly societies should have separate provision for expenses of management. He

was asked if there was no means of forming a rough estimate of the financial position of a society by a person not an Actuary; but he thought it could only be done by an Actuary with full particulars before him for a valuation. He referred to a case in which a valuation of a friendly society had been made by three separate actuaries, and in which very different results had been brought out by all of them, owing to their having used different data. He thought that, if correct data were provided, friendly societies would right themselves. He thought it was unnecessary to distinguish between what was really sickness and what was not. Whatever a society has to pay for should, he considered, be called sickness.

Mr. Sprague was examined entirely in regard to the valuation he had made, as at December 1867, of the Royal Liver Society. He stated that he satisfied himself that the London business of the Society might be considered as a fair specimen of the whole business, and he calculated the amount the Society should have in hand to meet the liabilities in the London branch. Then, taking the proportion that the business of the London branch bore to that of the whole Society, he arrived at the amount the Society should have in hand to meet the whole of its liabilities. He had adopted this course because it would have been a very tedious operation, involving great expense, to have valued the whole of the Society's business.

The Commissioners awarded thanks (inter alia) to Mr. Neison and Mr. Ratcliffe in particular as among the witnesses who "have supplied us with a great amount of interesting and "valuable matter throwing light upon the subject of our "enquiries."

Among the recommendations of the Commission were-

That the registration of friendly societies should continue.

That a competent actuarial staff be attached to the Registrar's office.

That tables of premiums for sickness and death claims be prepared and published by Government from time to time, but that such tables should not be compulsory.

That the power of the Registrar to prosecute be made more effective.

That the system of Post Office Insurance should be extended. That a new Friendly Societies Act be passed consolidating and amending the law in conformity with the recommendations of the Commission.

The general question, or policy or impolicy, of "State Interference," in the matter of friendly societies is ably summed up in the following paragraphs of the Report of the Commission—

"It is obvious that it is possible to follow either of two "conflicting lines of argument. It may be urged on the one "hand that it is the true policy of the State to encourage the "formation of habits of providence among the people; that the " friendly societies afford to the industrious and the thrifty an "excellent means of making provision against death, sickness, " old age, and the various casualties of life; that, if founded on "proper principles, and honestly adminstered, they will enable "the great bulk of our labouring population to maintain their "independence of eleemosynary and statutory assistance, and " may, to a great extent, supersede the action of the Poor Laws, "and enormously diminish the pressure of the poor rates: that, "however, these institutions are liable to gross perversion, and "may, partly through ignorance, and partly through fraud, be " made engines of mischief rather than of good to those for whose "benefit they are nominally established; that the members, if "left to themselves, have no means of guarding themselves "against these abuses, which they often cannot discover till "irreparable mischief has been done; that, consequently, many "persons are discouraged from joining them, and that others "who do join them find themselves ruined by doing so; that "the State has the means, if not of absolutely preventing all "these evils, at least of averting some of them, and of placing "the members of the various societies in a position which will "enable them, by the exercise of an ordinary amount of care and "vigilance, to discover whether their affairs are being properly "conducted, and to protect themselves against mismanagement " and fraud; and that consequently it is the duty of the State to "take such measures as may be reasonably adapted for attaining "this important end. It may be added that, though the direct " action of the Registrar's office upon the friendly societies is not " very powerful, its indirect influence is considerable; that it is "valued by the best societies, and that it affords a machinery "capable of being turned to even better account than at present.

"On the other hand, it may be said that it is in theory "objectionable to assign to the State what are called paternal functions; that it is not the duty of the Government to look after the affairs of individuals; that, by attempting to do so, it

"discourages the exercise of individual prudence and watchful"ness; that, after all, it can only do the work imperfectly, and if
"it is unsupported by such individual prudence and watchfulness,
"it cannot do it at all, so that it will fail, with discredit to
"itself, and with disaster to those whom it has educated to rely
"upon it. Further, it may be said that, as a matter of fact, the
"certificate given by the Registrar in the case of friendly
"societies does not give any assurance against the principal dangers
"to which these societies are exposed; that most of these
"dangers are of a character which it is impossible for the State
"to guard, or at all events to guard effectually; and that the
"certificate is therefore delusive and ought to be done away
"with."

The Commissioners then proceed to review the "limits of feasible Government interference", as follows—

"Now, the evils to which friendly societies are most subject "may be divided into (1) those which arise from the want of " proper information, and which generally affect the principles "on which the society is founded; and (2) those which arise "from improper management, which may be the result either of "ignorance, or of negligenee, or of fraud. As regards the first " class of evils, we believe that it is in the power of the Govern-"ment to do a great deal to correct it, without inconvenience to "the public, and without any undue interference with the liberty " of the promoters of societies. As regards the second class, " there is more difficulty, though we believe that something may "be done in the way of securing publicity, and of providing " readier means for the detection and punishment of fraud. "Possibly too, it may be found feasible and desirable for the " Government itself to enter directly into competition with certain "classes of societies, and to afford the public an alternative "provision for certain classes of objects. But this is a matter " to which we shall refer hereafter."

The Commissioners devoted a chapter to the consideration of the question referred to in the last paragraph, namely, as to the propriety of the State undertaking the business of friendly societies. The conclusion they arrived at was that the State ought not on any account to undertake sick pay; but they thought it would be desirable if the State could undertake the other parts of the business of friendly societies, namely death pay, deferred annuities, and endowments. Further enquiries, however, convinced the Commissioners that the difficulties in the

way of the State conducting such a class of business were so great that they thought it useless to pursue consideration of the subject further.

Actuarial Commission was appointed. Its members were Messrs. Pattison, R. P. Hardy, and A. J. Finlaison. They were asked to advise as to the forms to be issued by the Chief Registrar for the purposes of the various returns, and as to the best available data for valuations pending the construction of Tables of Sickness and Mortality from the Government returns. They also prepared a full set of Forms and Books to meet the requirements of friendly societies. The Tables they submitted for temporary use were based on Mr. Ratcliffe's Tables of Sickness and Mortality deduced from the experience for the years 1866–1870 of the Manchester Unity of Oddfellows, rural, town and city districts combined, and the following benefits were provided for in them.

- (1) Sickness allowance until 65 of £1 per week.
- (2) Sickness allowance until 70 of £1 per week.
- (3) Deferred Annuity of £1 per week, commencing at age 65.
- (4) Deferred Annuity of £1 per week, commencing at age 70.
- (5) Assurance of £10 at Death, the contributions being payable during life.
- (6) Assurance of £10 at Death, the contributions ceasing at age 65.
- (7) Assurance of £10 at Death, the contributions ceasing at age 70.

The Commissioners recommended that a provision should be inserted in the rules of any society adopting these tables for sickness allowance, to the effect that the allowance be reduced to half pay after 12 months' full pay has been allowed, and further reduced to quarter pay after 12 months' half pay has been allowed.

In the event of application being made for a Table of Contributions to provide an allowance in sickness throughout the whole of life, the Commissioners recommended that the Registrar should advise the adoption of a table providing for sickness allowance ceasing at a certain age (65 or 70) combined with a table providing for a deferred annuity of, say, half the amount of the sickness allowance.

Finally, the Commissioners state that "we have recommended

"the Manchester Unity Tables of Sickness and Mortality as a basis for rates of contribution, but we are of opinion that it would not be advisable to recommend any tables for the purpose of valuation. No doubt the Manchester Unity Tables would be suitable for a very large number of societies, but the actuary employed should be guided by the circumstances in each case in selecting the proper tables to be used."

As a result of the labours and Report of the Royal Commission just dealt with, a Bill was introduced into Parliament in 1874 to amend the law relating to friendly societies; but it was afterwards withdrawn in favour of a more comprehensive measure, which was introduced by Government in the next Session, namely, "The Friendly Societies Act 1875." That Act repealed all the Friendly Societies Acts previously in force, and continues to the present time to be the main Act for the regulation of such societies. When it was being drawn up the then Chancellor of the Exchequer consulted the Institute with reference to certain provisions to be inserted in it, and also in regard to the preparation of tables for friendly societies. Since the 1875 Act was adopted, four other Acts have been passed, but all of them were repealed by the Act of 1887. "The object of this " latter Act was in part to correct some grammatical errors of the " Act of 1875 (called the principal Act), and in other parts to "amend the provisions of that Act. This has been done "throughout by a series of 'additions, omissions and substitu-"tions', which 'The Friendly Societies Act 1887' required "should be embodied in any copy of the principal Act printed " after the passing thereof."

working of the worked, in the main, very smoothly for all parties concerned. In connection with the practical working of such a measure, however, there was bound to be some little doubts at the start as to the true meaning of some of its requirements. Even at the present day, there are differences of opinion as to the proper information to be given in reply to some of the questions in its fifth and sixth Schedules. The Act had not, however, been long in existence when at least two such cases of difficulty were brought to light. The one was when the Board of Trade applied to the Institute for an opinion as to what items should be included under the head of "Expenses of Management." Their question originally had reference to a particular Company, and the Institute replied that they were strictly a scientific body,

and made it a rule not to discuss the affairs of any individual Life Assurance Society. The Board of Trade then put some more general queries, irrespective of any particular office, and asked for information on the subject. To this communication the Institute replied, giving the opinion asked, and for which they received a letter of thanks from the Board of Trade.

The other matter, to which reference has to be made, was of much greater importance than the one just mentioned. In 1874, Messrs. Malcolm and Hamilton, Assistant Secretaries to the Board of Trade, issued a Report, which is addressed to the President of the Board of Trade. It begins as follows—"In "compliance with your instructions, we have the honour to submit "to you the following observations upon the Accounts and "Statements submitted to the Board of Trade in pursuance of "the Life Assurance Companies Act 1870."

The Report then proceeds, among other things, to give tables of certain particulars in regard to the various assurance companies drawn from their returns. It likewise enters into explanations of what a valuation is, and gives certain opinions on various methods of valuation, on the rate of interest to be assumed, and the portion of the premium to be reserved for future expenses and profits.

It is further stated in the Report that the valuation returns sent in by certain offices had been rejected as being manifestly incorrect and misleading, and that amended returns had been requested in such cases; also that the returns of other offices "have been accepted with considerable hesitation."

It is evident from this Report that the Board of Trade were under the impression that the 1870 Act required them to exercise pretty considerable supervision of the accounts and statements deposited with them by the assurance companies. A different opinion seems, however, to have been held by the Scotch Actuaries at least, on the point, and it appears that they, through the Managers' Association, sent a deputation to London to confer with the President of the Board of Trade on the subject. The views of the deputation seemed to be new to the Board of Trade; but, at the same time, it would appear that they were very welcome to them, as fitted to relieve them from a position which they were finding to be one of difficulty and anxiety. This conference has evidently been the means of making the Board of Trade reconsider their position all round toward assurance offices. The only part of the Act which lends any colour to the supposition

that the Board of Trade are required to supervise, as they apparently proposed to do, the returns sent into them by the offices, is Section 18, which imposes certain duties upon the Board in the ease of companies "which make default in complying with the requirements of the Act." By Section 9 they have, in addition, power to alter, with the consent of any particular company, the forms contained in the Schedules to the Act, in order to adapt them to the circumstances of such company. The Act otherwise limits the province of the Board of Trade simply to accepting deposit of the statements required from the offices. How far, however, Section 18 constitutes the Board of Trade judges as to whether the proper particulars are contained in the statements deposited with them, or whether it is simply intended that they should merely report cases in which no returns at all are made, is perhaps a matter of opinion. Certainly, where there is default, as also in cases of amalgamation and winding-up, the Board of Trade have no power to deal with the companies, but all power in these cases is vested in the "Court."

Appointment of Actuary of Friendly Societies, reference is made several times to the appointment of Mr. Sutton as Actuary to the appointment of Mr. Sutton as Actuary to the appointment of Mr. Central Office in pursuance of the 1875 Act, "as "marking a very important step in the history of the Friendly "Society System of this country." Several specific duties, it is stated, are assigned to the Actuary under the provisions of that measure, especially the accepting from the societies the materials for valuation and causing their assets and liabilities to be valued and reported on. But the Chief Registrar says "Over and above " these stated functions, however, the Actuary's assistance has been " found extremely valuable in framing forms of returns; adapting "them to particular groups of societies; giving advice to "societies as to forms of their accounts; scrutinizing the "returns of particular societies, &c. He serves as the natural "medium of communication between the Registry Office and the "Public Valuers, and all letters bearing upon actuarial subjects " are naturally referred to him. So valuable, indeed, is his help, "and so well does it fit in with the work of the office, that it "would be a matter of wonder how it should ever have done "without him had it not been for the fortuitous circumstance "that the Assistant Registrar for England had been trained " originally for the actuarial profession, and had thus been able "to supply aid in matters lying altogether beyond his statutory

"qualifications as a barrister." In the past, Government has benefited greatly from the services of Actuaries in different branches of the service, and we shall endeavour to show later on how they may in future derive even greater benefits by availing themselves more largely of the services of members of the profession in other directions.

"Instructions to Public Valuers appointed by the Triendly Societies' were issued early in the year 1877. According to the first regulation, the valuers were not to exceed fourteen in number, and it may be interesting to give the names of those who were first appointed. They were, for England, Messrs. Adler, Ansell, junr., R. P. Hardy, Hatton, Newman, Pattison, Ratcliffe, Ambrose Smith and Strachan. For Seotland, Messrs. Meikle and Prentice; and for Ireland, Messrs. Englebach and Hancock.

It may not be generally known that, as stated in 1880. Married the Journal, "The Married Women's Policies of Women's Policies of Assurance (Scotland) Act 1880" was prepared and Assurance (Scotland) Act. promoted by the Association of Managers of the Scottish Life Offices; and that the passing of the measure, in a peculiarly busy Session of Parliament, was mainly due to the tact and perseverance of Mr. Holms, M.P. for Paisley. connection with this and other kindred legislation, there is no doubt that an assurance M.P. would be extremely useful and beneficial to the profession and to insurance business generally, and it is worth the serious consideration of the offices whether they should not promote the candidature of one of their managers for a seat in the House of Commons.

There being still no improvement in the volume of business being done through the Post Office in Life Government Assurances and Annuities, a Select Committee of the House of Commons was appointed in March 1882 to enquire into the matter. After hearing evidence, the Committee considered the causes of the failure of the scheme to be—

- 1. The absence of solicitation, and parties having to go to Post Offices to pay their premiums.
- 2. The limited number of Insurance and Annuity Post Offices and the early hours at which they close.
- 3. The limitation of the amounts both as regards maximum and minimum policies.
- 4. The complicated character of the formalities to be gone through in effecting assurances and annuities.

## And the Committee proposed as remedies—

- 1. That the insurance and annuity business of the Post Office should be connected with the Post Office Savings Banks, so that deposits in any Post Office Savings Bank might, on the request of the depositor, be applied to the payment of premiums for life assurances and annuities, whether such deposits have been made specially for the purpose or otherwise.
- 2. That the number of Insurance and Annuity Post Offices should be greatly extended.
- 3. That the existing minimum, both for assurances and annuities, should be abolished.
- 4. That medical examination might be dispensed with, under certain conditions, for small assurances.
- 5. That the assurable ages should be extended downwards to age 8, and upwards to age 65.
- 6. That annuities should be granted as low as age 5.
- 7. That the maximum amount of assurances and annuities should be raised to £200.

Among the witnesses examined by the Committee were Messrs, A. J. Finlaison and Marcus N. Adler.

Mr. Finlaison saw no objection to reducing the minimum amount of assurance, and to doing away with a medical examination under certain precautions. These precautions were (1) If the death of the assured occur in the first year, only a proportion of the premium paid to be returned. (2) After the lapse of one year and under two years, a proportion of the sum assured might be paid, and it should take two years before the person should be fully insured under his contract. (3) The premiums payable for such policies to be limited to age 60. A valuation of the Post Office scheme was made as at 31 December 1880, showing liabilities on assurance account £52,084, and the assets £83,329; and on deferred annuity account, liabilities £96,000, and assets £93,485. Mr. Finlaison showed that the seeming loss on the annuity account was apparent rather than real, as a very large proportion of deferred annuities never mature, the assurers getting back their money before the annuity matures. In all cases the liability is reserved in case every annuity matures. Further, many deferred annuities are paid for by single premium, and it is not found out that parties are dead till the annuity comes to be payable. In these cases also full reserves are made. Mr. Finlaison was

asked whether it could not be arranged to give bonuses to assured under Government policies by increasing the premiums slightly, but he answered that it would be difficult to deal with the question of profit. The Committee seemed to have difficulty in comprehending why, if women were charged higher prices for annuities, they should not be charged less premiums for assurances. Mr. Finlaison's answer was that assurers and annuitants are two entirely different classes of people.

Mr. Adler suggested that friendly societies should be limited to the granting of sick allowances, and that, for the rest, assurances and deferred annuities, they should act simply as agents for Government. As illustrating certain of his views, he read extracts from papers read by him and printed in the Journal of the Institute of Actuaries, xii, p. 14, and another in 1865. He insisted, however, in order to have control over the members by friendly societies, that all who effected annuities or assurances must also be members of the sickness fund. He would reduce the amount of assurances to £1 or 10s. if the premiums could be collected for these. He thought that reducing the assuring age below 16 would render Post Office assurances more popular, as whole families often assured at once; and he did not think that the extension of the limit of assurances to €200 would affect the ordinary assurance offices. He also referred to a drawback, as he considered it, in Post Office policies, that if a man afterwards engaged in a dangerous occupation he became liable thereafter for a higher premium, whereas assurance companies did not increase the premium in such cases. He found, however, that practically the rule was not acted on, and he thought it would be as well to mitigate the rigor of the rule.

When the Government "Annuities and Assurance Bill 1882" was brought before Parliament, the Scotch actuaries, it appears, brought all the influence they could to bear upon members of the House of Commons to support an amendment by Mr. Fowler to the effect that it was undesirable to extend the maximum limit of Government annuities and assurances. No doubt as a result to some extent of this action, the Act as passed fixed the limit of Savings Bank Insurances at £100, and of Savings Bank Annuities at £100 per annum.

The fifth section of the Government "Annuities Act 1882" directed the preparation of tables for the purpose of carrying the Act into effect. Mr. A. J. Finlaison accordingly brought down to 1875 the experience

derived from the mortality of Government annuitants, and constructed the necessary tables therefrom. In his report, dated 10 February 1883, he gives a full and clear account of the method he employed, and it is unnecessary to enter here into details of the construction of the tables.

In the Treasury Minute of 7 September 1883 it is stated that the practical importance of the conclusions in Mr. Finlaison's report is very considerable, as upon their accuracy must depend for many years the financial working of large transactions between the State and individuals, and of a system which is about to be considerably expanded. The importance of securing justice to both parties in the case seemed so great that the Lords of the Treasury thought it their duty to obtain, from an independent quarter, some confirmation or criticism of Mr. Finlaison's conclusions. For this purpose they applied to Mr. T. B. Sprague, at that time President of the Institute of Actuaries, and he furnished them with a most careful and valuable report.

Mr. Sprague commences his report by saying that he has satisfied himself that, "assuming the accuracy of the figures "given, Mr. Finlaison's reasoning and conclusions may be "accepted by the Government with a reasonable assurance of "safety."

Mr. Sprague added a few criticisms on Mr. Finlaison's tables. He thought the effect of selection was not exhausted at the end of four years as assumed in the report, and he would have preferred the graphic method of adjustment in place of that employed by Mr. Finlaison. He accounted for the difference in the results of the old and new tables by the important effect in the former of a large number of recently selected lives and certain specially selected male lives. In conclusion, however, Mr. Sprague repeats his conviction that Mr. Finlaison's results may be safely accepted.

The Lords of the Treasury considered that the criticisms which accompanied Mr. Sprague's general approval of Mr. Finlaison's conclusions were such as to add to the weight of his favourable judgment, and they therefore accepted Mr. Finlaison's report as a basis for the calculation of the new annuity tables.

Mr. A. J. Finlaison's new tables were adopted by the Government in April 1884, superseding the tables of Mr. John Finlaison, which had been in use from 1830 up to that date.

In the "Revenue and Friendly Societies Bill", 1883 brought before Parliament in 1883, it was proposed to introduce a clause whereby life assurance companies would be precluded from paying the sums assured by their policies on a title established by probate or letters of administration obtained elsewhere than in the United Kingdom, and this notwithstanding that they might have power to do so by private Acts. Efforts, which for a time were successful, were, it would seem, made by the managers of the Scotch offices to keep this clause out, but, in the end, it was incorporated as part of "The Revenue Act 1884." Subsequently, however, as the result of further negotiation, the clause was repealed, and in place of it a clause was inserted in "The Customs and Inland Revenue Act 1889", enacting that, where an assured shall die domiciled elsewhere than in the United Kingdom, the production of a grant of representation from a Court in the United Kingdom shall not be necessary to establish the right to receive the money payable in respect of such policy.

After an interval of nearly one hundred years, national provident insurance again came to engage the serious

1885. National Provident

society.

attention of Parliament. In 1885, a Committee of the House of Commons was appointed, and continued its enquiries during that and the two following years. The Committee were to consider the various schemes for national insurance which have been proposed. In the end, however, they confined their attention and enquiries almost exclusively to the Rev. Canon Blackley's scheme, the substance of which shortly is that every individual in the nation shall be liable by law, after reaching the age of 18 years, and before he reach age 21, to contribute, either in one sum or by instalments, £10 or thereabouts to a national sick and pension benefit society, which would secure to him or her, when prevented by sickness from earning his or her usual wages, a sum of 8s. a week for however long sickness may continue at a time, until 70 years of age; and after age 70, a cessation of the sick pay, but a pension for life of 4s. a week. Those who do not earn wages have no claim on the fund so long as they do not earn wages, though they are called upon to contribute to it. The contributions are proposed to be collected by, and the sick pay distributed through, the Post Office. Canon Blackley's scheme is based on the assumption that four per-cent

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compound interest will be realized on the funds of the national

In 1885, the Committee examined, among others, Messrs. Ludlow, Reuben Watson, and Sutton.

Mr. Ludlow, Chief Registrar of Friendly Societies, was of opinion that the scheme of national insurance is absolutely

impracticable in England as now constituted.

Mr. Watson stated that, in order to provide for the benefits of Canon Blackley's scheme, at his rate of contribution, 40 per-cent of the contributors must be of the class who would not need to claim the benefits of the scheme. He also thought that the adoption of such a scheme would necessarily lead to all friendly societies closing their doors.

Mr. Sutton gave it as his opinion that for a compulsory scheme of insurance like Canon Blackley's "all the actuaries in " existence and all the data in existence would not enable sound "actuarial calculations to be properly made." Canon Blackley states that the larger the society the less loss there is from malingering, and therefore a national society would be largest and would have least malingering of all. Mr. Sutton, on the other hand, proved that "the wider the area extends over which a society "grants sick pay, the greater in the long run becomes generally "the sick pay rate at the various ages." Mr. Sutton says Canon Blackley evidently made the mistake, which has been made by others, of supposing that the Oddfellows and Foresters are large societies, instead of a union of a great many small societies, the fact being that, as regards sick pay, every court of these societies is in the same position as any small village society. Mr. Sutton stated that, according to the combined experience of the Manchester Unity, or according to the experience of the Ancient Order of Foresters, at 3 per-cent interest, a contribution of £10 at age 21 would only provide 8s. per week sick pay to age 70, and 4s. per week pension thereafter to 52 out of every 100 contributors (or, in other words, a contribution of about £19 would be required instead of £10 to give the full benefits according to these experiences and 3 per-cent interest). Mr. Sutton thinks that what Canon Blackley calls the "plunder of poor men's providence" should more properly be stated as the "blunder of poor men's providence", seeing that the members of friendly societies have one with another got what they paid for, though that requires that some of them should get nothing at all on account of others having received much more than their rightful share. Mr. Sutton is convinced that the best remedy for whatever evils may surround existing

societies is the diffusion of elementary actuarial knowledge among the rising generation. He thinks an elementary text-book upon the principles upon which friendly societies are carried on should be introduced into the public schools. Mr. Sutton goes on to say, "I believe there is now a machinery "provided by which in the future the diffusion of this actuarial "knowledge can be made still greater and more effective. The " Institute of Actuaries has lately become incorporated by Royal "Charter, and although hitherto it has been composed of the " leading officials of the Life Insurance Companies, it now under-"stands that, in becoming incorporated, it has to some extent taken " upon itself public duties; and it is anticipated by the Council " of that Institute that, in the future, one of their public duties " will be to examine and test the efficiency of all persons coming " as students to that Institute, and desiring to show their attain-"ments in actuarial matters. It is believed that that will yield "in the future very great help to the friendly society system." Mr. Sutton read a letter, addressed to the Chief Registrar of Friendly Societies, from the Secretary of a certain friendly society, stating that the Committee of the said society were of opinion that it would be a sheer waste of money to employ a certified Actuary to value their assets and liabilities, as all their funds were in bank and in their treasurer's hands. Mr. Sutton despairingly remarks, though we have been sending out for the past 10 years, pamphlets explanatory of valuations, "this man still thinks the valuation required is the valuation of an auctioneer." As regards the results of valuations, Mr. Sutton remarks: "True to human " nature, in cases where a surplus is disclosed on a valuation, the " most implicit faith in the valuer and his valuation generally "becomes apparent", and great care has to be exercised by the managing members against such surplus being appropriated. On the other hand, when the valuation discloses a deficiency, there is a strong tendency to question, or, at any rate, to ignore the valuer's results. Mr. Sutton had prepared for the Committee a few notes on Canon Blacklev's scheme, and these formed the foundation for his examination. Most of the points noticed in his "Notes" are referred to again in his evidence given above. In addition, Mr. Sutton deals very fully in his "Notes" with the difficulty of defining sickness (when a man is really ill); the difficulty of obtaining data to fix rates of contribution; the difficulty of accumulating large funds, especially at 4 per-cent; and the unfairness of requiring future entrants, not only to pay

a higher rate of contribution for themselves than those who entered at first, if experience showed that to be necessary, but also to make up for the deficient contributions of those who entered previously.

The witnesses connected with Life Assurance who were 1886. National examined before the 1886 Committee, were Messrs. George H. Pinekard and Ralph P. Hardy. Mr. Hardy's evidence is very specially worthy of perusal.

Mr. Pinckard thought the present friendly societies would earry out the sick portion of the work best, and that, if you could get any system by which the poor man could come in for a certain income at a certain age, you would by that means reduce the contribution to the Poor Law relief by one half. He thought superannuation more important than sick pay.

Mr. Hardy did not think the contribution of £10 required by Canon Blackley's scheme would be sufficient according to any basis with which he was acquainted, unless it could be shown satisfactorily that the number of abstentions from claims, or those who became ineligible under the scheme, would be such as to make the balance a sufficient payment. Like Mr. Sutton, he thought that the further you get away from the controlling influence, the more difficult it is to keep down sickness. Instead of at once applying Canon Blackley's scheme to the whole country, he thought it should be commenced in a district first to see how it would work. He concurred in the whole of Mr. Sutton's memorandum. He thought sickness was a thing the State should never touch; and, if everybody has to pay towards a deferred annuity, he did not see why the benefit should not be charged upon the taxes at once, and save the cumbrous machinery of Canon Blackley's scheme. He thought friendly society meetings afforded admirable training in the elementary duties of citizenship. Asked whether such a scheme as Canon Blackley's would increase providence or improvidence, Mr. Hardy answered-"A moral "effect is brought about upon a man by his recognition of " circumstances, and not by any State action outside. If you " want to change a man's heart you must bring about actions " which are founded upon convictions. Pauperism is a habit of " mind, and you will never get rid of it until these people get to " see what a distressing and degrading thing it is. You will not "get rid of pauperism by any compulsory system of providence; "the thriftless man will remain thriftless and a ne'er-do-well "still." In reply to another similar question, Mr. Hardy

replies—"I hold that the financial and the moral questions are "distinct, and they must be kept so. I hold the opinion that the "mere money improvement in a man's condition is unimportant "compared with the moral improvement in the man; you will "get rid of a good deal of Poor Law, and a good deal of taxes if "you make the people better. I aim at that first, and leave the "money to take care of itself."

The Committee of 1887 examined, among other National Provident Insurance. Witnesses, Mr. Frederick Hendriks. He considered that a contribution of £20, in place of £10, would be required under Canon Blackley's scheme. He said there were four elements to be considered—

- (1) The rate of sickness at each age from 21 to 70;
- (2) The rate of mortality from 21 to the end of life;
- (3) The rate of interest; and
- (4) The rate of expense of management.

He thought it impossible to determine (1) (2) or (4), and he thought the highest rate of interest that could safely be counted on would be 3 per-cent. He shared the opinion of the late Professor De Morgan that Actuaries had more to fear in the future from a fall in the rate of interest than from a rise in the rate of mortality, as a disturbing element in their calculations. His opinion regarding Canon Blackley's scheme is—"On "financial and actuarial grounds, quite apart from my repugnance "to the scheme from the air of social compulsion which pervades "it, and from its infraction of individual liberty, I cannot but "view the scheme as, in spite of its good intentions, a mistake, "and incapable of practical accomplishment; nor do I consider "that it would either do much to alleviate poor's rates, or to "encourage thrift if it could really be carried out."

The Committee, in reviewing the evidence (given in 1885, 1886, and 1887), considered the following as the objections to the scheme—

- (1) The difficulty of preventing malingering.
- (2) The difficulty of many persons between 18 and 21 providing £10.
- (3) The proposal that only wage earners should participate creates a distinction that would be unworkable.
- (4) The upper and middle classes would strongly object to this form of compulsory charity.

- (5) The scheme would not teach habits of thrift, seeing employers would be required to deduct contributions from wages.
- (6) Absence of actuarial certification that the £10 at 21, and invested at 3 per-cent, would suffice.
- The Committee did not think that a higher rate of interest than 3 per-cent could be prudently assumed.
- The Committee further considered it highly desirable that the Legislature, which has made education compulsory, should cause instruction in sound principles of thrift, and insurance to form part of that education.

They also thought that the financial objections to the scheme did not press with such force against the deferred annuity part as against the sick pay part of the scheme; but still they were disposed to wait further development of public opinion before advising adoption of a general obligatory system of superannuated pay.

In his Presidential Address, delivered in November 1886, Mr. Day remarks that "the Rev. Canon Blackley, an enthusiastic " supporter of a somewhat Utopian idea, had had the rare good " fortune to obtain for his plans the consideration of a Committee " of the House of Commons in two successive Parliaments (with " a recommendation for the re-appointment of such Committee in "the newly-elected chamber); and it must be conceived, there-" fore, that much could be urged in support of the compulsory "system of which the reverend Canon is an advocate. It was no "doubt thought", he says, "by many of our actuarial friends that " the evidence given by that distinguished expert, the Actuary to "the Chief Registry of Friendly Societies, before the Committee " of 1885, had gone far towards extinguishing the chance of a "favourable report, had time been available for its preparation "before the dissolution." Mr. Day refers to the fact that, in their evidence before the Committee of 1886, both Mr. Pinckard and Mr. Hardy, as well, apparently, as Canon Blackley himself, attach greater importance to the provision for pension after 70 than to the sick pay prior to that age; and therein, he says, lies the essential difference between this plan and the compulsory national insurance of the German Empire, which provides (1) for sickness benefit, and (2) for compensation in case of accident, with no deferred annuity, or payment at death, which benefits, Dr. Ashchrott, who was examined as to the German system, considered were distinctly objectionable.

In the same Address, Mr. Day refers to the recent failure of a certain Life Assurance Association, and asks, "What is the "lesson that we may learn here? Is it that a system of "Government supervision, glibly recommended by irresponsible "writers, should be introduced? Is it that the Act of 1870 has "failed? I think not: and, for myself, would strongly deprecate "panic legislation or any additional interference by Government "Departments." He remarks, further, that the vast mass of our life assurance societies are beyond suspicion, and suggests that all that is required is a regular and continuous audit by independent professional accountants. To such an audit, he attributes the immunity of the Scottish offices from failures.

In December 1886, the Actuary to the Chief Registry of Friendly Societies, Mr. Sutton, addressed a letter to the Council of the Institute, stating that he had been authorized to receive applications from Fellows of the Institute to be approved as Actuaries to certify Annuity Tables for Friendly Societies under Section 11 (5) of "The Friendly Societies Act 1875." Such applications, he mentioned, must be made in writing, and be addressed to the Chief Registrar, and must show (1) That the applicant has been a Fellow of the Institute for at least five years after passing the necessary examinations; and (2) That the applicant has, from the time of becoming a Fellow, exercised the profession of an Actuary. A similar letter was at the same time addressed to the Council of the Faculty of Actuaries.

From the Journal of the Institute of Actuaries, Str John Lubbock's vol. xxvii, we learn that, in 1888, a Bill was brought into Parliament by Sir John Lubbock and others to consolidate and amend the laws relating to collecting friendly societies. The Bill did not make much progress, and ultimately a Select Committee was appointed to enquire into the subject.

Committee on That Committee was accordingly appointed, and Friendly Societies.

The Select Committee appointed to enquire into, and report upon, the operation of Section 30 of 'The Friendly Societies "Act 1875', as amended by subsequent Acts, and into the "organization or general condition of Societies and Companies "to which the said section applies, and to suggest what "amendment of the Law (if any) is required to ensure the better "management of such Societies and Companies, and the more

"complete protection of the interests of their members." Have agreed to the following Report—

"Your Committee are of opinion that at this late period of the Session, it will not be in their power to conclude their investigations. They have, therefore, agreed to report the evidence already taken to the House, and to recommend that a Committee on the same subject should be appointed in the next Session of Parliament."

The Committee only examined four witnesses in all, and only one of these, namely Mr. Sutton, was an Actuary.

Mr. Sutton, in his evidence, strongly advocated amending the law in the interests of the members of collecting friendly societies, and gave instances of the great difficulty now experienced in obtaining investigations of societies known to be in a failing condition. He was disappointed as to the power his office was considered to have in its endeavours to regulate the affairs of collecting societies. He would have resorted to prosecution in some instances, but it was supposed the Registrar's Office had no power to prosecute.

The year 1889 also had its Committee of the House of Commons on friendly societies. Before that Committee, Mr. Thomas C. Dewey gave a great deal of important and interesting evidence relating to industrial insurance, and more especially in regard to the experience and practice, in that branch of its business, of the great and notable institution of which he himself is one of the Managers.

Mr. Dewey's evidence went to show that the impression that children are frequently ill-treated or their deaths otherwise hastened for the sake of the insurance money is quite unfounded. He showed that, according to the 1881 Census Returns for England and Wales, there were probably 5,000,000 children of the industrial class, alive, under ten years of age. As 4,000,000 of these same children were assured by the various industrial companies and in friendly societies, the statement that in inquests on children, 50 per-cent of them were assured was not only not surprising, but in fact, if the number had even been given as 80 per-cent, it was only what was to have been expected.

In July 1890, the Select Committee on the Children's Life Insurance Bill, examined the Managers and Secretaries of various Friendly Societies and Burial Societies as to the number of children assured by them, and as to whether

such insurance, in their opinion or experience, led to the neglect of children; or even, in some cases, as had been suggested, to their murder for the sake of the insurance money.

Next year (in July 1891), two Scotch Actuaries,

Mr. Marr and Mr. Turnbull Smith, gave evidence
before the Select Committee on Town Holdings, with
the view of defending feu duties against taxation.

In the same year (1891) the Board of Trade wrote the Proposal to Amend 1500 Act. Various insurance organizations, namely: The Institute of Actuaries, the Actuaries' Club, and the Life Offices Association, in London, and the Faculty of Actuaries, and the Managers' Association, in Edinburgh, suggesting the desirability of amending some of the provisions of the Life Assurance Companies Acts 1870-72. In his Presidential Address to the Institute, delivered on the 26 November 1892, Mr. Augustus Hendriks gives a full and interesting account of the supposed intentions of the Board of Trade on that occasion, and we cannot do better than quote from his Address on the subject. He says, "it is already within your knowledge that the Board of "Trade, some months since, addressed a communication to this " Institute suggesting that the time had arrived for contemplating " amendments of the Life Assurance Companies Acts of Parliament, " and asking for our advice thereon.

"It seemed desirable on our part to enquire of the Board of "Trade upon what points relating to these Acts they were desirous " of receiving advice, so that our attention might be directed more " especially to giving our views in the direction in which amend-" ments might be deemed necessary or desirable. At this juncture, "however, the Board of Trade decided not to proceed further in "their research, and it can well be understood that, coming as it "did on the last day of a moribund Parliament, the question " should be temporarily shelved. It was, however, fully understood "that the question was to be revived on the advent of the now "re-elected House of Commons. It is believed that the Board of "Trade is not likely to desire any grave or heavy changes, either in "the schedules or in the statistics, or in the information supplied " under the existing Act, which Act, taken as a whole, has been "admirable in its working during the 22 years of its existence. "It has been practically taken as a model, with unimportant " variations, by nearly all of the Colonies, with the sole exception "to the contrary of Canada, which, guided no doubt by its " propinquity, pronounced in favour of the system of supervision

" adopted in the various States of North America. It would seem, " so far as one can gather, that the attention of the Board of "Trade is directed more particularly to the defects in the Act " which give such unrestricted scope to waste and spoliation when "a company is being wound-up, or its contracts diminished, by "the provisions of the Act; and it seems almost incredible, but "it is too true, that we should still be able to witness the disgrace " of the possibility of one captious shareholder, if indeed he "ever was a shareholder, being able to inflict, in abortive law "proceedings, an expenditure of upwards of £30,000 upon a "company already wrecked by malversation, but struggling to " give as good salvage as possible to its unfortunate policyholders. "Words cannot be too strong to condemn any system which can " possibly give a loop-hole to such cruel wrong, and one can well " imagine the desire of the Board of Trade to protect the public " against any recurrence of such a grave scandal as the one now " commented on. When the time arrives for a general enquiry " into the working of the Act, this Institute, and the other bodies " referred to by the Board of Trade, will give their advice and "their cordial support to the Government authorities in their "endeavour to perfect a system which has already conferred "great benefit both on the public and on insurance companies."

Though, perhaps, not strictly a Government matter, it may be noticed that, in 1891, Messrs. Sprague and Sutton gave evidence as to the sufficiency of rates, &c., before the Select Committee in connection with the School Board for London (Superannuation) Bill.

Colonial Probates Act. When the Bill, which resulted in "The Colonial Probates Act 1892", was introduced into Parliament, it is understood that the Association of Scotch Managers endeavoured to get its scope extended so as to include all the Colonies. This effort was, however, unsuccessful.

In the report presented to the annual general meeting of the Institute, held on 9 June 1894, several matters of importance and interest are referred to.

Government Jin the first place, it is mentioned that the ComJoint-Life Amounty Trables missioners for the Reduction of the National Debt, and
their Actuary, Mr. A. J. Finlaison, C.B., have kindly placed at the
disposal of the Institute, tables relating to Joint-Life Annuities,
calculated in 1884 upon the observations of the Mortality of
Government Annuitants. These valuable tables, as is well known
to all, have, since that date, been published in a very handy form.

In the next place, it is stated that negotiations were entered into with the Imperial Institute, and have resulted in the President of the Institute of Actuaries becoming ex-officio a member of the governing body of the Imperial Institute.

The other point we would notice is the reference to the modification secured by the Institute and the Life Offices Association of one of the clauses of the Finance Bill then before Parliament. The clause in question proposed to enact that a certificate must be obtained showing that estate duty had been paid before the person entitled to receive the sum assured by a policy could give a good discharge for the same. It was felt by the assurance profession that the proposed clause would cause considerable delay in payment of claims, and that it would lead more particularly to misapprehension in the mind of the public in the sense that they might suppose, however erroneously, that the insurance companies, in place of being most willing to meet their engagements, were making needless delay in the settlement of A letter explaining matters was, accordingly, addressed to the Chancellor of the Exchequer, and was followed by the right hon, gentleman giving an interview which resulted in the promise being given that the clause complained of would be withdrawn.

The last matter we have to refer to, under this part of our subject, is the appointment, last year, of a Committee on Old Age Pensions. This Committee is under the presidency of Lord Rothschild, and among its members are the President and one of the Vice-Presidents of the Institute, and the Actuary of a large friendly society. The terms of the remit to the Committee were as follows-"To consider any "schemes that may be submitted to them for encouraging the "industrial population, by State aid or otherwise, to make " provision for old age, and to report whether they can recom-"mend the adoption of any proposals of the kind, either based "upon, or independent of, such schemes, with special regard, in "the case of any proposals of which they may approve, to "ascertain their cost and probable financial results to the "exchequer and to local rates; their effect in promoting habits " of thrift and self-reliance; their influence on the prosperity " of friendly societies; and the possibility of securing the " co-operation of these institutions in their practical working."

A report of one of the initial meetings of this Committee states that "the Committee discussed the numerous schemes which "had been sent in. They were inclined to think that those "which involved compulsory contributions (after the German "system), and those which involved a system of universal free "pensions, without any kind of contribution, or evidence of thrift "on the part of the beneficiaries, were not within the scope of the "reference." The enquiry will apparently, therefore, only deal with voluntary schemes. From all that has gone before, one would imagine that it should now be becoming apparent to most people that compulsory old age pensions are no more likely than compulsory military service to be submitted to in this country, under present circumstances, at least. The present Commission will no doubt make this clear once for all.

The Earl of Glasgow, lately speaking at a friendly society meeting, stated that there had just been passed (by the House of Representatives) in New Zealand, an Act to give every one over 65 years of age an old age pension of £38 per annum. Legislative Council have since, however, rejected this Bill.) this country, he said, the question had occupied great attention, and now the Government, he was told, had come to the conclusion not to bring in a measure on the subject without having the greatest amount of information first thrown upon it. For that purpose they had ordered a Commission to go into the question, and so anxious were they to arrive at a right conclusion, that they had put the very best men they could upon that Commission, and, in order that justice should be done to friendly societies, had appointed men who were experts with regard to friendly societies, well acquainted with their circumstances, and likely to do justice to them. Such testimony to the character of this Commission on the part of an outsider, must be gratifying to every member of our profession.

It may just be mentioned that two private Bills, somewhat similar in character, were brought into Parliament, in the month of January 1897, for the purpose of providing small pensions after the age of 65 to deserving poor persons. These pensions, in both cases, are proposed to be paid out of a special rate. It is not likely, however, that more will be heard of either of these Bills.

Suggestions as to extending the Actuary in the past in assisting and advising the Actuary.

Legislature in matters connected chiefly with life assurance and the business of friendly societies. We shall now proceed to make a few suggestions as to the manner in which the sphere of the Actuary may be extended in these, and other

directions, in the future, with benefit both to the community and to the profession.

Now that both the Institute of Actuaries and the Faculty of Actuaries are firmly established and empowered by Royal Charter, it is to be expected that there should be a much wider recognition of the profession in the future than in the past, and also that new fields of usefulness will hereafter open up to the Actuary, if he will only avail himself of the opportunity of entering into them.

The ambition of the Actuary ought surely to be to evolve order out of confusion; to trace law in seeming disorder; and to endeavour, to a great extent, by the aid of what has been, to foretell what shall be.

In the past, Actuaries have, of course, rendered such services, outside of assurance matters, to the Government and to the community.

Shortly after Mr. John Finlaison entered the service of the Government, he was asked to consider the state of the records and dispatches of the Admiralty, and, if possible, to devise some remedy for the inextricable confusion into which they had fallen. After nine months of incessant application, we are told he produced a magnificent system of digesting and indexing the records and correspondence of the Admiralty, which still works with such perfection that, should information or precedent be needed on any subject under discussion, all that has been received concerning it can be immediately known; or, should the name of some individual be called for, all that relates to him may be instantly produced. A full and interesting account of his system is given in the Journal of the Institute, vol. x. Work of this description seems by no means unworthy of the Actuary's skill and training.

Mr. Finlaison also compiled the first official Navy List, which is said to continue to be issued still in the form in which he originated it; and he compiled for Government various digests in regard to the fleets and arsenals, &c., of foreign nations.

He also instituted important reforms in the system of accounts of various Naval Departments, which resulted in very great saving of money, as well as in much greater ease in the production and auditing of the accounts themselves.

Dr. Farr remarked many years ago that "the whole commerce" of the country turns on contingencies which demand the "application of scientific observation and calculation; and as

"English agriculture has its chemists, English commerce must, to "keep pace with it, ultimately employ Actuaries to calculate the "risks which are now only roughly guessed at, and thus extend "the useful sphere of an important scientific class of men at "present almost peculiar to this country."

In instituting the "Samuel Brown Prize", Mr. Brown stated that his intention was to lead members of the Institute "beyond our ordinary professional subjects", to discuss new questions in which the doctrine of probabilities may be applied beyond the mere practice of insurance. For example—"Various questions "arising out of population statistics; emigration questions and "results; commercial matters—the law of recurrence of bankrupt—"cics, the law with which they follow certain conditions of credit, "or certain phases of commercial or trading speculations. "Questions connected with fire and marine insurance, which must "depend on the constant recurrence of events which must be "regulated by definite laws. Effects of hazardous employment on "health; the growth and strength of man; the regularity of "action of the human will, and similar topics."

In his Presidential Address, delivered to the members of the Institute on 26 November 1888, Mr. Sutton remarks as follows—
"The developments of ordinary insurance are only a portion of the many exemplifications of the principle of combination; and, wherever this principle of combination is put in force, the collection of facts in which the laws of average come into play furnishes the opportunity for the Actuary. Loss of wages through sickness; loss of wages or of life arising from accidents, whether in discharge of a man's ordinary vocation or not; loss of wages through loss of employment; pensions during spinsterhood; endowments on marriage; will at once occur to you among these many exemplifications, and the list is constantly increasing."

"Comparatively recent legislation", Mr. Sutton goes on to say, "has done a great deal towards increasing the demand for actuarial advice, and many private Acts of Parliament contain provisions for calling in, from time to time, Actuaries to investigate and advise upon the affairs of pension funds, sickness funds, &c. Not only are such provisions inserted in companies' private Acts, but in those regulating the powers of Municipal Corporations."

Referring to the German Scheme of State Insurance, Mr. Newbatt, in his Presidential Address of 1890, asks: "What "do the English people or its legislators, or what even does the "Institute of Actuaries, know of this great scheme? Cannot "we do something in regard to this great question, which, while aiding our Senators, will earn for our Institute that place in the esteem of our nation of which I have already spoken?" The seeming reproach cast upon the members of the Institute in these sentences, was, as we all know, in the following year effectively wiped out by Mr. Young's very able and exhaustive paper on "The German Law of Insurance against Invalidity and Old Age." The principle inculcated by Mr. Newbatt, however, still remains, namely, that the actuarial profession should ever be forward to give the State and the nation the benefit of its skill and experience in examining and explaining such national schemes of a technical character as may be brought before the public from time to time.

We have already referred generally to the fact that, prior to 1820, the Government had no regularly appointed Actuary in its employment. The need for such an officer was, however, strikingly demonstrated about that time by the revelation of the enormous loss that had been entailed upon the country through the grant of Government Annuities on ruinous terms. Previously to the year 1820, Mr. John Finlaison had rendered himself extremely useful to the Government in various matters connected with finance and otherwise, and, his aptitude for such work clearly marked him out for the position of first Actuary to the National Debt Commissioners, to which he was appointed by Act of Parliament in 1821. The appointment of an Actuary at this crisis could not, with any reason, have been delayed, and it is certain that the nation has saved millions sterling since 1821, through the institution of the office.

We have also referred to the appointment, in pursuance of the great Friendly Societies Act of 1875, of an Actuary to the Chief Registry of Friendly Societies, and to the very great benefit that has resulted from such appointment. In connection with this matter, it must occur to everyone as being a very great omission that no provision was made for the appointment of a somewhat similar officer at the Board of Trade in connection with the working of the even more important Life Assurance Companies Act of 1870. We do not suggest such an officer as is described so forcibly in the following passage from Mr. King's paper, read before the Institute in 1892, "On Legislation affecting Life Assurance Companies." Mr. King there says,—

speaking of State supervision in details,-"To have a "Government official, who has probably been educated in red "tape, interfering at every turn, would soon paralyze all energy " and enterprise. He might be officious and fond of magnifying "his office, or prejudiced and narrow-minded, or incompetent, or "even corrupt; and it is difficult to say which class of men "would exert the worst form of influence. At the best, he would " be narrow and ignorant as compared with the aggregate of the " profession outside the Government bureau, because he, from the "nature of the case, would not have had the education and "experience that are gained only by daily intercourse with the "nublic: and he could not appreciate the benefits of, or " necessities for, the changes of plan and the new features which "naturally present themselves to the active man of business. "The Government official, therefore, even with the best intentions." " could not help being more or less an obstructive."

The foregoing description refers, of course, to an official who is supposed to possess more ample powers than, and to be assigned very different functions from, those we would assume to be conferred upon, or entrusted to, the officer we suggest for the Board of Trade. We would not contemplate greater powers being entrusted, or other functions being assigned, to that officer than are already possessed by the Board itself. It seems reasonable, however, to believe that it would tend to even greater efficiency in the working of the 1870 Act, and to, if possible, increased confidence in the Board of Trade, if the official openly responsible for its insurance department were a Fellow of the Institute of Actuaries, or a Fellow of the Faculty of Actuaries.

Besides, however, the National Debt Office, the Registry of Friendly Societies, and the Board of Trade, there are many other Government departments that would no doubt benefit by the services of one or more Actuaries. Some of these departments are apparently, of their own accord, paving the way for the introduction of the Actuary officially, for, on referring to the list of members of the Institute, it will be found that there are Fellows of the Institute at the India Office, and at the Office of the Ecclesiastical Commission, and Associates at the War Office and the Scottish Office respectively, in addition to members also at the National Debt Office and the Registry of Friendly Societies. In all, thirteen members of the Institute seem to be already engaged in Government departments at home, and fifteen in various Government departments in the Colonies and India.

At the War Office there would seem to be an enormous amount of material fitted for treatment by the skilled Actuary:—records of the mortality of the army at home, and, more especially, statistics of the mortality at various foreign stations, and in the numerous frontier expeditions in which our army is continually engaged. Records of the mortality from climate at various ages would be useful, as showing to which stations only mature soldiers should be sent. Both at home and abroad such comparisons would be useful as pointing out, very often, the condition of insanitary towns or barracks. In an army, recruited with such difficulty as ours is, such information would, no doubt, amply repay, in the mere saving of men, the expense necessary for its compilation. Such records would also furnish a reliable basis for the calculation, by assurance companies and others, of the extra risk incurred of residence in various parts of the globe.

Similar remarks apply, with even greater force, to the Admiralty, seeing that our ships are stationed, or are cruising, in every corner of the earth. If the Government Actuaries at the War Office and the Admiralty had not enough work in connection with our own army and navy to keep them fully engaged, they might usefully employ any spare time to compile, as we have seen Mr. John Finlaison did, statistics in regard to the armies and navies of foreign powers, &c.

Very similar, and many additional, statistics might be prepared in regard to our mercantile marine; and, in that case, much useful and interesting information might be supplied regarding the increase or decrease in the number of foreign seamen in our Merchant Service, and the superiority in many directions of our own, as compared with foreign, sailors.

There is no doubt that any actuarial calculations required by many of the smaller Government departments would be best done at the office of the Actuary of the National Debt, and, in view of such calculations, that officer ought to have an officially appointed assistant Actuary under him, and a competent actuarial staff to carry through the work. Any Government, or semi-Government, publications issued would then carry more weight with them in certain quarters than they may do at present. As an instance of what we refer to, we may cite the case of a very useful set of tables issued some years ago for the repayment of loans by instalments. The author of that publication is now a Fellow of the Institute, but, on the title page, he simply describes himself as of the Public Works Loan

Office. To those who do not know the author otherwise, the addition to his name of the letters F.I.A., or the words, "Actuary at the Public Works Loan Office", would naturally give greater confidence in the use of the tables.

Of all the Government departments, however, the one which seems peculiarly to require the direction and supervision of the Actuary is that of the Registrar-General. It is admitted on all hands that the reports of the Registrar-General are not usually given in such a shape as to be read by the general public. There seems no reason, however, why the important facts contained in these papers, which have been collected at great expense and trouble, should not be presented in such a form as to be made most interesting and instructive, and practically useful to the various sections of the community. Instead of a single large volume, which no one cares to begin to read, let there be a series of small separate tracts dealing with the various branches of the report, and these could also be bound together in one volume, as at present, for the use of those who desire to peruse the whole report. The mere statement of facts is, however, of little real use, even although it may be put in suitable form. As has been well remarked, "The actual condition of the country is by no " means obvious to the superficial observer, and the causes of the " various phenomena which it exhibits lie very deep, and can only " be elicited with the aid of extensive investigations conducted "with skill and judgment as well as with mathematical exacti-"tude. The statistics of education, of crime, of pauperism, of " labour, of health, of trade, of agriculture, of manufactures, and " of every one of the details which enter into the survey of our "national condition and prospects, are closely connected one "with another. Population is the statistical element of greatest "importance. The classification, according to age, supplies the "means of establishing useful tables of population, of forming "correct views on mortality, on the efficient resources at the "disposal of the State in case of necessity, and of fixing ratios "between the useful portion, which actively contributes to the " general well-being, and the weaker portion which yet requires "assistance and support to become, in its turn, useful. " classification by profession, trade, and occupation, indicates the "means by which the population provides for its subsistence, " and allows the legislator more particularly to concentrate his "attention on the separate details of the complicated machine "confided to his care. Classification by civil condition, by

"education, by origin, &c., furnishes the administration with no "less valuable information to assure internal good order, and to "facilitate the execution of the laws." Who, we may ask with confidence, is so well fitted as the Actuary to make such classification in proper form, and to make the facts "tell their own tale" truly and interestingly? Not to carry the classifications to their fullest extent—if the mere expense is the only barrier—is a policy that may truly be said to be "penny wise and pound foolish."

There are, no doubt, other departments of the State, besides those we have specially referred to, that would do well to have the advice and assistance of the Actuary. Supposing then each such department to have its officially appointed actuarial adviser, it seems natural to infer that all these Actuaries together would form an advisory board to give assistance to Government in connection with legislation regarding Old Age Pension Schemes; all life assurance and friendly society matters, whether as regards existing companies or the winding-up or reconstruction of insolvent societies; currency proposals; many matters connected with taxation, &c. These Actuaries, being all Fellows, either of the Institute or of the Faculty, would be quite in touch with these bodies, and the advice they might offer to the Legislature would usually be likely to be approved by the profession generally. Hence, Government measures, involving actuarial or financial matters, would, in most cases, come forth matured and likely to be favourably received by the actuarial profession, and, through them, by the public at large.

But, beyond the Government departments, there are other wide spheres into which the Actuary ought to be introduced. Why should there not be an actuarial professor in each of our Universities for the instruction very specially of medical and law students? The medical profession is daily being brought more intimately into connection with the business of life assurance. Each office has one or more principal medical officers at its head office, besides thousands of medical examiners all over the country. How very few of these gentlemen know anything of the principles of life assurance, and do not most of them recommend, say additions to age in the case of underaverage lives, at the veriest random? Again, how many medical contributions on life assurance subjects are simply spoiled by the want of skill in treating tables of facts properly? All these and many other matters might be rectified if the

medical course at our universities included a certain amount of actuarial instruction.

Again, in regard to the law profession, how very much at sea are most of our barristers—and many of our judges even—when actuarial matters have to be dealt with by them; and how difficult it is for them to account for the difference that sometimes occurs in the opinions of various Actuaries that may be brought before them. It is to the credit of our assurance companies that the law courts are so comparatively seldom occupied with assurance cases; but that of itself is no excuse for our lawyers being so little instructed in the principles of assurance science.

But, coming down from lawyers and doctors, what dense ignorance of the rudiments of actuarial science still pervades the great mass of our people. The only remedy in that case appears to be the one suggested by Mr. Sutton, in his evidence, already referred to, before the Committee of 1885 on National Provident Insurance, as to the provision in our public schools of suitable elementary text-books on thrift and assurance and actuarial matters. The diffusion of such knowledge among the rising generation would no doubt result in the benefits offered by friendly societies and assurance companies being even more largely taken advantage of than at present, and in the consequent lessening of the amount payable by the country in poor's rates, and the saving of much of the distress and anxiety that exists in many cases which may never come to be chargeable to such rates. It is, consequently, in the interest both of the Government and the actuarial profession that such text-books should be adopted; and it seems worthy of the attention of the Institute whether they might not have such books prepared, say by offering prizes for those of a suitable form. If the matter were brought before them in a complete and taking style, it is not unlikely that Government might be induced to arrange for the introduction of these text-books into the schools. Suitable tracts might also be prepared under the authority of the Institute, for general circulation among the public, and there seems little doubt that the assurance offices would gladly avail themselves of these for distribution among their agents, in place of the very indifferent publications which are available for the purpose at present.

One other suggestion we would add to those already made. In Scotland, at least, it is usual for the various law societies to appoint some of their junior members to act for the poor. Could

the Institute not take a hint from this, and appoint certain of their junior members to be valuers for small friendly societies? The complaint was frequently made before various committees appointed to enquire into friendly society matters, that the expense of the actuarial valuation deterred many societies from having their affairs investigated and put upon a proper basis. Whether the expense be the true excuse or not in such cases, it seems desirable that the Institute should, where circumstances really require it, offer to appoint a qualified valuer at a merely nominal fee, or even without any fee at all. We feel sure there are many among the younger members of the Institute public-spirited enough to be willing to undertake such duties at the call of the Council. Every one may not be willing or able at once to submit papers to the meetings of the Institute, but this is another way in which all may, to some extent, repay the debt they owe to their profession, and, at the same time, benefit themselves in no small degree.

In the treatment of our subject, we have confined ourselves throughout almost entirely to our own country; and, owing to the extent and importance of the enquiry, it would seem to be impossible to deal as well, at all satisfactorily, in the scope of a single essay, with the relation of our profession to the State in the Colonies, the United States of America, and various foreign countries.

As regards the United States, we should have to begin with the year 1778, when Dr. Price received a formal invitation from Congress to become an American citizen, and to give his assistance in regulating the finances of the American Republic. The history of the relation of the actuarial profession to the State in America alone, could not fail to be most interesting and instructive, and it is to be hoped that some of our professional brethren in the States will see their way to undertake the task, It is somewhat difficult for one, acquainted only with the customs of our own country, to grasp intelligently the phases of American business, and, for that reason, perhaps, there has been apparently very little written on the subject by writers in this country. Those who have sought information from American sources direct seem—owing to the number of different States—to have experienced considerable difficulty in obtaining the particulars they desired. The most complete statement that we know of, dealing with the requirements of the various

States of America as regards life assurance companies, is contained in the *Journal of the Institute of Actuaries*, vol. xxxiii, in a schedule prepared by Mr. Chisholm, and appended to a paper by Mr. Sheppard Homans in that volume. That schedule is very concise, and it is to be regretted that it, and the valuable remarks accompanying it, were not published more prominently than simply as part of the discussion on Mr. Homans' paper.

On the whole, however, it is perhaps well that we have dealt at this time only with our own country, seeing that the second International Congress of Actuaries is to meet in London in a few months, and that it is expected that Actuaries from our various Colonies and from foreign countries will submit papers, on the same subject, as regards the different countries they represent. They will, no doubt, in this way treat the subject better and more completely than any stranger could do; and, by leaving the field entirely open, we may hope that each writer will thereby be induced and encouraged to deal the more exhaustively with his part of the subject.

In our attempt to sketch the history of our profession and its relation to the State, in our own country, we have noticed very specially the ability and business qualities of its members; and there is no doubt that these qualities have contributed very largely to the success and prosperity of life assurance business in our midst. We must not forget, however, that there is another quality which was largely possessed by the founders of our profession, and which, we rejoice to think, has continued to be characteristic of their successors to the present day, and, without which, all the valuable discoveries of our science, and all the admirable laws passed by our Legislature, would have been entirely futile as regards the stability and success of our assurance institutions,—we mean the quality of lofty integrity of character. Such character shines out most conspicuously in nearly all the biographical sketches of Actuaries we have perused; and we can entertain no better or surer hope for the prosperity of the Institute, and the actuarial profession in future, than that their members individually will continue to maintain this same high standard of character; and that, as a result, it may come to be said of many, or all of them, as it was said of the late Mr. John Finlaison, "He died, "as he had lived, with the courage and resignation of a " Christian Gentleman."

## Discussion.

The President called attention to some admirable remarks upon "The future Work of the Actuary", contributed by Mr. Ryan to the current number of the International Journal of Actuaries. hailed with very special satisfaction the extension of the compass of their studies which was indicated by this paper. No portion of the earlier numbers of their Journal was more rich in interest, or more deserving of imitation at the present day, than the transgression of purely mathematical and technical limits, and the incursion into fields of general fiscal and economic applications which were made by Mr. Charles Jellicoe, Mr. Peter Hardy, Mr. Samuel Brown, Mr. Arthur Bailey, and others. A profession accustomed to restricted studies, or circumscribed within a limited compass inevitably tended to become dwarfed and infertile. By a steadier and more persistent return to that former method of their forefathers, in attempting the fruitful application of their professional principles to current questions of a more diffused and public character, they would not merely amplify the area of their power, but at the same time they would utilize in a larger measure that capacity of service which the profession is competent to render to the State in the consideration and solution of general practical problems.

Mr. Wm. Hughes said the fact that the essay was awarded the prize so generously offered by Mr. Chisholm had prepared them for a production of more than usual interest, and now that they had heard it read they were not disappointed. The historical part of the essay could, he thought, hardly have been better. author seemed to have taken account of nearly every event he could put his hand upon. There was something, he thought, for every consecutive year from 1868 to the present time. Occasionally, very occasionally, the author might have travelled a little beyond the limits of the syllabus, but there was very little that he had omitted. Yet, strange to say, he seemed to have overlooked altogether one matter which was of very great interest to himself (the speaker) in particular, and indeed to all of them, namely, the important subject of industrial assurance. Although the author had given much evidence in detail, or the effect of the evidence which was laid before the various Commissions on Friendly Societies, he had most strangely overlooked the fact that a great deal of evidence was given with reference to industrial assurance and the assurance of children's lives. That was a matter which was certainly as germane to the general subject as many of those points which had been inserted. He thought Mr. Nicoll had been a little severe, perhaps, upon the medical and legal professions. Some communications which he had received from medical gentlemen, where they had touched upon actuarial matters, had, he must say, displayed a very lamentable degree of ignorance, but he did not think they ought to judge the whole profession by some of the expressions of individuals. whole, the medical profession was an intelligent one, and really did understand more than Mr. Nicoll gave them credit for. Certainly as to the legal profession, he had been very much struck not with their ignorance but with the amount of knowledge they displayed, although they did not possess the advantages of the course of study which actuaries had gone through. He thought the suggestions as to the teaching of elementary principles were very valuable, and it had often occurred to him that one thing which the Institute might very properly undertake would be the preparation of an elementary text-book or primer of actuarial science, which, as the essayist had said, might be used in the Universities and in the upper forms of schools. He did not mean a merely popular exposition of actuarial matters; he thought it should be a serious document, but one that was within the comprehension of any person who had a schoolboy's

knowledge of algebra and reasonable common sense. Mr. James Chisholm said it could not but be gratifying to him that this prize, which, through the kindness of the Council of the Institute, he was enabled to offer under its auspices, should have called forth such a valuable paper as the one Mr. Nicoll had read to them that evening. The subject was a large one, and was perhaps too wide to be compressed within the limits of a single paper, but Mr. Nicoll had at least given them a broad and deep foundation to build upon, and had devoted himself first of all and mainly to the origin, growth, and past history of the actuarial profession. Nothing could have been more interesting and suggestive than such a procedure, for it had reminded them that alike in the time of Ulpianus under the Roman Empire, and in that of De Witt, the Grand Pensionary of Holland, on to the times of Dr. Price and the three Finlaisons, reasons of State had demanded the calculation of values of annuities on lives, and this was recognized as work which formed the peculiar province of actuaries. Then they found that where it had been a question of friendly societies, or old age pensions, or national mortality tables, the advice or services of actuaries had been required by the State. Now, there were two pregnant sentences in Mr. Nicoll's paper to which he would direct attention. of the annual Budget proposals of the Government and similar subjects, the author said, "We do not remember to have seen any papers on such subjects which have been read at the Institute, and it is indeed very rarely that even reference is ever made to them now-a-days at the meetings. Since a Charter was granted, the public are entitled, even more than in earlier days, to look to the Institute for guidance in regard to proposals of the Government in matters of finance and taxation, and it seems very desirable, therefore, that more attention should be given to such subjects in future by members of the Institute than has of late been bestowed upon them." It was for this reason chiefly that he desired to draw out papers in competition on the subject of the wider relations of the actuarial profession to the State, so that the minds of all the members might be directed to the importance of not letting go that wider interest in things national which was more commonly found at their meetings in earlier days than it was now. The Institute itself recognized the high public duties resting on it and had striven nobly to fulfil them. Mr. Sutton, quoted by Mr. Nicoll, and speaking in 1885, said, "The Institute of Actuaries has lately become incorporated by Royal Charter, and although hitherto it has been composed of the leading officials of the life assurance companies, it now understands

that, in becoming incorporated, it has to some extent taken upon itself public duties; and it is anticipated by the Council of that Institute that, in the future, one of their public duties will be to examine and test the efficiency of all persons coming as students to that Institute, and desiring to show their attainments in actuarial matters." If, then, there was a consensus of opinion that there was valuable work in relation to the State that actuaries could undertake, and the Institute on its part was doing its duty as a teaching and examining body, why was it that subjects of that wider interest so seldom came before them? The reason seemed to him to be that the area from which they draw their members is too eramped. The profession as at present constituted was a very limited one, as it was supported mainly by members of the staffs of the life assurance offices, and the ever-increasing exigencies of business would not as a rule permit them to stray much out of their own field. Was there, then, no prospect of any outlet? He had great hope that one might be found in their municipal institutions and county councils. Mr. Clare spoke to them in that room a few days ago of the immense development of representative local government. It was a fact which they had as yet perhaps hardly sufficiently realized. Taking the local debts alone, they found that between 1875 and 1894 the total amount had increased from 93 millions to 224 millions. Then the duties thrown on local authorities included poor relief, education, drainage, public health, recreation grounds, &c. Those local bodies were, in fact, little States or Republics within the greater State. Was there no room for the services of a trained actuarial expert on the staffs of such bodies in connection with the debt obligations incurred, the superannuation of the large staffs employed in municipalities, school boards, &c., the local valuation of friendly societies, &c.? For his part, he believed that in the century which they were so nearly approaching it would be as strange to find their great corporations, like Birmingham or Glasgow, without an actuarial officer of health as it would be now to find them without a medical officer of health. Taking the question of friendly society valuation, he did not suggest that they should adopt any other system than that applied to the great assurance institutions—freedom and publicity but it should be applied in the centre where the society was known. Their life assurance companies had to stand the fierce public light of a great commercial centre like London, where they were all well known and had a special press devoted to criticising their accounts. A small local friendly society would be lost in that erowd. Why should there not be an actuarial bureau, small or large, as circumstances might require, attached to each borough or county council, which for a small fee would make a valuation, or register a valuation already made by a competent person, of any friendly or benefit society in the district? The local register and the local valuation would direct public opinion in the locality to those societies which did not care to undergo the test, and subject them to criticism in their own district where it would be most efficacious. Taking again the subject of poor law relief, which was intimately associated with the question of old age pensions, it was greatly to the honour of a high Officer of State like Mr. Chamberlain that he should take

such a close interest in a question that affected principally the poor and the unfortunate. The community would be destitute of moral feeling and on a level with those savage tribes which exposed their aged members to starvation and death as soon as they could not work, if it did not occur to them to endeavour to do something to mitigate the weight of misfortune. The subject was a thorny and difficult one. A Royal Commission, which included two of their honoured members, was now considering its report, but he was not in their confidence. He thought, however, that they all acknowledged that the task of providing old age pensions would be a gigantic one for the State to undertake, and for his own part he considered that, unless local knowledge and local opinion and local responsibility for outlay was brought to bear, it would be impossible to separate the deserving from the undeserving, or to check malingering under any system that might be devised, and even then it would be very difficult. Was there, at any rate, not there some possibility of an outlet for actuarial services? Would it not also be a considerable gain to local bodies themselves that they should be served, to a certain extent at least, by members of a professional body who were bound by its laws to a high standard of professional honour? Their profession was at present like an inverted pyramid. It rested upon its apex. The apex was the relatively small number of life assurance institutions from which its members were drawn. Its base was the potentiality of work contained in the ever-increasing number of its trained and professionally-examined members. All that these asked was the opportunity to evolve the capacities that were in them, and fit themselves for higher and higher responsibilities. When they had an annual Congress of British Actuaries, at which would be found actuaries appointed by all their great corporations, boroughs, and public bodies, as well as the actuaries of their life assurance offices, then they might say that their profession was established on a truly national basis.

Mr. Geo. King, in congratulating Mr. Nicoll and Mr. Chisholm and the Institute of Actuaries upon the essay submitted to them that evening, said he would confine himself to the last few paragraphs, which touched upon the Congress. He thought there they had an indication that they were travelling more or less in the direction so eloquently pointed out to them by Mr. Chisholm. He thought that the sphere of the actuary so far from narrowing was really widening. And it was not only at home that they found actuaries on the road of progress. There was a federation of actuaries taking place throughout the world, and he was glad to think that their own Institute was one of those causes which was producing that great result. The President himself had a few minutes ago mentioned a paper in what, after all, could not but be considered a remarkable journal, the International Actuarial Journal. And they were to have immediately an International Actuarial Congress. The appearance of Mr. Nicoll's paper seemed to him, therefore, very opportune, the subject being "The Relation of the Actuarial Profession to the State." There was no set paper upon that subject at the Brussels Congress, but it was referred to, and he did not know whether it was there that Mr. Chisholm had got his inspiration to suggest the subject for a prize essay, but he thought it was a very happy suggestion. He believed that the International Congress would do a great deal of good, he would not say to the profession, because the profession could take care of itself, but to the public, who would come to know the uses to which it could turn the profession. And if he might, without breach of confidence, mention some of the indications they had had, he thought the view he had ventured to express would be accepted. They had Governments officially recognizing from that point of view, for the first time, he might say, the actuarial profession. Mr. King then proceeded to mention some of the subjects to be discussed at the forthcoming Congress, and referred to the gentlemen who were to represent the various Governments at its proceedings.

A vote of thanks was unanimously passed to Mr. Nicoll, for

his paper.

Mr. J. NICOLL, in reply, thanked the members for the very kind reception which they had given to his rather dry paper. experience of Scotch actuaries generally, that they received nothing but kindness in all their dealings with the Institute, or with any of the members of the Institute. The only criticism, he thought, had come from Mr. Hughes, who said there was no reference in the paper to industrial assurance. There were two references on pages 174 and 232, but his idea was that, in the paper, he was to deal with the relations of the actuarial profession to the State, and anything in which no actuaries were involved he had cast aside. There was a good deal of interesting matter which he might have added to the paper, but where the secretaries or managers of industrial companies were examined and not actuaries he held that did not come within the scope of his paper, and he had left it out on that ground simply. He thought the remarks as to the finances of municipal bodies, by Mr. Chisholm, were very suggestive indeed, and he wished they could be added to the suggestions at the end of the paper, because he thought they would in a measure complete the paper.

On the Mortality in the British Navy and Army, as shown by the Official Reports. By James J. McLauchlan, F.F.A., Secretary of the Scottish Equitable Life Assurance Society.

THE Annual Reports of the Registrar-General for England contain each year, tables giving information as to the mortality in the Navy and Army for a long series of years. My object in undertaking this paper was to compare the actual deathrates, as shown in these tables, with the rates prevailing among assured lives, with a view to ascertaining the extra mortality to which persons serving in the Navy and Army are subject. I subsequently found it desirable to extend the scope of the

paper so as to include an examination of the recent mortality of the Navy, as shown by the Statistical Reports of the Health of the Navy, and of the Army, as shown by the Reports of the Army Medical Department.

## NAVY.

The table in the 58th Annual Report of Death-rates Registrar-General for England, relating to for last 40 years. mortality of the Navy, gives the death-rates in the Service afloat from all causes, from disease and from violence respectively, in each year from 1858 to 1895 inclusive. From this table, with the addition of figures for the years 1856 and 1857, taken from the 56th Report, I have made up the following Table A, which contains the average death-rates from disease, from violence, and from all causes respectively, for four periods of 10 years each, ending with the year 1895. These rates are the central death-rates for each group of ages; and the word death-rate is used throughout this paper as meaning the central death-rate. The rates of invaliding and impairment, hereafter mentioned, are also central rates.

Table A.—Average death-rates per 1,000 in the Service affoat.

| 10 Years ending<br>31 December | From Disease | From Violence | From all Causes |
|--------------------------------|--------------|---------------|-----------------|
| (1)                            | (2)          | (3)           | (4)             |
| 1865                           | 11.7         | 4.2           | 15.9            |
| 1875                           | 6.8          | 3.5           | 10.3            |
| 1885                           | 5.4          | 4.0           | 9.4             |
| 1895                           | 4.4          | 2.6           | 7.0             |

The most remarkable feature in the above table is the large decrease in the mortality from disease, as shown in column (2). This is principally due to the intelligent care which has now come to be exercised in regard to sanitation, zealous co-operation between the executive and medical officers being the almost universal rule. Sanitation is here taken in its widest sense, and includes the selection of anchorages, and the getting away as soon as practicable from unhealthy conditions of climate or locality, and from the neighbourhood of epidemics. Much is, no doubt, also due to improvement in the conduct of the men and in the way they

are treated. This point was referred to last month by Lord Charles Beresford, in an address to the Harrow boys on the Navy. He said that when he went first to sea, he thought it a common thing to help to hoist aboard fifteen or twenty men daily, all dead drunk.

Note.—See Statistical Report of the Health of the Navy for 1887, p. 17.

For the purpose of calculating the expected death-rates—ages of officers and men. in Table A, it is necessary to have some information as to the ages of the officers and men constituting the Service afloat during the period of forty years covered by the figures in the table. This has been obtained from the Parliamentary Papers containing the results of each of the last five censuses, which contain tables showing the number of men of all ranks serving in the Royal Navy and Royal Marines at the date of the census, classified according to age. These tables have been used to calculate the expected deaths in a year in the Service afloat at the date of each census. One-half of the Royal Marines are assumed to be included in the Service afloat.

The question of the Mortality Table to be used in Mortality Table to be calculating the expected deaths is involved in considerable difficulty. Dr. Sprague, in a paper on "The Rate of Mortality in the Navy", published in 1884, points out the difficulty of comparing the rates of mortality in the Navy with those exhibited by any of the ordinary tables. He says: "The officers and men under observation "are all healthy lives at the time when the observations " commence; and the average standard of health among the "lives under observation is maintained by the constant " invaliding of those persons who have fallen into chronic bad "health. These conditions are quite different from those "which prevail among the bodies of lives, from whose "experience ordinary mortality tables have been compiled, "and any comparison with the figures furnished by them "would, therefore, be misleading. A comparison may be " more appropriately made with the mortality among recently " selected lives, that is to say, among insured lives in the year "immediately following their insurance; and my Select "Life Tables (J.I.A. xxii, 391) enable us to make such a " comparison."

Expected death-rates according to Select-Life Tables.

Following the method indicated by Dr. Sprague, I have calculated the expected deaths in a year according to his Select-Life Tables. The expected deaths have been obtained by multiplying the number living (and constantly at risk) in each group of ages,

by the proper central death-rate calculated according to the formula given by Dr. Sprague. The results are given in the following table:

Table B.—Number of officers and men serving on board Her Majesty's ships at each of the last five censuses, with the corresponding expected deaths in a year, and expected deathrates per 1,000 according to the Select-Life Tables. First year of Assurance.

| Year<br>of Census | Number<br>enumerated | Expected Deaths<br>in a Year | Expected<br>Death-Rate |
|-------------------|----------------------|------------------------------|------------------------|
| (1)               | (2)                  | (3)                          | (4)                    |
| 1851              | 30,403               | 146                          | 4.8                    |
| 1861              | 57,088*              | 272                          | 4.8                    |
| 1871              | 47,455†              | 221                          | 4.7                    |
| 1881              | 50,992               | 246                          | 4.8                    |
| 1891              | 59,777               | 310                          | 5.2                    |

The expected death-rates in Table B, may fairly Comparison be taken as applicable to the corresponding periods of actual and expected in Table A. The following Table C gives a death-rates. comparison of the actual and expected death-rates on this basis:

Table C.—Average annual death-rates per 1,000 in the Service afloat compared with the rates to be expected according to the Select-Life Tables.

| 10 Years ending<br>31 December | Actual<br>Death-Rate | Expected<br>Death-Rate | Excess of Actua<br>Death-Rate<br>(2)-(3) |  |
|--------------------------------|----------------------|------------------------|--|--|
| (1)                            | (2)                  | (3)                    | (4)                                      |  |
| 1865                           | 15.9                 | 4.8                    | 11.1                                     |  |
| 1875                           | 10.3                 | 4.7                    | 5.6                                      |  |
| 1885                           | 9.4                  | 4.8                    | 4.6                                      |  |
| 1895                           | <b>7</b> ·0          | 5.2                    | 1.8                                      |  |

<sup>\*</sup> Excluding 2,674 whose ages were not stated.

<sup>†</sup> Excluding 1,228 whose ages were not stated.

The remarkable decrease in the extra mortality as shown in column (4), is principally due to the decrease in the mortality from disease as shown in Table A, column (2).

The figures given in the Registrar-General's Table Deaths by are taken from the statistical reports of the health of the Navy, and I have carefully examined the statistics given in these reports for each of the 10 years 1886-1895. Each statistical report gives a table showing the deaths by violence in the Royal Navy in the year, classified under different heads. The following Table D is made up from the information in those yearly tables:

Table D.—Deaths by violence in the Royal Navy in the 10 years 1886-1895.

| Falls from  | aloft   |       |      | <br>47    |
|-------------|---------|-------|------|-----------|
| Wounds, fr  | actures | , &c. | <br> | <br>203   |
| Wounds in   | action  |       | <br> | <br>37    |
| Drowned     |         |       | <br> | <br>1,045 |
| Suicide     |         |       | <br> | <br>63    |
| Other cause | š       |       | <br> | <br>31    |
|             |         | Total | <br> | <br>1,426 |

Note .- The deaths by drowning include S0 lives lost by the wreck of the "Wasp" in 1887; 173 lives by the wreck of the "Serpent" in 1890; and 358 lives by the disaster to the "Victoria" in 1893.

The death-rate from violence in the Navy during the 10

years 1886-1895, deduced from the figures in Tables D and E  $\frac{1,\pi-9}{55,630}$ , or 2.6 per 1,000, being the same as given in Table A. It will be observed that a very large proportion of the deaths by violence were caused by drowning, the death-rate from or 1.9 per 1,000; and that the this cause being number of deaths from wounds in action was only 37, the

death-rate from this cause being '07 per 1,000.

Comparison of actual and expected death-rates at different

Each of the Statistical Reports gives a table showing the mean force between certain ages in the year, with the corresponding deaths. From this information, the following Table E has been prepared, for the purpose of showing how the mortality at different ages compares with that to be expected according to Dr. Sprague's Select-Life Tables, and the Institute H<sup>M</sup> Table respectively.

Table E.—Total force between certain ages and corresponding deaths in the 10 years 1886-1895; also the death-rates per 1,000, compared with those to be expected according to the Select-Life Tables and the Institute H<sup>M</sup> Table respectively.

|          | .,            |             | DEATE           | Excess of<br>Actual |                            |                           |
|----------|---------------|-------------|-----------------|---------------------|----------------------------|---------------------------|
| Ages     | Mean<br>Force | Deaths      | Actual (3) ÷(2) | Expected<br>Select  | Expected<br>H <sup>M</sup> | Select<br>Rate<br>(4)—(5) |
| (1)      | (2)           | (3)         | (4)             | (5)                 | (6)                        | (7)                       |
| 15-25    | 31,496        | 1,898       | 6.0             | 4.6                 | 5.7                        | 1.4                       |
| 25-35    | 17,521        | 1,267       | 7.2             | 4.6                 | 7.6                        | 2.6                       |
| 35-45    | 5,641         | <b>56</b> 6 | 10.0            | 5.9                 | 10.2                       | 4.5                       |
| Above 45 | 972           | 179         | 18.4            | 11.9                | 23.9                       | 6.2                       |
| All .    | 55,630        | 3,910       | 7.0             | 5.2                 | 8.1                        | 1.8                       |

It will be seen that the extra mortality per 1,000 for the total force, when the actual death-rate is compared with that according to the Select-Life Tables, is 1.8, being the same as given in Table C, but that, when the lives are grouped according to age, the rate rises from 1.4 for the youngest group. to 6.5 for the oldest. The question now arises: Do these figures lead to the conclusion that the extra risk of service in the Navv increases with age? I think the figures do not lead to this conclusion, but rather show that the Select-Life Tables are not suited for calculating the expected deaths of the lives in the separate age groups given in Table E. The standard of healthiness existing among the officers and men of the Navy corresponds, on the average, fairly well to that existing among a body of insured lives in the first year of insurance; but it evidently varies according to length of service. Service in the Navy is an occupation that requires a considerable standard of bodily strength and vigour, and there is no inducement for any man to enter it who does not consider himself strong and well. The effect of this, in conjunction with the medical examination of entrants, is such, that the standard of bodily health in those who have just joined the Navy, is considerably higher than among a body of insured lives in the first year of insurance. The system of invaliding out of the Service men whose health and strength fall below the necessary standard, prevents the average standard of health falling so much as it would otherwise do, but is not sufficient to keep it up to the level of that among lives in the first year of insurance, and it gradually declines as the length of service increases.

Comparison of death-rates I think we shall get a better idea of the extra in total force mortality at successive ages by taking the mortality at different on the home station, exclusive of deaths from ages, with death-rates injuries, as representing the normal rate of mor-(excluding Injuries) on Home Station. tality, and comparing that with the rate of mortality from all causes in the whole force. This is based on the view that the extra risks of naval service are entirely due, either to foreign service, or to the risk of death by violence. I shall, when convenient, use the term "natural" applied to deaths and invalidings, to signify that deaths and invalidings arising from injuries are excluded.

Tables showing the mean force between certain ages with the corresponding deaths, are given in the statistical reports for each station, and those tables show separately the deaths from the different classes of disease and from injury. The following Table F is made up from the tables relating to the home station.

Table F.—Total Force between certain ages in the 10 years, 1886–1895, on the Home Station, with the corresponding actual deaths and death-rates per 1,000.

|          |               |                    | Deaths           |                                    |                           | DEATHS PER 1,000              |                              |  |
|----------|---------------|--------------------|------------------|------------------------------------|---------------------------|-------------------------------|------------------------------|--|
| Ages     | Mean<br>Force | From all<br>Causes | From<br>Injuries | Excluding<br>Injuries<br>(3) - (4) | From all Causes (3) ÷ (2) | From<br>Injuries<br>(4) ÷ (2) | Excluding Injuries (5) ÷ (2) |  |
| (1)      | (2)           | (3)                | (4)              | (5)                                | (6)                       | (7)                           | (s)                          |  |
| 15-25    | 16,095        | 790                | 238              | 552                                | 4.9                       | 1.5                           | 3.4                          |  |
| 25 - 35  | 7,806         | 482                | 145              | 337                                | 6.2                       | 1.9                           | 4.3                          |  |
| 35 - 45  | 2,959         | 267                | 41               | 226                                | 9.0                       | 1.4                           | 7.6                          |  |
| Above 45 | 686           | 124                | 7                | 117                                | 18.1                      | 1.0                           | 17:1                         |  |
| All .    | 27,546        | 1,663              | 431              | 1,232                              | 6.0                       | 1.5                           | 4.2                          |  |

The expected death-rate at all ages, according to the Select-Life Tables, is  $\frac{128.9}{27,546}$  = 4.7 per 1,000. This very slightly exceeds the corresponding rate in column (8) 4.5 per 1,000, and confirms the view that the natural rate of mortality on the home station is practically at the normal rate.

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The following Table G gives the death-rates per 1,000 on the home station, excluding deaths from injuries, compared with the death-rates from all causes in the total Force.

Table G.—Death-rates per 1,000 in the 10 years 1886-1895.

| Ages     | Home Station,<br>excluding<br>Injuries | Total Force,<br>all Causes | Excess of<br>General<br>Death-Rate<br>(3)-(2) |
|----------|--|----------------------------|---|
| (1)      | (2)                                    | (3)                        | (4)   |
| 15-25    | 3.4                                    | 6.0                        | 2.6   |
| 25-35    | 4.3                                    | 7.2                        | 2.9   |
| 35-45    | 7.6                                    | 10.0                       | 2.4   |
| Above 45 | 17:1                                   | 18.4                       | 1.3   |
| All .    | 4.5                                    | 7:0                        | 2:5   |

It will be seen from Table G that the extra mortality, as shown in column (4), does not increase with age.

The rate for the whole Force, 2.5 per 1,000, is greater than the figure in Table C, 1.8. This was to be expected, as the method of calculation assumes that all the deaths by violence in the Navy are an addition to the expected number, whereas, in strictness, a deduction should be made of the number of violent deaths that might be expected to occur in a corresponding body of select lives.

Dr. Sprague, in his paper "On the Rate of Mortality in the Navy", referred to above, points out that the persons invalided in the Navy correspond to the damaged lives of his Select-Life Tables. Referring to the invalidings on the home station in 1878, he says: "No "information whatever is given as to the ultimate fate of the "642 persons who were invalided. Probably some of them "regained their health and rejoined the Service; and if the " number of them were stated in the report, which could, I " suppose, be done with comparatively little trouble, the value " of the statistics would be considerably increased. "remainder or the greater number of them would, I suppose, "be incapacitated, through ill-health, from further naval "service, and many of them would die, within a few years, "from the diseases for which they had been invalided; and, " although the survivors might so far recover as to be able to "undertake light work, they would probably be always

"subject to a much higher rate of mortality than the normal "rate for their age. They would, in fact, be what we "technically call damaged lives."

Dr. Sprague goes on to compare the number invalided in 1870, with the numbers becoming damaged according to the Select-Life Tables. Referring to the rate of invaliding on the home station, he says: "At ages 15-25 the percentage of the "selected lives that become damaged is only 38, while the " percentage of invaliding in the Navy is almost ten times as "large, namely, 3.74. This proportion becomes gradually "less with increasing age, but at the highest ages the "percentage of the Navy is about four times that for the "select lives. In making this comparison, we must not "forget that all the damaged lives, according to my table, "die within five years, whereas, as already mentioned, some " of those who are invalided probably regain their health; but, "when the largest possible allowance is made for this, it is " clear that the rate of invaliding in the Navy will be several "times greater than the rate at which select lives become "damaged; and the unavoidable inference is that, if we could "trace those lives which have become invalided, so as to " include the deaths among them with the deaths among those "who remain in the Service, we should get a death-rate " very much higher than that which prevails among insured " lives."

Finally, referring to the figures relating to the Force, exclusive of the Irregular Force, Dr. Sprague says: "The rate "of invaliding at all ages is about seven times the rate of "impairment among the insured lives."

Tables are given in the statistical reports which show the invalidings in the total Force for each year from different classes of diseases and from injury. Similar tables are given for each station. Adding together the figures for the 10 years 1886–1895, I find that the invalidings in the total Force were 14,914, or 26.8 per 1,000. The number to be expected according to the Select-Life Tables, calculating by the use of Dr. Sprague's rates of impairment, is 2,539, or 4.6 per 1,000. The actual rate of invaliding is thus rather less than six times the expected rate, showing some improvement as compared with the rate for 1870. But the reports now give, besides the number of persons invalided, the number of these finally invalided out of the Service. The former figure includes men

temporarily invalided from foreign stations, many of whom, on arrival in England, or after treatment in home hospitals, were again able to join the active Force. The number of persons finally invalided out of the Service in the 10 years 1886–1895 was 9,499, or 17·1 per 1,000. This is rather less than four times the number to be expected according to the Select-Life Tables.

In dealing with the subject of invaliding in the Navy, it will be convenient to use the phrase "rate of impairment" to denote the ratio which the number of men finally invalided out of the Service in a year bears to the total mean Force. I shall also use the phrase "active naval lives" to denote the lives at any given time actually in the Service, and the phrase "mixed naval lives" to denote the whole body of existing lives, made up of those who are serving in the Navy, and those who have formerly served. Now, for life insurance purposes, the question we want to answer is, as Dr. Sprague indicates: what is the death-rate among mixed naval lives? In order to answer this question it is necessary to ascertain (1) by how much does the rate of impairment of active naval lives exceed what, having regard to the constitution and management of the Force, may be called the normal rate? and (2) what effect has the extra rate of impairment upon the rate of mortality of mixed naval lives?

The best method that occurs to me of answering the first question, is to assume that service on the home station involves no impairment of health beyond what may arise from violence; that is to say, that the number and causes of the natural invalidings on the home station are such, that the mortality of the mixed naval lives of the home station is equal to that prevailing among insured lives. Having regard to the great decrease that has taken place in recent years in the mortality from disease, as shown in Table A, and keeping in view the fact that men are rendered unfit for a Service like that of the Navy by failure of strength or activity and other causes not necessarily tending to shorten life, I think this assumption is not unreasonable. consistent with the assumption already made, that the extra risks of naval service are entirely due either to foreign service or to the risk of death by violence. On this further assumption, then, the excess of the rate of invaliding from all causes in the whole Force, over the natural rate of invaliding

on the home station, will represent the extra rate of impairment to which lives serving in the Navy are subject.

The total invalidings on the home station in the 10 years 1886–1895 were 6,360, of which 463 were on account of injuries, leaving 5,897 due to other causes. It is reasonable to assume that the proportion of those finally invalided out of the Service is the same as that prevailing in the whole Force,

namely,  $\frac{9,499}{14,914} = .637$ . This gives 3,756, and makes the natural rate of impairment on the home station 13.6. The rate of impairment in the total Force being 17.1, the required extra rate of impairment is 3.5 per 1,000.

We now come to the question of what effect this extra rate of impairment has on the mortality among mixed naval lives. It has already been assumed that, leaving injuries out of account, there is no extra mortality or impairment of health on the home station. Now, we know that the natural mortality among the active naval lives of the home station is in accordance with Dr. Sprague's Select-Life Tables; and we have assumed that the mortality among the mixed naval lives of the home station is the same as that of assured lives: in other words, is in accordance with the Institute H<sup>M</sup> Table. It follows that having regard to the constitution and management of the Force, the natural rate of impairment, as existing on the home station, is such as to produce an increase in the mortality equal to the difference between the Select death-rate and the Institute H<sup>M</sup> rate. The Select rate for the home station being

4.7 per 1,000, and the Institute  $H^{M}$  rate  $\frac{1,864}{27,546} = 6.8$ , this

gives an extra death-rate of 2·1, corresponding to a rate of impairment of 13·6. There must obviously be a proportion between the rate of impairment of active naval lives and the rate of mortality among the corresponding mixed naval lives; and I think it may, for our present purpose, be fairly assumed, that those rates are in the ratio of 13·6 to 2·1, or say, that a given rate of impairment among active naval lives corresponds to a death-rate among the corresponding mixed naval lives, of 15 per-cent of such given rate of impairment. Now, we have seen that the extra rate of impairment among the active naval lives of the whole Force is 3·5 per 1,000; and it follows that the corresponding extra death-rate to be expected among the mixed naval lives is  $1.5 \times 3·5 = .5$  per 1,000.

The extra death-rate among mixed naval lives may be ascertained directly from the extra rate of invaliding among the corresponding active naval lives, by taking  $10 \ (= \cdot 637 \times 15)$  per-cent of the latter. Thus, in the case of the whole Force, we have the extra rate of invaliding per  $1,000 = 26 \cdot 8 - 21 \cdot 4 = 5 \cdot 4$  (see Table I), 10 per-cent of which is 5, as above.

War Risk. We have now dealt fully with the question of the mortality among active naval lives during the ten years 1886–1895, and with the subject of the extra mortality among mixed naval lives, due to extra impairment of health among active naval lives. But before we can form an estimate of what the extra mortality in the Navy is likely to be in future, we must take account of the fact that, during the ten years 1886–1895, the naval forces of the Crown were engaged in no active operations of any importance. This appears from the figures given in Table D. It is, therefore, necessary to make an estimate of the probability of death in active service in time of war; and the average death-rate from this cause must be added to the death-rates already obtained.

Messrs. Smee and Ackland, in their well-known joint report, made in 1890, to the Directors of the Gresham Life Assurance Society, discuss the subject of war mortality under the two heads of "small wars" and "national wars." As regards "small wars", they find that during the 15 years 1872 to 1886, on the average, somewhat more than 6 per-cent of the total strength of the Army abroad was annually engaged in small wars. They estimate the annual mortality from small wars at  $1\frac{1}{2}$  per-cent of the Force engaged, and the total probability, in the case of both Navy and Army, for a life now stationed at home, of death in active service abroad in a

small war at  $\frac{1}{2} \times \frac{6}{100} \times \frac{15}{1,000} = 00045$ .

On the question of the rate of liability to active service in a "national war", they say: "After carefully weighing the "available data, and giving consideration to the conditions of "modern warfare, and the present strength and efficiency of "European armies and navies, and also to the diminished "mortality in respect of climate risks, we propose, for the "purposes of the report, to estimate that the British Army "and Navy will be engaged once in 40 years in a national "war, involving the calling out on active military service, for "a term not exceeding three years, of two-thirds of the whole

"force of officers and men in both branches of the Service." It is interesting to notice, in connection with this point, that Lord Rosebery, speaking some weeks ago before the Association of Chambers of Commerce, reminded his hearers that the country had arrived at the dangerous epoch, for it had not been engaged in a European war for forty years. Messrs. Smee and Ackland further assume that the mortality from "national" wars may be estimated at 20 per-cent of the total force engaged in the campaign. On this basis they calculate the total annual probability, in respect of a life now subject to the home or normal mortality, of death in active service in a "national" war at  $\frac{1}{40} \times \frac{2}{3} \times \frac{1}{5} = .00333$ .

The total probability of death in active service in time of war is thus 00045 + 00333 = 00378, corresponding to a deathrate of, sav, 3.8 per 1,000, which I shall adopt for the purposes of this paper.

We have now seen, first, that the extra death-rate among the active naval lives of the whole Force, rate from all during the 10 years 1886-1895, was 1.8 per 1,000 (Table C); second, that the extra death-rate among mixed naval lives, due to impairment of health among the active naval lives, may, judging from the experience of the same period, be fairly taken at 5 per 1,000; and, third, that the risk of death in active service in time of war may be taken as equivalent to a death-rate of 3.8 per 1,000. Adding together these rates as a sufficiently correct method of obtaining the desired result, we arrive at 1.8 + .5 + 3.8 = 6.1per 1,000, as the extra rate of mortality which may, for life insurance purposes, be considered as incident to service in the Navy.

Smee and Ackland's result.

This result differs somewhat from that arrived at Comparison with Messrs. by Messrs. Smee and Ackland in the report mentioned above. Their conclusion is, that the extra risks of service in the Navy and Army are the same, and may be taken as equivalent to an extra deathrate of 7.5 per 1,000, of which one-half, or  $3\frac{3}{4}$  per 1,000, represents the addition in respect of foreign service in time of peace (or climate risk), and one-half, or  $3\frac{3}{4}$  per 1,000, the addition in respect of active military operations (or war risk). My estimate is, therefore, lower than theirs by 1.4 per 1,000. This 1.4 is the difference between their estimate of the extra mortality in time of peace, say, 3.7 per 1,000, and mine, 1.8, less the addition, 5, which I have made for the extra mortality of lives invalided out of the Service. It is to be observed, however, that Messrs. Smee and Ackland's estimate is based on the experience of the years 1871–1887 inclusive, during which the average death-rate in the Service afloat was 9 per 1,000; while mine is based on the experience of the years 1886–1895, during which the average death-rate was 7 per 1,000.

Extra deathrates and invaliding rates on the Foreign Stations.

The tables given in the statistical reports referred to above, give the deaths, as well as the invalidings, for each station; and from the information thus furnished, the following Tables H and I have been

made up, to show the healthiness or unhealthiness of the various stations. Table H shows the average strength, deaths, and invalidings in each station during the ten years 1886–1895, also the deaths and invalidings from injuries. Table I gives the death-rates and invaliding-rates per 1,000 (1) from all causes, and (2) from causes other than injuries; also the extra invaliding-rates and extra death-rates from causes other than injuries; such extra rates being taken as equal, in the case of each station, to the excess of the death-rate or invaliding-rate over the corresponding rate for the home station.

Table H.—Average Strength, Deaths, and Invalidings on the various Stations, in the 10 years 1886–1895.

|                               |                     | Di    | EAD              | lnvalided |                  |
|-------------------------------|---------------------|-------|------------------|-----------|------------------|
| Station                       | Average<br>Strength | Total | From<br>Injuries | Total     | From<br>Injuries |
| (1)                           | (2)                 | (3)   | (4)              | (5)       | (6)              |
| Home                          | 27,546              | 1,663 | 431              | 6,360     | 463              |
| Mediterranean                 | 7,732               | 817   | 441              | 3,174     | 183              |
| North America and West Indies | 2,670               | 137   | 57               | 636       | 50               |
| South-East Coast of America.  | 579                 | 35    | 22               | 123       | 12               |
| Pacific                       | 1,601               | 68    | 24               | 370       | 37               |
| West Coast of Africa and Cape |                     |       |                  |           |                  |
| of Good Hope                  | 2,290               | 184   | 59               | 835       | 90               |
| East Indies                   | 2,018               | 193   | 37               | 1,145     | 81               |
| China                         | 3,745               | 282   | 107              | 908       | 57               |
| Australia                     | 2,140               | 141   | 61               | 444       | 52               |
| Irregular                     | 5,309               | 390   | 171              | 919       | 101              |
| Total Force                   | 55,630              | 3,910 | 1,410            | 14,914    | 1,126            |

Table I.—Death-rates and Invaliding-rates per 1,000 on the various Stations, in the 10 years 1886-1895, showing also the Death-rates and Invaliding-rates from causes other than injuries.

|                               | DEAT            | H-RATE P                   | ER 1,000                              | lnvalii               | ING-RATI                   | EPER 1,00                              |
|-------------------------------|-----------------|----------------------------|---------------------------------------|-----------------------|----------------------------|--|
| Stations                      | From all Causes | Ex-<br>cluding<br>Injuries | Extra<br>due to<br>Climate<br>(3)-4.5 | From<br>all<br>Causes | Ex-<br>cluding<br>Injuries | Extra<br>due to<br>Climate<br>(6)-21.4 |
| (1)                           | (2)             | (3)                        | (4)                                   | (5)                   | (6)                        | (7)                                    |
| Home                          | 6.0             | 4.5                        |                                       | 23.1                  | 21.4                       |  |
| Mediterranean                 | 10.6            | 4.9                        | .4                                    | 41.0                  | 38.7                       | 17:3                                   |
| North America and West Indies | 5.1             | 3.0                        |                                       | 23.8                  | 21.9                       | •5                                     |
| South-East Coast of America . | 6.0             | 2.2                        |                                       | 21.2                  | 19.2                       |  |
| Pacific                       | 4.2             | 2.7                        |                                       | 23.1                  | 20.8                       |  |
| West Coast of Africa and Cape | 1               |                            |                                       |                       |                            | į                                      |
| of Good Hope                  | 8.0             | 5.5                        | 1.0                                   | 36.5                  | 32.5                       | 11.1                                   |
| East Indies                   | 9.6             | 7.7                        | 3.2                                   | 56.7                  | 52.7                       | 31.3                                   |
| China                         | 7.5             | 1.7                        | .2                                    | 24.2                  | 22.7                       | 1.3                                    |
| Australia                     | 6.6             | 3.7                        |                                       | 20.7                  | 18.3                       |  |
| Irregular                     | 7.4             | 4.1                        |                                       | 17:3                  | 15.4                       |  |
|                               |                 |                            |                                       |                       |                            |  |
| Total Force                   | 7.0             | 4.5                        |                                       | 26.8                  | 24.8                       | 3.4                                    |

Table I shows that a heavy invaliding-rate has prevailed on the Mediterranean Station, and heavy death-rates and invaliding-rates on the East Indies Station. The high rates on the Mediterranean Station appear to be due to the insanitary condition of many of the Mediterranean towns and harbours. Considerable improvement has been effected in regard to this of late years; and if the period of time covered by Table I be divided into two equal parts, the extra invaliding-rate is 22.0 per 1,000 in the five years 1886–1890, and 14.2 per 1,000 in the five years 1891–95.

On the East Indies Station, the extra death-rate increased from 2.7 per 1,000 in the first period of five years to 4.1 in the second period. The extra invaliding-rate, on the other hand, fell from 43.4 in the first period to 16.5 in the second. If we assume that the rule stated above is applicable to the East Indies Station, and that the extra rate of mortality for insurance purposes may be approximately obtained by adding together the death-rate and 10 per-cent of the invaliding-rate, we obtain for this station an extra rate of mortality due to climate of 2.7 + 4.3 = 7.0 per 1,000 in the five years 1886-1890, and 4.1 + 1.7 = 5.8, in the five years 1891-1895.

## ARMY.

Death-rates for last 40 years. The table in the 58th Annual Report of the Registrar-General for England, relating to the mortality of the Army, gives the strength in each year from 1868 to 1895 inclusive, with the corresponding deaths and death-rates per 1,000. These figures are given for the total Army, the Army in the United Kingdom, and the Army abroad, respectively. From this table, with the addition of figures for the years 1866 and 1867, taken from the 56th Report, I have made up the following Table J, which contains the average strength, total deaths, and average death-rate per 1,000, for three periods of 10 years each, ending with the year 1895.

Table J.—Strength and Mortality of the British Army.

|  |                                       | 1866-75              | 1876-85             | 1886-95            |
|--|---------------------------------------|----------------------|---------------------|--------------------|
| Average  | In United Kingdom . Abroad            | 94,263<br>96,746     | 94,624<br>95,080    | 105,380<br>107,015 |
| Strength   | Total Army                            | 191,010              | 189,704             | 212,395            |
| Total  | In United Kingdom . Abroad            | 9,970<br>16,334      | 7,696<br>17,026     | 5,746<br>14,220    |
| Deaths   | Total Army                            | 26,304               | 24,722              | 19,966             |
| $\begin{array}{c} \text{Death-rate} \\ \text{per} \\ \text{1,000} \end{array} \bigg\}$ | In United Kingdom . Abroad Total Army | 10·6<br>16·9<br>13·8 | 8·1<br>17·9<br>13·0 | 5·5<br>13·3<br>9·4 |

It will be observed that a remarkable decrease has taken place in the rate of mortality of the army in the United Kingdom; the rate having decreased from 10.6 in the first period included in Table J, to 5.5 in the last.

The Parliamentary papers containing the results of death-rates. each of the last four English censuses, contain tables showing the number of men of all ranks serving in the British Army at the date of the census, classified according to age. These tables have been used to calculate the expected deaths in a year, at the date of each census. For reasons similar to those explained in the case of the Navy, I have used

Dr. Sprague's Select-Life Tables (first year of insurance), in the calculations. The results, with the corresponding deathrates per 1,000, are given in the following Table K:

Table K.—Number of men of all ranks serving in the British Army, at each of the last four censuses, with the corresponding expected deaths in a year, and death-rates per 1,000, according to the Select-Life Tables.

| Year Number of enumerated |          | Expected<br>Deaths<br>in a Year | Death-Rate per 1,000<br>$1,000 \times \frac{(3)}{(2)}$ |  |
|---------------------------|----------|---------------------------------|--|--|
| (1)                       | (2)      | (3)                             | (4)  |  |
| 1861                      | 229,518* | 1,077                           | 4.7  |  |
| 1571                      | 193,607  | 907                             | 4.7  |  |
| 1581                      | 186,428  | 965                             | 5.2  |  |
| 1891                      | 222,859  | 1.044                           | 4.7  |  |

Comparison of actual and expected death-rates.

The following Table L gives a comparison of the actual and expected deaths in the total Army, as given in Tables J and K respectively.

Table L.—Average annual Death-Rates per 1,000 in the Army, compared with the Rates to be expected according to the Select-Life Tables.

| 10 Years<br>ending<br>31 December | Actual<br>Death-Rate | Expected<br>Death-Rate | Excess of<br>Actual Rate<br>(2)-(3) |
|-----------------------------------|----------------------|------------------------|-------------------------------------|
| (1)                               | (2)                  | (3)                    | (4)                                 |
| 1875                              | 13.8                 | 4.7                    | 9.1                                 |
| 1885                              | 13.0                 | 5.2                    | 7.8                                 |
| 1895                              | 9.4                  | 4.7                    | 4.7                                 |

Statistics in the Army Medical Reports. The figures given in the tables by the Registrar-General, upon which Tables J and K are based, are stated to be furnished by the Commander-in-Chief.

The figures relating to the average strength of the Army correspond with those given in the General Annual Return of the British Army. They include officers, warrant-officers, non-commissioned officers and men, and appear to include also the colonial corps. The figures relating to strength and mortality given in the yearly reports of the Army Medical Department, relate to warrant-officers, non-commissioned officers and men, and do not include the colonial corps. I have,

<sup>\*</sup> Excluding 44 whose ages were not stated.

nevertheless, thought it well, following the course adopted in the case of the Navy, to carefully examine the statistics given in these reports for the ten years 1886–1895. I have not, however, thought it necessary to make up tables showing the death-rates at different ages. Considerable labour would be required to make up such tables from the information given in the reports, and, indeed, I doubt whether the necessary materials are given.

The average strength of the Army at home and abroad, during the ten years 1886–1895, as given in the Army Medical Reports, is 197,817. The total deaths amount to 18,262, being at the rate of 9.2 per 1,000 per annum. This corresponds pretty closely with the figure given in Tables K and L, namely 9.4.

The Army Medical Reports give, for the United Kingdom and each of the foreign commands, the number discharged as invalids each year. The total number so discharged during the ten years 1886–1895, was 28,400, being at the rate of 14·4 per 1,000 per annum. This corresponds to what I have termed the rate of impairment in the Navy, 17·1 per 1,000.

Comparison with rate of impairment in the Navy. The seaman class in the Navy is kept up to its strength by the entry of boys at ages 15 or 16, who must agree to serve for 12 years from the age of 18. In the Army, on the other hand, the mass of men enter for seven years' service with the colours, or eight years if the period of Army service expires when the man is abroad. As might be expected, the average age of the men in the Navy is greater than of those in the Army. Thus the expected death-rates per 1,000 per annum in the two Services in 1891, compare as follows:

|      |      |      | Select     | Institute H <sup>M</sup> |
|------|------|------|------------|--------------------------|
| Navy | <br> | <br> | $5\cdot 2$ | 8.1                      |
| Army | <br> | <br> | 4.7        | 6.9                      |

In these circumstances, it is quite to be expected that the rate at which men are invalided out of the Army should be less than the coresponding rate for the Navy. As it happens, the proportion between the rates of invaliding is exactly the same as that between the corresponding death-rates according to the Institute  $H^M$  Table,  $17\cdot1\div14\cdot4$  being equal to  $8\cdot1\div6\cdot9$ .

It is not practicable to ascertain the extra invalidingrate in quite the same way as was done in the case of the Navy. The bulk of the Army consists of the territorial regiments. Each of these consists of two battalions, one of which is-or is supposed to be-at home, and the other Soldiers are trained in the home battalion, which has to send each year a draft of trained and serviceable men to fill the vacancies in the foreign battalion, caused by men being sent home on the expiration of their service with the colours or as invalids or otherwise. In these circumstances, the best way that occurs to me of obtaining a normal rate of invaliding for the Army, similar to that furnished by the home station in the case of the Navy, is to assume that the experience of the Gibraltar command furnishes such a rate. The rule is not to send soldiers under 20 years of age to foreign stations. But of late years, the pressure put upon the Army has made it necessary to consider Gibraltar, for this purpose, a home station. This is one reason for supposing that the troops stationed there during the ten years 1886-1895, afford, as regards age and length of service, a fair sample of the whole Army. This view is confirmed, when we observe that the rate of mortality at Gibraltar during the period above mentioned was 4.5 per 1,000, as against a general expected rate of 4.7 (Table K). The rate of impairment at Gibraltar during the period above mentioned, was 8.5 per 1,000 against 14.4 for the total Army. This gives an extra rate of impairment of 5.9 per 1,000.

Comparison with extra rate of impairment in the Navy.

By proceeding as follows, it can be shown that the rate is consistent with that obtained from the Navy. The ratio between the extra death-rate in the Navy

and the total rate, is  $\frac{1.8}{7.0}$ , and the ratio between the corres-

ponding rates of impairment is  $\frac{3.5}{17.1}$ . If the same proportion be assumed to exist between the corresponding quantities in the Army, we shall have—

$$\frac{1.8}{7.0} : \frac{3.5}{17.1} : : \frac{4.7}{9.4} : \frac{x}{14.4}$$

x being the rate of impairment in the case of the Army: this gives x=5.7. Putting the thing in another way, if the proportion between the extra rates of impairment in the Navy

and Army were the same as that between the total rates, the extra rate for the Army would be  $\frac{14\cdot4}{17\cdot1}\times3\cdot5=2\cdot95$ . But the extra death-rate in the Army bears a much greater proportion to the total rate than in the case of the Navy; and it is reasonable to expect a similar proportion in the case of the rates of impairment. Making allowance for this, the extra rate of impairment in the Army becomes  $\frac{4\cdot7}{9\cdot4}\div\frac{1\cdot8}{7\cdot0}\times2\cdot95=5\cdot7$ , as above.

In ascertaining what effect this extra rate of Extra deathimpairment among active Army lives, has on the rate of mixed mortality among mixed Army lives, I shall apply the same principle as in the case of the Navy. We have assumed that there is no extra mortality among the troops in the Gibraltar command; that is to say, that the death-rate among the active lives is in accordance with the Select-Life Tables, and that among the mixed lives, in accordance with the Institute H<sup>M</sup> Table. It follows that the rate of impairment among the active lives in the Gibraltar command, is such as to produce an increase in the death-rate among the mixed lives equal to the difference between the Select rate of mortality and the Institute HM rate. The Select rate is  $\frac{21.63}{4,683} = 4.6$  per 1,000, and the H<sup>M</sup> rate  $\frac{30.92}{4,683} = 6.6$ , and this gives an extra death-rate of 2.0 per 1,000 corresponding to a rate of impairment of 8.5. Consistently with what was done in the case of the Navy, I assume that the rate of impairment among the active lives of the Army, and the death-rate among the corresponding mixed lives, are in the ratio of 8.5 to 2.0; or, say, that a given rate of impairment among the active lives corresponds to a death-rate among the corresponding mixed lives, of 24 per-cent of such given rate of impairment. It follows that the extra death-rate to be expected among the mixed Army lives, as a whole, is  $24 \times 5.9 = 1.4$  per 1,000.

During the period of ten years under observation, the British Army was never engaged in active operations of any consequence. It will therefore be necessary, in determining what the extra mortality in the Army is likely to be in future, to make an estimate of the risk of liability to active service in time of war. I shall, as in the case of the Navy, adopt Messrs. Smee & Ackland's conclusion, and assume that the

risk of liability to active service in time of war may be taken as equivalent to a death-rate of 3.8 per 1,000.

We have thus ascertained first, that the extra Extra deathdeath-rate among the active lives of the total Army rate from all during the ten years 1886-1895, was 4.7 per 1,000 (Table L); second, that the extra death-rate among mixed Army lives, due to impairment of health among the active Army lives, may, judging from the experience of the same period, be fairly taken at 1.4 per 1,000; and third, that the risk of death in active service in time of war may be fairly taken as equivalent to a death-rate of 3.8 per 1000. Adding together these rates, we obtain 4.7 + 1.4 + 3.8 = 9.9 per  $1{,}000$ , as the extra rate of mortality which may, for life insurance purposes, be considered as incident to service in the Army.

Smee and Ackland's result.

The corresponding figure given by Messrs. Smee Comparison with Messrs. and Ackland is 7.5 per 1,000; so that my estimate exceeds theirs by 2.4 per 1,000. Of this, 1.4 represents the addition which I have made for the extra mortality among lives invalided out of the Service; while the remaining 1 per 1,000 is the difference between 4.7, my estimate of the extra mortality in time of peace, and 3.7, the corresponding figure given by Messrs. Smee and Ackland. The 3.7 is obtained by making the following assumptions, namely, (1) that the rate of mortality in the Army abroad in time of peace may fairly be represented by an average annual rate of 15 per 1,000; (2) that the home mortality is at the normal rate, and may be taken on the basis of the Institute of Actuaries' H<sup>M</sup> Table, at 7½ per 1,000; (3) that one-half of the Army will be engaged on foreign service at any particular time. The combined probability for a life, now stationed at home, of death in time of peace from causes due to climate

thus comes out  $\frac{1}{2} \times \frac{75}{10,000} = 00375$ , say 0037.

Now the two assumptions mentioned under (2) are neither of them consistent with the results I have obtained above. I think it has been clearly shown that the normal rate of mortality of active Army lives is much less than that shown by the Institute HM Table, and may fairly be taken as on the average equal to that shown by Dr. Sprague's Sclect-Life Tables. Again, although I have made no separate calculation of the expected rate of mortality of the Army at home during the period 1886-1895, I think it may be fairly concluded.

comparing the figures in Tables J and K, that it is distinctly higher than the normal rate, and having regard to the working of the linked battalion system, this is to be expected.

If I am correct, it follows that Messrs. Smee and Ackland. in taking the climate risk in time of peace at 15-7.5 or 7.5 per 1,000, have under-rated it; and that a more correct estimate would have been, say, 15-5=10 per 1,000; this would raise the extra mortality of the Army arising from foreign service in time of peace, during the period to which their investigation applies, to 5 per 1,000—the corresponding rate for the period 1886-1895 being 4.7 (Table L). Having regard to the fact that the mortality of the Army abroad, taken by Messrs. Smee and Ackland at 15 per 1,000, fell to 13.3 per 1,000 during the latter period, on which my results are based; we might look for a difference of  $\frac{1}{2}$  (15-13.3), or say, 8 between the extra death-rates in time of peace, instead of  $\cdot 3$  only  $(5-4\cdot 7)$ , the actual difference. It is to be remembered, however, that the method adopted by me, gives effect to the facts (1) that the mortality of the Army at home is somewhat in excess of the normal rate, and (2) that, from the increasing pressure upon the Army of late years, a somewhat larger proportion than one-half has been engaged on foreign service.

Comparative mortality among the officers from disease was much lighter than among the men. Having regard to the great improvements in the conditions of service for privates, and the consequent decrease in the general death-rate, this is not, I think, likely to be the case at the present time. The following Table M, made up from information given in the Army Medical Report for 1886, gives information as to the comparative mortality among officers and men in India.

Table M.—Mortality and Invaliding in India, in the ten years 1875–1886.

| Rank   | Average<br>Strength      | Average<br>Number of<br>Deaths<br>in a Year | Average<br>Number<br>Invalided<br>in a Year | Death-<br>Rate per<br>1,000<br>(3) ÷(2) | Invaliding-<br>Rate<br>per 1,000<br>(4) ÷ (3) |  |
|--|--------------------------|---|---|---|---|--|
| (1)  | (2)                      | (3)   | (4)   | (5)                                     | (6)   |  |
| Commissioned Officers Non-Commissioned Officers Privates | 2,292<br>6,975<br>51,830 | 38·80<br>147·00<br>865·60                   | 132·30<br>209·10<br>1,926·30                | 1·7<br>2·1<br>1·7                       | 5·8<br>3·0<br>3·7                             |  |

Death-rates and invalidingrates in the different commands. The tables given in the Army Medical Reports show the number died and discharged as invalids, in each of the commands in which the troops were stationed; and, from the information thus furnished,

the following Table N has been made up. Table N shows the average strength, number died, and number discharged as invalids in each command during the ten years 1886-1895, also the death-rates and rates of impairment per 1,000.

Table N.—European Troops—Average Strength, Number Died and Number Discharged as Invalids, in the ten years 1886– 1895, with the corresponding Death-Rates and Rates of Impairment.

| Command               | Average<br>Strength | Died   | Discharged<br>as<br>Invalids | Death-rate<br>1er 1,000<br>(3) :- (2) | Rate of<br>Impairment<br>Per 1,000<br>(4) ~(2) |
|-----------------------|---------------------|--------|------------------------------|---------------------------------------|--|
| (1)                   | (2)                 | (3)    | (4)                          | (5)                                   | (6)  |
| United Kingdom        | 99,519              | 5,057  | 16,154                       | 5.1                                   | 16.2   |
| Gibraltar             | 4,686               | 211    | 400                          | 4.5                                   | 8.5  |
| Malta                 | 6,907               | 515    | 756                          | 7.5                                   | 10.9   |
| Egypt and Cyprus .    | 5,300               | 864    | 788                          | 16.3                                  | 14.9   |
| Canada                | 1,373               | 59     | 172                          | 4.3                                   | 12.5   |
| Bermuda               | 1,383               | 148    | 133                          | 10.7                                  | 9.6  |
| West Indies           | 1,159               | 107    | 157                          | 9.2                                   | 13.5   |
| South Africa and St.  |                     |        |                              |                                       |  |
| Helena                | 3,381               | 223    | 508                          | 6.6                                   | 15.0   |
| Mauritius             | 519                 | 85     | 95                           | 16.4                                  | 18:3   |
| Ceylon                | 1,204               | 137    | 142                          | 11.4                                  | 11.8   |
| China and the Straits |                     |        |                              |                                       | 9  |
| Settlements           | 2,543               | 241    | 281                          | 9.5                                   | 11.0   |
| India                 | 67,324              | 10,451 | 8,814                        | 15.5                                  | 13.1   |
| On Board Ship         | 2,522               | 164    |                              | 6.2                                   |  |
| Total                 | 197,817             | 18,262 | 28,400                       | 9.2                                   | 14.4   |

In the case of Egypt and Cyprus, the death-rate fell from 21.6 in the five years 1886–1890 to 9.7 in the five years 1891–1895, while the corresponding rates of impairment were 17.8 and 11.2 respectively. The death-rate in the separate Egyptian command, in the year 1886, was 36.9 per 1,000, of which 21.8 was due to enteric fever, caused principally by the bad sanitary condition of the towns. The improvement in the death-rate and rate of impairment in later years, appears to be due to the practice of sending out corps composed, as far as practicable, of seasoned men, and to improvement in the general sanitary conditions under which the soldier is placed.

In the case of Bermuda, the death-rate decreased from 12.4 per 1,000 in 1886–1890, to 9.2 in 1891–1895, while the rate of impairment decreased from 13.6 to 6.0.

## Conclusion.

My final conclusion as to the average extra rates of mortality which may for life insurance purposes be considered as incident to service in the British Navy and Army, is, that these may be fairly taken at 6 per 1,000 for the Navy, and 10 per 1,000 for the Army. It would not, of course, be prudent to fix on this basis the extra premiums for lives in any of the unhealthy stations or commands, or engaged in dangerous service; nor for lives which have more than the usual prospect of being ordered to an unhealthy station or on dangerous service. I do not propose to enter, in this paper, upon the further question of what are the extra premiums corresponding to these rates, which ought to be charged for the different kinds of policies.

## LIST OF BOOKS AND PAPERS REFERRED TO.

Fifty-sixth and Fifty-eighth Annual Reports of the Registrar-General of Births, Deaths, and Marriages in England.

Census of Great Britain, 1851. Population Tables II, Vol. I.

Census of England and Wales, 1861. Vol. III. General Report.

Census of England and Wales for the year 1871. General Report. Vol. IV.

Census of England and Wales, 1881. Vol. IV. General Report.

Census of England and Wales, 1891. Vol. IV. General Report.

Statistical Reports of the Health of the Navy for the years 1886 to 1895 inclusive.

Army Medical Department Reports for the years 1886 to 1895, inclusive.

General Annual Return of the British Army for the year 1896.

Paper in the Journal of the Institute of Actuaries for October 1884, "On the Rate of Mortality in the Navy", by Dr. T. B. Sprague. (Vol. XXV, p. 49.)

Joint Report to the Board of Directors of the Gresham Life Assurance Society, "On the Assurance Risks incident to Professional Military and Naval Lives", by A. H. Smee and Thomas G. Ackland. London, 1890.

#### Discussion.

The President said the paper was evidently one of those wellconducted and carefully arranged statistical enquiries, which always proved of practical value for deductive purposes to the profession. He considered it was simply and exclusively the fault of the data themselves—which, like all other Government statistics, ought first to have applied to them the actuary's art for the purpose of elucidating their full, and especially their discriminative, value—that we did not find that distinction of facts relating to officers and men which would enable actuaries to employ the results with confidence in connection with their practical problems of extra premiums. Referring to the experience of the navy, and looking to the social rank to which the ordinary seamen belonged, and to the nature and social scale of the pleasures in which they were apt to indulge in port, he was disposed to question the suggestion made by the author in connection with what he termed the extra rate of impairment.

Mr. H. W. Manly thought there were two sections of the paper to which special reference might be made. First, there was that portion in which Mr. McLauchlan discussed the suitability of Dr. Sprague's Select Life Tables as a standard of measurement of the normal home rate of the mortality in the navy. There was no doubt that the examination, although a very careful one, of the candidates for the services, was not quite so severe as the one which people were put through when they applied for insurance. Also, as the President had suggested, the pleasures in which seamen indulged in times when they were in ports must of necessity increase the impairment beyond what was experienced in insurance business. Perhaps, on the other hand, the anxiety of a delicate or unhealthy person to enter the service was not quite so great as that of the same class of person to get insured, nor was there the same temptation to deceive the doctor -a deception which, in life assurance, he thought sometimes was successful. These two considerations probably counterbalanced each other; and if, at the outset, as Mr. McLauchlan seemed to consider, Dr. Sprague's Select Life Tables did fairly represent the experience, he thought that the deterioration would be so rapid that the rate of mortality in the navy would necessarily, even after excluding those who were invalided, soon cease to be represented by the select rates of mortality for the first year of insurance. Then, he thought, careful attention should be given to the manner in which Mr. McLauchlan had deduced the extra rate of impairment and the extra rate of mixed lives. The methods described were deserving of very careful study. To his mind it was an admirable piece of deductive reasoning. The arrangement of the data was ingenious, and, in his judgment, sound. Coming to the practical conclusions to be drawn from the investigation, Mr. McLauchlan hesitated to say what ought to be charged, and he thought Mr. McLauchlan was right. Circumstances must necessarily differ according to the time and place, and judgment must be exercised as to what should be charged for each case as it arose. If the whole of the navy and the whole of the army were to ask to be insured, actuaries could at once fix a common rate for the whole, but the selection was always against the office. The

officer who expected to be ordered to an unhealthy climate was more desirous to insure than those who led a pleasant life at home; and he was always anxious to persuade the company that the station to which he was ordered should be classified as a health resort, no matter where it was. It was impossible to have a rate under such circumstances which could be universally applied. Another point had occurred to him which Mr. McLauchlan had not referred to. It was now a very common practice with the offices, to cover all risks of climate and war by a commuted annual charge, to be paid so long as the insured remained in the service, and if that rate were made up of the various extras which Mr. McLauchlan said the services were subject to, and it certainly should be, there was one portion, a rather large portion, which was intended to cover the deferred risk of a great war. In the navy it amounted to about 6s. Sd. per-cent, and about 7s. 6d. in the army. Would it not be wise when that commuted annual extra premium was charged, to set aside that portion representing the deferred risk to accumulate against the time when the risk would actually be incurred? He thought it would be a wise precaution to take, for otherwise if a large war did break out and the offices were suddenly called upon to pay the insurances on which they had previously received such commuted extra premium it might very much affect the surplus at the end of the

quinquennium.

Mr. George King thought the subject of the paper a very difficult one to speak on, because it was very complicated, and the necessary data for deducing sound conclusions were wanting. agreed with what Mr. Manly had said as to the very elegant methods and ingenious hypotheses put forward by Mr. McLauchlan. And if one could not altogether accept the conclusions as sound it was not the fault of the author. Both the army and the navy were in a period of transition, and the figures brought forward by Mr. McLauchlan amply proved it. At the conclusion of his paper the author said that the facts showed throughout the great reduction that had of late years taken place in the mortality of disease, and that was a matter on which the authorities might very well congratulate themselves. It showed that the science of health was making most decided progress. From other quarters the same facts were gathered. The mortality of the army in India in the early fifties was something like seventy per 1,000; he did not quite know what the figures were reduced to now, but they were well below twenty. It was the same thing in other districts. But there was also a transition state from another point of view, and that was that it was not possible to forecast what the mortality would be in any future war either with regard to the navy or the army. There had been no great war since weapons of precision were so much perfected, and since the fleet had been entirely revolutionized, and the wooden ship had given place to the ironclad. What would happen in the next great naval battle none could say. It was quite possible that all the ironclads would float, but it was also possible that they might all go to the bottom. The result could not really be forecast. Consequently from statistics of the past no sound conclusions could be drawn as to the future. The paper showed the difficulties that were met with from those circumstances, and

Mr. McLauchlan had set himself in the most able manner to overcome them, but he was afraid the difficulties were really iusuperable, and while the paper was a most useful one from the point of view of showing how such enquiries ought to be set about, he very much doubted himself whether any financial results deduced from it would be of any great weight. He had intended to sav a few words with regard to the two methods which had been followed for arriving at the extra mortality, namely, that of using the select tables and that which Mr. McLauchlan had introduced of taking the home station, and making that the basis of his comparison, but Mr. Manly had already taken up that ground, and there was little left to say. He might, however, remark that as to the select table he doubted very much whether the first year of insurance was a suitable one to adopt. It showed too light a rate of mortality, and probably the extra mortality derived therefrom was an over-statement. He did not think that limiting the assured lives to one year was a proper arrangement, because it was known that in that year the rate of mortality was exceptionally low, and he did not think that the conditions of the army and navy were such as to bring selection into such very marked play. With regard to using the home station, that did not have the same objection, and probably Mr. McLauchlan's conclusions derived from that were much more to be trusted. Nevertheless, he doubted very much whether the impairment in the army and navy were quite comparable to the damaged lives amongst the insured, for the simple reason that what would render a man quite incapable of further service need not necessarily diminish the value of his life. The loss of a hand, for instance, would prevent either a soldier or a sailor from being again employed, but would not in any way interfere with the longevity of the life. Therefore damaged lives were really much worse from a life assurance point of view than the impaired lives in the army and navy would be on the average. It would be very difficult of course to analyze the returns so as to deduce the correct figures from that point of view, and he merely pointed out the difficulty—not the difficulty arising from Mr. McLauchlan's treatment, but the innate difficulty of the subject. Then, again, he would ask whether the HM Table was really a suitable one for comparisons, and whether it could be called an exponent of the true mortality amongst insured lives? Personally, he did not think so. In the early years of life, up to, perhaps, age 35, the mortality was very considerably under-stated on account of the admission of newlyselected lives. On the other hand, at the older ages, from above ages 45 or 50, the mortality was over-stated through the survival of the damaged lives entering earlier, and there was an undue proportion of damaged lives in that part of the aggregate table. It was difficult to say what table should be employed, and the more the question was gone into the more hopeless did it seem to get anything in the way of trustworthy statistics. A man with little knowledge thought it very easy—simply take a mortality table that suited, and apply it. But the more experienced men knew the great difficulties in selecting such a table, and they knew the faults of even the most carefullyconstructed tables; not faults of construction, but faults arising from the data themselves—faults in principle that seriously vitiated the

conclusions. From the life office point of view, it seemed to him that the statistics of the whole forces were not suitable, but it would be much preferable to get, if it were possible, the statistics of the officers alone. It was not the men that came for insurance, but the officers, and it was to the officers' mortality, therefore, that companies would look. He doubted very much whether the mortality of officers, in the army, at any rate, was comparable in the aggregate with that of the men. There was no such thing as short service among the officers. They entered the army or the navy to follow the profession as a career, and the average age, therefore, must be considerably higher than that of the men. From Mr. McLauchlan's tables it was seen that a very large proportion in the service was composed of very young men, but that would not be so among the officers—they, on the average, would be much older, and therefore the normal rate of mortality would be much higher with them, and unless in the measure of the extra rate this was specifically allowed for, the conclusions would be vitiated. He wished it had been possible to give more attention to the officers. There were only a few lines devoted to them in the paper, and if Mr. McLauchlan had an opportunity to go into that question further the Institute would welcome another contribution from him.

Mr. T. G. ACKLAND said the Institute was much indebted to Mr. McLauchlan for a very useful paper on a practical subject. The subject was one of peculiar difficulty, and that difficulty arose not only from the absence (or the paucity) of data in the official returns, but also from the obstacles in the way of forming a judgment as to many of the factors which were involved. It seemed very desirable that offices should, if possible, provide, by way of an equal annual charge, for the extra risks of professional military or naval lives, and so avoid the inconvenience of heavy, and perhaps unequal, extra premiums. But at the same time, the actuary who faced the problem had to "make bricks without straw", because he found the data very deficient, although, he thought, less so of later years. In many matters, also, the personal equation involving questions of pure judgment and chance, had inevitably to come in. Mr. McLauchlan had furnished valuable conclusions based upon thirty years of official returns, and they appeared to strengthen somewhat the opportunities for arriving at a reliable judgment. One very marked result indicated in the tables was the materially diminishing mortality among the military and naval lives of late years. Ten years ago it would not have been in any way permissible to assume that the rate of mortality in the army, in the home stations, was comparable with the select H<sup>M</sup> Table; but apparently the decline had been such that, in some cases and at some groups of ages, the rate experienced was actually below that of Dr. Sprague's Select Tables. He thought also that the data now available as to the invaliding of lives were The question of invalidity was of very great importance, and the difficulty that arose, which Dr. Sprague had pointed out, and which must have occurred to many others, was that they could not in any way follow up the lives which left the service and never returned to it so as to deduce the "rate of impairment", as Mr. McLauchlan had most happily termed it. There was a

necessity for making some assumptions as to this, and Mr. McLauchlan had boldly and skilfully faced the question, and, if he understood him, had assumed that the minimum rate of mortality, not necessarily in the home station, but at a normal station, accorded with that of the Select HM Table, and that the rate of impairment arising from the mortality of those lives so stationed who permanently left the service might be represented by the difference between the Select and Ordinary H<sup>M</sup> Tables. He had not been able to see the evidence or the data upon which Mr. McLauchlan had arrived at that particular conclusion, that the measure of the invalidity lies between those two tables; undoubtedly it was a very happy method of dealing with the difficulty, if it could be considered as a final solution, but it was a question whether fresh investigations, if the Government were able and would be good enough to furnish the data, would not somewhat vary those conclusions as to the true measure of the risk in respect of invalidity. The assumption made by Mr. McLauchlan as to the extra mortality in respect of those who left the service invalided from foreign ports, whether in the army or navy, was based upon the initial assumption, and the two would stand or fall together. The author's final conclusions were that in the navy the extra mortality was somewhat less than had been previously deduced, and in the army greater, rising to an extra rate of about ten per 1,000. He thought the members were prepared to find that in the navy the rate was somewhat low, but personally he should consider that in the army the rate of ten per 1,000 distinctly over-stated the extra mortality. This was, he thought, practically equivalent to an extra premium of about 16s. per-cent per annum, payable until retirement, which was somewhat above the average rate charged, so far as he was aware, by leading life offices. In considering these conclusions they had to remember that Mr. McLauchlan had dealt with a very limited body of data, and he mainly formed his conclusions from the years 1886 to 1895, undoubtedly the most recent obtainable but of a very special character as regards the rate of mortality experienced. One could not help thinking that (as Mr. King had suggested) the present was a transitional period, and that the experience might be somewhat varied later on. Mr. McLauchlan had been good enough to refer to a report of 1890, and to mention his name in connection therewith. He had the honour to assist in arriving at some results, included in that report, in connection with the present problem. thing that surprised him was that Mr. McLauchlan was so kind as to accept the conclusions arrived at as to the estimates made in that report for "small" and for National wars, because they were confessedly purely matters of judgment, and while it seemed quite essential to form some conclusion on the subject, the particular conclusions arrived at would, he thought, almost necessarily vary with the computer who took the question in hand. In connection with Mr. Manly's remark on war reserves, he happened to know that in some continental assurance companies, where an extra premium was charged annually to cover the risks of war, the accumulations appeared in the balance sheet as a reserve fund, set aside in anticipation of future payments. He questioned whether Mr. Manly had quite correctly interpreted Mr. McLauchlan in his reference to the mortality experienced amongst naval lives. He (Mr. Ackland) had, in a measure, understood Mr. McLauchlan's conclusion rather to indicate that the vitality of the navy was actually superior at the outset to that of Dr. Sprague's Select Tables, but it might be that he was under some misapprehension in that respect.

On the motion of the President, a cordial vote of thanks was

passed to Mr. McLauchlan for his paper.

Mr. McLauchlan, in reply, thanked the members for the kind reception they had given the paper. Although he had not the honour to be a member of the Institute of Actuaries, he had always taken the greatest interest in its proceedings. Mr. Manly had referred to the question of the difference between the rate of mortality to be expected among the officers and among the men respectively, and as he had said, the statistics that were given by the Government did not afford the means of obtaining separately the mortality in the case of officers. In the old times—even ten years ago—the mortality of the whole force was not a good guide to the mortality to be looked for among lives effecting insurances, but it appeared that a very great improvement had taken place in the conduct of the men and the care which was taken of them, and therefore in the mortality amongst them. It was not, of course, so satisfactory to have to take the mortality of the whole force as the basis for calculating the extra premium to be charged in the ease of an officer, yet the mortality of the whole force had gradually become such as to be a much better guide in that way than it formerly was. With regard to the remarks that had been made as to the comparison of the mortality in the different age groups with that of the select tables, he certainly thought that the mortality at the outset in the navy was considerably lighter than that of select lives in the first year of insurance. He believed the examination of men or boys applying to join the navy was exceedingly stringent. It was stated in the House of Commons a few weeks ago that a very large proportion of the applicants were rejected. Not only was a sound constitution required, but physical activity was also necessary. He believed that amongst other things, great attention was paid to the subject of a man's teeth, and a considerable number were rejected because their teeth were not up to He believed a candidate must be able to cat a ship's the standard. biscuit. Mr. Manly had referred to the question of the office premiums to be charged, and there was a great deal of truth in what he had said as to the selection that existed against the office. He had no doubt that an officer in the army proposing for insurance had a better idea than the officials of the company as to his prospects of being ordered abroad. They would require to be exceedingly familiar with the regulations and practice of the army in order to be in as good a position as the officer himself to judge of his chances of going on dangerous service, and he thought that this kind of selection must certainly tell against the office in the long run. Mr. King had referred to the present being a transitional period, not only with regard to the attention paid to the health of the men in the two services, and the consequent mortality amongst them, but also in regard to the manner of carrying on operations of war, particularly in the navy. With regard to the former point, it occurred to him that any person who took the trouble to examine into the subject would be able to say what the causes were that had contributed to the great improvement in the rate of mortality, and his general impression was that that improvement was not likely to go much further. With regard to what the next naval war was to be like, of course nobody knew what would happen—whether the ironelads would all sink one another, or, on the other hand, whether the proportion of easualties would be pretty much the same as it had been in the old times. His own idea was that, in the absence of anything better, the assumption made in Messrs. Smee and Ackland's report was a reasonable one, and was as good as any that could be made. Mr. Ackland had remarked that he had come to the same conclusion as himself on the subject of the risk of liability to active service in time of war. This was a difficult subject, and he must say that he did not see his way to do anything better than Mr. Ackland had already done. That gentleman had collected his facts from a wide area, and had had exceptional experience in connection with the subject, and he felt that he could not do better than adopt his estimate. withstanding what Mr. King had said, he thought that estimate was such as might—in the absence of anything better—be properly adopted for practical purposes at the present time. Of course, as regarded the whole paper, he quite admitted it could only be considered as a contribution to the subject. There were a variety of points in connection with it that remained to be further investigated and discussed, and all he could hope was that the paper might for a time be of some assistance to the profession, and that the subject might by and by be taken up and carried further by some other actuary. It seemed highly probable that a war was now about to be waged which would furnish a new and important set of facts. and perhaps upset some of the conclusions derived from the experience of the past. Mr. King had asked whether, as the ages of the officers were on the average greater than those of the men, he had calculated the expected deaths, having due regard to that fact? In reply, he would say that the expected deaths were calculated from information given in the census returns, that those census returns included, in the ease of both services, the whole force, officers and men together. Due allowance was therefore made for the fact that the ages of the officers were greater than those of the men. In the Army Medical Reports the figures did not include the officers at all, but this made a very small difference in the death-rate of the whole force. Mr. King's remark on the differences existing between the lives invalided out of the army or navy and the damaged lives of the select tables was correct. Men were invalided out of the navy and army for a variety of causes that did not exist in the case of the select tables, and no doubt one of the most important causes was that which Mr. King had referred to-namely, a failure of physical strength, or some disablement or other which would make it impossible for the man to serve, whereas it would not necessarily shorten his life.

## THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, APRIL 1898.

Examination for Admission to the Class of Associate (Part I).

Examiner—Prof. S. L. Loney, M.A. Supervisors—Messrs. W. Hughes and G. J. Lidstone.

## First Paper.

1. A landlord has an estate which brings him in £4,000 a-year, but the repairs, &c., cost him 15 per-cent of the gross rental. He sells the estate at 30 years' purchase on the gross income, and invests the price in  $2\frac{\pi}{4}$  per-cent Consols at  $112\frac{\pi}{4}$  (brokerage  $\frac{1}{8}$ ). Find to the nearest penny the resulting difference in his net income.

+2. If 
$$p=x-\frac{1}{x}$$
 and  $q=y-\frac{1}{y}$ , prove that 
$$pq+\sqrt{(p^2+1)(q^2+4)}=2\left(xy+\frac{1}{xy}\right).$$

3. Solve the equations:

(1) 
$$\frac{10x+17}{18} - \frac{12x+2}{13x-16} = \frac{5x-4}{9};$$

and

(2) 
$$x^2 + xy + y^2 = 109$$
  
 $x - y = 2$ .

4. Find the sum of n terms of an Arithmetical Progression of which a is the first term and d the common difference.

If b+c, c+a, and a+b are in H.P., prove that  $a^2$ ,  $b^2$ , and  $c^2$  are in A.P.

- 5. The men in a regiment can be formed in a solid square and also in a hollow square four deep. The number of men in the front of the latter formation exceeds twice the number in the front of the former formation by four. Find the number of men in the regiment.
- 6. Find the number of permutations of n things taken all together, when p are of one kind, q of a second, and r of a third.

A Committee of eight is to be elected from 14 men, of whom six are British and eight Canadians. In how many ways can the Committee be selected so that the British may not be outnumbered?

1898.7

7. Find the present value of an annuity of £P, the first payment to be made at the end of p years from the present time and to continue for q years.

A man has to pay £10,000 at the end of 12 years from now, and he has £2,000 in hand; what sum of money must he save per annum for the next 12 years so that, by investing all his capital at 4 per-cent, he may be able to pay his debt at the proper time, his year's savings being supposed to be invested at the end of each year.

Log 104 = 2.0170333, and log 1601.03 = 3.2043996.

8. Assuming the expansion of  $\log_{e}(1+x)$ , show how to calculate the values of logarithms, and find the value of  $\log_{e}2$ .

For all ordinary rates of interest prove that the time in which a sum of money will double itself at compound interest is very nearly  $\frac{70}{r} + 35$  years, where r is the rate per-cent per annum.

- 9. Given  $\log 2 = 30103$  and  $\log 3 = 4771213$ , find after how many years the number of trees in a plantation will be less than one-third of the original number if, in each year, there be cut down one-sixteenth of the number standing at the commencement of that year.
- 10. A and B play a match at a certain game, the winner being the one who first wins three games, no games being drawn. A's chance of winning any particular game is three-fifths. Find the chance of his winning the match after B has won the first game, and prove that the odds against him are nearly 11 to 10.
- 11. Given one solution in integers of the equation ax + by = c, find the general solution.

In how many ways can £100 be paid by using half-crowns and sovereigns only?

12. If the sum of the squares on two adjacent sides of a triangle be equal to the square on the third side, prove that the angle contained by the two adjacent sides is a right angle.

Enunciate the corresponding propositions when this angle is (1) acute, and (2) obtuse.

# Second Paper.

1. Construct a Balance Sheet having given the following particulars; Paid-up Capital £25,000, Cash in Bank £518. 3s. 10d., Accrued Interest £298. 11s., Sundry Debtors £1,966. 3s. 3d., Sundry Creditors £4,933. 1s., Investments £38,059. 19s., Reserve Fund £19,004. 6s. 8d., Loans on Securities £8,094. 10s. 7d.

Explain clearly the difference between a Revenue Account and a Cash Account.

2. Show that the difference between the Commercial Discount and the True Simple Discount on a sum of money is equal to the interest on the True Simple Discount for the same time.

If the True Discount and the Commercial Discount on a sum of money due 7 months hence be respectively £8.15s. and £8.18s.  $0\frac{3}{4}d$ ., find the sum and the rate of interest.

- 3. A cyclist travels the first half of the distance from P to Q at the rate of 10 miles an hour; he then rests 15 minutes and travels the remainder of the distance at the rate of 15 miles per hour. Had he travelled the whole distance without stopping at the rate of 11 miles per hour he would have taken exactly the same time. Find the distance from P to Q.
- 4. If I take any number, such as \$4796205, and subtract from it a number formed of the same digits, such as 60594287, I am sure to find a remainder consisting of a multiple of nine and having the sum of its digits a multiple of nine. Prove this to be true for all numbers and explain the reason.
- 5. Assuming the truth of the Binomial Theorem for a positive integral exponent prove its truth, with a certain condition, for a negative or fractional index.

Write down the first five terms of  $(1+x)^{\frac{1}{2}}$ , and hence show that

$$\sqrt{101} = 10.0498756...$$

- 6. Find the sum of n terms of the series
  - (1) whose *n*th term is (3n-1)(n+2);

(2) 
$$\frac{3}{1.2.4} + \frac{4}{2.3.5} + \frac{5}{3.4.6} + \dots$$

- 7. A is three times as old as B was when A was as old as B is now; and in 24 years' time B will be twice as old as A was when B was half as old as A is now. Find the ages of A and B.
- 8. Define a logarithm, and prove that  $\log_a(m^n) = n \log_a m$  and  $\log_b a \times \log_c b = \log_c a$ .

A spendthrift borrows £600 from a money lender. The bill is renewed every six months at an increase of 18 per-cent. Find what time will elapse before he owes £5,000, given  $\log 2 = 30103$ ,  $\log 3 = 4771213$  and  $\log 59 = 1.770852$ .

9. An estate, the clear annual value of which is £800, is let by a college on a lease for 20 years, renewable every seven years on payment of a sum of money, at a rent of £300 per annum. Interest being reckoned at 6 per-eent, what sum should the tenant pay on renewing his lease, given  $\log 106 = 2.0253059$ ,  $\log 4.688385 = .6710233$  and  $\log 3.118042 = .4938820$ ?

10. Explain the mathematical meaning of the word Probability, and prove that the chance of the happening of any number of independent events is the product of the chances of their severally happening.

Four cards are drawn from a pack of playing cards; find the chance that there is one of each suit.

- 11. The a priori odds against a certain event having happened are 10<sup>12</sup> to 1; if 13 independent witnesses, each of whom makes a correct assertion nine times out of ten, assert that it happened, prove that the probability that it did happen is now about  $\frac{5}{7}$ .
- 12. If two chords of a circle cut one another within the circle prove that the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

Prove that the shortest chord of a circle that can be drawn through a given point within it is perpendicular to the straight line joining the given point to the centre.

Examination for Admission to the Class of Associate OR TO THE CLASS OF FELLOW (PART II).

Examiners-Messrs. W. P. Phelps, F. T. Mason Byers, E. R. STRAKER, and A. LEVINE.

## First Paper.

1. State and prove Makeham's formula for the rate of interest yielded by a redeemable bond.

What should be the present price of Consols in order to yield an effective rate of  $2\frac{1}{2}$  per-cent per annum exactly?

2. Explain the following symbols:

$$egin{array}{c} \mathbf{A}^2_{xyz} \ \mathbf{A}_{xy:z} \end{array}$$

and give convenient expressions for their solution.

3. Give a verbal interpretation of the formula

$$(P_x+d)$$
  $(1+a_{x+n})+{}_nV_x=1$ ,

and show what modification must be made

(i) For Endowment Assurances.

(ii) For Whole Life Policies with limited number of premiums.

State for what purposes the formula may be advantageously employed.

- 4. Cards have been written for the purposes of a mortality experience, on which the nearest duration has been entered for the existing and withdrawals and the curtate duration for the deaths. It is desired to ascertain the experience by ages attained, but to separate it into two parts, that for the first five years, and the subsequent experience. Explain in detail how you would proceed and how you would check your work. What special point arises in regard to the eards on which the duration has been entered as five?
- 5. An office proposes to grant Endowment Assurances under a table of premiums reduced by anticipation of future bonus. Investigate a formula for the annual reduction when the discounted bonus is

(a) A quinquennial cash bonus.

(b) A simple reversionary bonus of £1 per-cent per annum declared quinquennially and vesting as soon as it is declared.

It may be assumed that the office does not allow interim bonuses.

- 6. Find, according to the H<sup>M</sup> Tables supplied, the value, at three per-cent interest, of a contingent annuity for the remainder of 30 years certain from the present time, the annuity to commence on the failure of the joint existence of two lives both now aged 30, but only in the event of such failure taking place after the expiration of 5 years, and before the completion of 10 years.
- 7. Obtain the numerical values of the probability that out of three lives, 30, 35 and 40,

(1) One, at least, will die in the 10th year.(2) Not more than two will fail in the 10th year.

.99999858681

(3) All will die within 20 years.

.0121876

8. Find the net premium for a policy issued under a reduced premium for the first five years, and state what precaution is necessary in fixing the amount of the reduced and subsequent office premiums.

Investigate a formula for the policy-value after it has been three years in force. How would such a policy usually be treated in a valuation?

- 9. A Copyhold Estate is held for two lives, each renewable at the end of the year in which it drops by a life aged 10 on payment of a fine of £8. Assuming the two lives to be now aged 30 and 35 respectively, find the present value of all the fines in perpetuity at 3 per-cent interest.
- 10. Obtain a formula for the value of a complete annuity, payable m times a year, to (x) after the death of (y), and state when the first instalment of the annuity becomes due as provided by your formula.

- 11. State the main distinctions between the Insurance clauses of
  - (i) The Married Women's Property Act, 1870. (ii) ,, ,, 1882.
  - (iii) The Married Women's Policies of Assurance Act (Scotland), 1880.

A policy effected under the 1870 Act on AB for the benefit of his wife, CD, has become a claim in 1898 by the death of AB, CD being still living, and no Trustee has been appointed. State whether a good discharge for the policy monies can be given by the Executors of AB, and give your authority.

12. Discuss the principal methods of treating redeemable Stocks and Bonds in the books of an Insurance Office. Which method do you recommend, and on what grounds?

## Second Paper.

1. Prove that, if  $a_{n|\vec{r}|}$  denote the value of a varying annuity of the rth order for n years, then  $(1+i)^r a_{n|\vec{r}|}$  is equal to the sum of the first (n-r+1) terms of the expansion of  $(1-v)^{-r}$  in powers of v.

Deduce the value of the perpetuity of the rth order.

- 2. A foreign Government has contracted for a loan by the issue of Bonds bearing interest at 5 per-cent (payable half-yearly) and redeemable by an accumulative sinking fund of 1 per-cent per half-year on the original amount of the loan applied by drawings. Arrangements have been made with a financial house to take up the whole of the loan at a price to pay them 6 per-cent per annum (receivable half-yearly). The financial house sell the Bonds in the Market at  $93\frac{1}{2}$  per-cent. Find
  - (a) The profit to the financial house.
  - (b) The rate of interest which the loan returns to the public.
- 3. Explain what is meant by the average age at death of persons aged x.

Deduce from the tables supplied you the average age at death of those persons who die between ages 25 and 42.

Express the ratio of deaths to population in terms of the complete expectation of life.

4. What do you understand by the expression "expected claims"?

How would you proceed to ascertain the amount of profit or loss from mortality experienced in a Life Office?

5. Give an account of the construction of the Northampton and Carlisle Tables, explaining their constitutional defects, and compare their effects when used for the calculation of annuities and assurances, with the results obtained from the H<sup>M</sup> Table.

6. A son wishes to secure for his mother an annuity for life, to commence at his death, and proposes to pay an annual premium for the benefit, subject to the condition that, should she predecease him, the premiums paid shall be returned without interest. Find a formula for the annual premium.

Having regard to the effects of selection, how would you

approximate to the premium in practice?

- 7. A Reversionary Life Interest to x after y is bought by an Insurance Company, with an option to the vendor to re-purchase on fixed terms at any time within the first n years. The Company sets up a Whole-Life Non-Profit policy at once for the minimum amount required to protect the Security during the term of n years, and has arranged that, if the option is not eventually exercised, a policy for the balance of the amount required can be effected at the end of n years at the rate of premium for age x+n, without medical examination. What is the ratio of the amount of the policy set up at once to the total assurance which will be eventually required if the option be not exercised?
- S. Deduce a formula for the net annual premium for an Endowment on a child now aged 3, to be payable on attaining 25, with the proviso that if the child's father, now aged 40, predecease the child before the attainment of 21 by the latter, no further premiums shall be payable until the child attains 21; and further, if the child predecease his father (but not otherwise), the office premiums are to be returned, but without interest.
- 9. A whole life policy has been in force some years, and a reversionary bonus has been added to the sum assured. The assured now wishes to surrender one-half the existing policy (and bonus) for a paid-up assurance, which would be non-participating, and to continue the other half on its present footing. How would you calculate the amount of the paid-up policy?

Draft a form of endorsement to be placed on the original policy.

10. Find the conditions under which two mortality tables will give equal policy-values.

Deduce that an increase in the rate of interest will reduce the

policy-value.

11. Discuss the broad principles which should guide an Actuary of a Life Office in the investment of the Assurance Fund. In what respects do the principles differ from those applicable to a Banking Institution? Would you consider the following suitable for a life office, and give your reasons.

(1) Discounting Bills.

(2) Loans on Bills of Lading.

(3) Loans on Stock Exchange Securities.

- (4) Loans on the Deposit of Title Deeds to Leasehold Property?
- 12. How were assignees of life policies affected by the Policies of Assurance Act, 1867, and the Judicature Act, 1873?

State the conditions required by the last-mentioned Act.

# Examination for Admission to the Class of Fellow (Part III, Section A).

Examiners—Messrs. T. G. Ackland, F. Schooling, S. G. Warner, H. Cockburn, E. Woods, and R. Todhunter.

## First Paper.

- 1. What estates can be held in Real Property? Give a short explanation of each of them.
  - 2. What is the meaning of the following expressions?

Legal Estate.
Equitable Estate.
Equitable Mortgage.
Distringas.
Stop Order.
Voluntary Assignment.

- 3. What alteration was made in the law relating to the avoidance of Voluntary Conveyances by the "Voluntary Conveyances Act, 1893"?
- 4. What were the proposals that the Government of the United States recently put before the British Government with regard to the silver currency? What would have been the effect had the British Government acceded to these proposals?
- 5. What is the amount of the Permanent Annual Charge of the National Debt, and to what uses is it put? State the reasons for and against the redemption of the National Debt.
  - 6. Explain fully the meaning of
    - (a) Specie point.
    - (b) Mint par of exchange.
    - (c) Bank rate.
- 7. Find an expression for  $u_n$  in a series consisting of  $u_0$  and its successive differences, without resorting to the method of separation by symbols.
  - 8. Given a series of equidistant values of a function, show how to insert, between each two consecutive values, (n-1) equidistant terms, so as to divide each interval into n equal intervals.

Apply the formula so deduced to the calculation of the values of  $p_{21}$   $p_{22}$   $p_{23}$   $p_{24}$ , given

 $p_{20} = .99428$   $p_{25} = .99293$   $p_{30} = .99229$  $p_{35} = .99138$ 

- 9. Given an even number of equidistant values of a function, deduce a formula for interpolation by central differences, and show that  $\mu$  (the force of mortality) is equal to  $\frac{2d_{-1} + 5d_0 d_{+1}}{6l_0}$  approximately.
- 10. State in detail the methods followed in the construction and graduation of either of the following Mortality Tables:—

Government Annuity Experience, 1883. American 30 Offices' Table.

- 11. Give some account of the several tables available for the valuation of sick benefits, and state briefly the data upon which they are respectively based.
- 12. Discuss the question of the graduation of Select Mortality Tables, and the suitability and applicability of the following methods, respectively: (a) Makeham's modification of Gompertz's formula, with or without special modifications; (b) Woolhouse's formula for interpolation; (c) the Graphic Method.

## Second Paper.

- 13. What change in the law was effected by "The Sales of Reversions Act, 1867"? Upon what grounds can sales of reversionary interests now be set aside?
- 14. State briefly the various provisions of "The Friendly Societies Act, 1896", as to (1) registration; (2) annual and quinquennial returns; (3) dissolution.
- 15. Has a person to whom a life policy is merely delivered, and who has no documentary evidence of title, any and what rights?
- 16. State briefly the main provisions of the Cape of Good Hope Life Assurance Act, 1891, and in what material respects they differ from the provisions of the Life Assurance Companies Acts, 1870-2.
- 17. The Bank of England Notes in active circulation show no tendency to keep pace with increasing trade or increasing population. The average amount in Notes in active circulation, for the years 1876–8, was £27.9 millions, for the years 1888–90, £24.4 millions. How do you account for this decline in the use of Bank Notes as a currency medium?
- 18. What are the powers of the Sceretary of State for India in Council to raise money on loan in the United Kingdom? Does the United Kingdom guarantee the Debt of India? Discuss the comparative expediency of India raising money in the United Kingdom or in India.

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- 19. Discuss the question of the liability of a Life Assurance Company to income tax in respect of (a) the interest on the Life Assurance Fund; (b) the profit as actuarially ascertained; (c) the payments to life annuitants; and illustrate your remarks by reference to legal decisions affecting the matter.
- 20. Given n values of a function which are not consecutive and equidistant, find any other value whose place is given.
- 21. Let  $u_1 u_2 \ldots$  denote a series of equidistant quantities, and  $S_x$  denote the sum of the first x of them,  $S_0$  being=0. Having given the values of  $S_n$ ,  $S_{2n} \ldots S_{rn}$ , show how to find the values of  $u_1 u_2 \ldots u_{rn}$ .

Show how the above method was applied by Mr. Berridge in the graduation of the Peerage Table.

- 22. State concisely the methods adopted by Dr. Sprague in the construction of Select Life Tables based upon the Institute  $H^{M}$  Experience.
- 23. State fully the method proposed by Mr. George King for ascertaining the mortality experienced by a company during an inter-valuation period, and discuss its advantages.
- 24. Upon what principles and methods would you proceed for the elimination of duplicates in the construction from the Experience of Assured Lives of (a) full Aggregate Tables, (b) Aggregate Tables with exclusion of the first t years of assurance? Discuss the method adopted for the elimination of duplicates in the construction of the  $\mathbf{H}^{\mathrm{M}}$  and  $\mathbf{H}^{\mathrm{M},5}$ . Tables, and state whether in your opinion the plan adopted actually secured, in the case of the  $\mathbf{H}^{\mathrm{M},5}$ . Table, the entire exclusion of the experience of the first five years of assurance.

# EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III, SECTION B).

Examiners—Messrs, T. G. Ackland, F. Schooling, S. G. Warner, H. Cockbern, E. Woods, and R. Todhunter.

# First Paper.

- $1.\ \mathrm{Discuss}$  the various methods of dealing with policies on rated-up lives;
  - (a) As regards Office Reserves.
  - (b) As regards Surrender Values.

2. How does a deferred or contingent bonus differ from what is ordinarily meant by a tontine bonus?

An office has a deferred-bonus scheme under which a policy is entitled to a bonus at each investigation after the assured has

survived his age-at-entry expectation; the policies issued thereunder are kept in a separate class. How would you deal with the surplus of the class at successive investigations?

- 3. What inquiries would you make with the object of bringing up to date, for the purposes of a valuation, the information given in the office books? In what ways would you expedite a valuation by work done in advance?
  - 4. Discuss the eligibility of

(a) Licensed property in England,

(b) Real property in a British Colony,

as security for the funds of a Life Office.

Draft, as regards one or other (not both) of these classes of security, a report setting forth a practical method of obtaining applications for loans, with arrangements for their investigation and management.

- 5. A Life Office proposes to commence Annuity business. Draft a memorandum of instructions for the calculation of a Table of rates for single lives.
- 6. An absolute reversion is offered for sale. Part of the fund consists of with-profit policies, at annual premiums, on the life of the life tenant. What enquiries would you make respecting the policies, and how would you deal with them, in valuing the reversion?
- 7. Obtain a formula for the value of a life interest in possession which shall accord with the conditions under which such an income is payable in practice. Show how the formula is modified should the investment be partly protected by an existing policy.
  - S. A, a female, aged 50, is tenant for life of the following fund: £5,000 India 3 per-cent Stock.

£4,000 Great Western Railway 4 per-cent Debenture Stock. £2,000 invested on mortgage of long leasehold property at  $4\frac{1}{2}$  per cent.

- B, her son, is absolutely entitled to the fund at her death. It is proposed to divide it, by mutual consent. State, in symbols, the proportions in which you would apportion it, giving the data upon which you would proceed.
- 9. Investigate the effect upon the reserves of an Office of the methods of adjustment usually adopted on the discovery, (1) within a few years of the issue of a policy, (2) on a claim arising, of an error in the assumed age of the life assured.
- 10. Explain what is meant by the integration of a function, and prove, from first principles, that if  $\frac{d\phi(x)}{dx}$  is finite and continuous for

all values of x between a and b,  $\int_{a}^{b} \frac{d\phi(x)}{dx} dx = \phi(b) - \phi(a).$ 

If the force of interest declines continuously, in such a way

that its value after any time t is  $\delta r^t$ , prove that the present value of 1 due t years hence is  $v^{\frac{r^t-1}{\log\epsilon}r}$ .

11. Obtain a formula for  $\overline{\mathbf{A}}_{xy}^1$  in terms of  $\bar{a}_{xy}$  and Makeham's constants.

If A and B are subject to the mortality exhibited by a Table graduated by Makeham's formula, prove,

- (1) That, as they grow older, the probability that A will die before B increases or decreases according as A is older or younger than B;
- (2) That the value of a continuous assurance on one life against the other, subject to a continuous premium, bears a constant ratio to the value of a similar assurance on the joint lives.
- 12. State Maclaurin's Theorem, and prove that

$$\int_{-n}^{n} u_x dx = \frac{n}{3} (u_{-n} + 4u_0 + u_n)$$
 approximately.

If the exact maturing dates of the endowment assurances maturing in the (n+1)th year after the date of a valuation are so distributed that the amount maturing on the expiration of  $(1+\frac{1}{2}+t)$  years exactly, for any value of t from  $-\frac{1}{2}$  to  $+\frac{1}{2}$ , is  $S\left(1+\frac{2t}{3}\right)dt$  (where S is the total amount maturing in the year), prove that the effect of assuming that all the endowments mature in the middle of the year is to over-estimate the value of the sums assured by  $\frac{S}{18}\left(A_{xn}-A_{x}\frac{S}{n+1}\right)$ , approximately.

# Second Paper.

[In answering the last three questions on the paper the Candidate is expected to obtain exact numerical results, by aid of the Actuarial Tables supplied, using, for Questions 23 and 24, such data therefrom as come nearest, in his opinion, to those which should be employed, and indicating what data he would use in practice].

13. Describe fully (but without reference to bases of interest or mortality) the principles upon which you would value the liabilities of a Life Office under its Assurance contracts.

Your answer should include a statement of the methods you would employ to determine valuation-ages, net premiums, and valuation annuities; to group the principal classes of policies; and to provide reserves for extra risk, early payment of claims, and paid-up loading.

14. A Mutual Life Office, making an H<sup>M</sup> 3 per-cent net premium reserve, distributes its profits as a uniform addition per-cent to the sums assured in respect of each yearly premium paid since the previous valuation. On what terms would you allow eash commutation of bonus? And why?

1

- 15. It has been the practice of an Office to declare a cash bonus by way of an uniform percentage on the premiums paid on each participating policy since its issue. Under what conditions, as regards premium rates and reserves, would such a system appear to you equitable and satisfactory? What points would you consider in reporting on a proposal to adopt the compound reversionary bonus plan for new business?
- 16. How would you deal, in a valuation and distribution, with guarantees and reassurances participating in the profits of other Offices?
- 17. Give an investment formula for the value of a complete annuity, and deduce a formula for the value of a reversionary life interest. In connection with the latter, obtain an expression for the amount payable by the reversioner on re-purchase of the life interest at the end of the year of the life tenant's death, a policy assuring the theoretically requisite sum being handed over with a premium due and unpaid.
- 18. On what bases of interest and loading would you prepare Tables of single and annual premiums for Leasehold Sinking Fund Policies, extending from one year to 100 years? What surrender values would you allow for such policies?
  - 19. What approximate methods would you employ to check
    - (1) A net rate of annual premium for a joint life endowment assurance, and
    - (2) An office rate of single premium for a reversionary annuity on a life aged 30 after the death of 60, provided the latter predecease 80?
  - 20. Demonstrate the process of integration by parts.

Write down the integral for  $\overline{A}_{xyz}^1$ , and show that on Makeham's hypothesis  $A_{xyx}^1 = \frac{\log_e s}{3\log_e s - \delta}$  for all values of x, y, z, satisfying the equation  $\frac{\delta}{\log_e s} = \frac{2\mu_x - \mu_y - \mu_z}{\mu_x + \log_e s}$ .

21. What conclusions would you draw from the fact that the first differential coefficient of a function is positive, zero, or negative; and whv?

Prove that the risk under a policy effected to secure a reversionary annuity to y after the death of x will be a decreasing risk if and when (approximately) the ratio of the first difference of  $a_{y+t}$  (taken positively) to the first difference of  $\mu_{x+t}$  exceeds the ratio of  $a_{y+t}$  to  $\mu_{x+t}$ .

22. Obtain the following values by HM 3 per-cent Table (Text-Book Graduation), showing your work in detail:

23. A, born in April 1877, is entitled, subject to his attaining 25 years of age, to the reversion, expectant on the death of the survivor of his uncles, aged respectively 60 and 65, to one-fifteenth share of £120,000 Bank of England Stock. He is a first-class life, about to proceed to and reside in Jamaica.

What price would you advise a Life Office to give for A's

reversionary interest?

24. Find the reversionary charge necessary to cover a cash payment of £1.500. and costs, secured on the reversion of a healthy male life aged 30 next birthday, contingent on his surviving his father aged 65 last birthday, to the fee-simple of property of ample value; with option of re-purchase during the first five years on the basis of an accumulation at 5 per-cent per annum compound interest with half-yearly rests.

## PROCEEDINGS OF THE INSTITUTE.—Session 1897-98.

First Ordinary Meeting, 29 November 1897.

The first ordinary meeting of the session 1897-98 was held at the Hall of the Institute, on the 29th day of November 1897.

The President (Mr. T. E. Young) in the Chair.

The President delivered an inaugural address.

# Second Ordinary Meeting, 20 December 1897.

The President (Mr. T. E. Young) in the Chair.

A paper "On the Treatment of Endowment Assurance Policies in Periodical Valuations", was read by the author. Mr. H. Archer Thomson, B.A.

The following gentlemen took part in the discussion:—Messrs. H. W. Andras, J. Sorley, G. King, H. W. Manly, G. F. Hardy, J. H. Barnes, W. A. Hutcheson, and the President.

# Third Ordinary Meeting, 31 January 1898.

The President (Mr. T. E. Young) in the Chair.

The President announced that the first prize of £30, offered by Mr. James Chisholm, had been awarded to Mr. John Nicoll, F.F.A., for his Essay on "The Relation of the Actuarial Profession to the State."

The President referred to the death of Mr. Sheppard Homans, Corre-

sponding Member.

The following gentlemen were duly elected: -Mr. Robert Stirling. F.F.A., as Fellow, and Messrs. Hugh Wylie Brown, F.F.A., and Henry John Pearce, F.F.A., as Associates.

A paper entitled "Some Remarks on the Valuation of Endowment Assurances in Groups", was read by the author, Mr. G. J. Lidstone.

The following gentlemen took part in the discussion:—Messrs. G. F. Hardy, F. Schooling, H. W. Manly, J. Burn, H. Archer Thomson, and the President.

## Fourth Ordinary Meeting, 28 February 1898.

The President (Mr. T. E. Young) in the Chair.

The prize essay, "On the Relation of the Actuarial Profession to the State", was read in abstract by the author, Mr. John Nicoll.

The following gentlemen took part in the discussion:—Messrs. W. Hughes, G. King, J. Chisholm, and the President.

## Fifth Ordinary Meeting, 28 March 1898.

The President (Mr. T. E. Young) in the Chair.

Messrs, Alexander Fraser, Jr., F.F.A., and Vyvyan Marr, F.F.A., were elected Associates.

A paper, entitled "Some Observations on Industrial Assurance", was

read by the author, Mr. C. H. E. Rea.

The following gentlemen took part in the discussion:—Messrs. G. H. Ryan, T. G. Ackland, A. H. Bailey, A. R. Barrand, J. Sorley, H. W. Manly, F. J. Vincent, and the President.

# Sixth Ordinary Meeting, 25 April 1898.

The President (Mr. T. E. Young) in the Chair.

A paper "On the Mortality in the British Navy and Army, as shown by the Official Reports", was read by the author, Mr. J. J. McLauchlan, F.F.A. The following gentlemen took part in the discussion:—Messrs. H. W. Manly, G. King, T. G. Ackland, and the President.

# The Fifty-first Annual General Meeting, 5 June 1898.

The President (Mr. T. E. Young) in the Chair.

The proceedings at the Annual General Meeting will be found on page 303.

#### REPORT, 1897-98.

The Council have pleasure to report to the members upon the progress of the Institute during the session of 1897-98, the fiftieth year that it has been in existence.

The increase in the number of members in the year has been 34, as compared with 38 in that which preceded it. At the end of the year in which the Institute was incorporated by the Royal Charter the number of members was 434, while five years later, at 31 March 1890, it was 601. Since that time it has grown as follows:

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1898.

|  | Honorary<br>Members | Fellows | Associates | Students | Corres-<br>ponding<br>Members | Total |
|--|---------------------|---------|------------|----------|-------------------------------|-------|
| i. Number of Members<br>in each class on<br>31 March 1897 .  | 1                   | 178     | 228        | 406      | 13                            | 826   |
| <ul><li>ii. Withdrawals by</li><li>(1) Death</li><li>(2) Resignation</li><li>(3) Default in pay-</li></ul> |                     | 1       | <br>1      | 1<br>17  | }                             | 54    |
| ment of Sub-<br>scriptions.  |                     | 1       | 2          | 28       | }                             |       |
| ii. Additionsto Membership   | 1                   | 175     | 225        | 360      | 11                            | 772   |
| (1) By Election . (2) By Order of Council (3) By Re-instatement  |                     | <br>    | 4<br>      | 80<br>2  | ··· }                         | 88    |
| v. Transfers   | 1                   | 176     | 229        | 442      | 12                            | 860   |
| (1) By Examination:  from Associates to Fellows  |                     |         | 9          |          |                               |       |
| (2) By Examination:  | 1                   | 185     | 220        | 442      | 12                            | 860   |
| from Students to Fellows .   |                     | -1      |            |          |                               |       |
| (3) By Examination:  | 1                   | 189     | 220        | 438      | 12                            | 860   |
| from Students to Associates .  |                     |         | <br>22     | 22       |                               |       |
| v. Number of Members<br>in each class on<br>31 March 1898 .  |                     | 189     | 242        | 416      | 12                            | 860   |

The Council have, with great regret, to report the loss by death during the year of one Fellow, Mr. David Chisholm; and two Corresponding Members, M. Martin-Dupray, of Paris, and Mr. Sheppard Homans, of New York.

Mr. David Chisholm was elected a Fellow of the Institute in 1854, and was a contributor to the *Journal* in the earlier years of its existence. He was also the compiler of a valuable and extensive set of Commutation and other Monetary Tables based on the Carlisle Table of Mortality.

Mr. Sheppard Homans, who for so many years held a prominent position among actuaries in the United States of America, became a Corresponding Member of the Institute in 1867. Among his communications to the Journal, perhaps one of the most interesting was his article in the 11th volume, explanatory of the system devised by him and adopted by a large number of American Life Companies, for the Distribution of Surplus on a Contribution Plan.

Monsieur Martin-Dupray was elected a Corresponding Member of the Institute in 1895. He was one of a Committee of four Actuaries who undertook the collation of the Mortality Experience of four great French Life Assurance Companies, which work has recently been published.

The Accounts for the year show that the total funds on 31 March last amounted to £7,080. 0s. 5d., being an increase during the year of £1,795. 16s. 5d.

This considerable increase is largely accounted for by the amount received from the Joint Mortality Experience Investigation Committee, as a refund of sums previously expended on their account by the Institute.

The Annual Subscriptions, together with admission and other fees, amounted to £1,731. 19s. 0d., showing an increase of £167. 14s. 0d. as compared with those of the previous year.

The total Income for the year was £3,328. 10s. 11d., and the total Expenditure £1,587. 14s. 6d.

The Revenue Account and Balance Sheet are given herewith (p. 302).

The stock in hand of the Institute publications on 31 March was as follows:

| No. of Copies | 3 |  | Description of Work                         |  |  |  |
|---------------|---|--|---|--|--|--|
| 111           |   |  | Text-Book, Part II.                         |  |  |  |
| 15            |   |  | Mortality Experience Tables.                |  |  |  |
| 792           |   |  | Government Joint-Life Annuity Tables.       |  |  |  |
| 868           |   |  | Select Life Tables.                         |  |  |  |
| 397           |   |  | Logarithm Cards.                            |  |  |  |
| 356           |   |  | Messenger Prize Essay (Friendly Societies). |  |  |  |
| 517           |   |  | Index to 10 Vols.                           |  |  |  |
| 48            |   |  | " to 20 "                                   |  |  |  |
| 1,035         |   |  | ,, to Vols. 21 to 30.                       |  |  |  |
| 9,302         |   |  | Parts of Journal.                           |  |  |  |
|               |   |  |   |  |  |  |

The following papers were submitted at the sessional meetings of the Institute, namely:

- 29 November 1897.—An inaugural address by the President, Mr. T. E. Young.
- 20 December 1897.—"On the Treatment of Endowment Assurance Policies in Periodical Valuations"—Mr. H. Archer Thomson.
- 31 January 1898.—" Some Remarks on the Valuation of Endowment Assurances in Groups"—Mr. G. J. Lidstone.
- 28 February 1898.—"The Relation of the Actuarial Profession to the State", being the Essay by Mr. J. Nicoll, to which a Prize was awarded from the Chisholm Prize Fund. (Read in abstract.)

- 28 March 1898.—"Some Observations on Industrial Assurance"— Mr. C. H. E. Rea.
- 25 April 1898.—"On the Mortality in the British Navy and Army, as shown by the Official Reports"—Mr. J. J. McLauchlan, F.F.A.

For the Examinations held in the United Kingdom on 22, 23, 25, and 26 April last, 163 candidates presented themselves, namely:

66 for Part I. 57 .. ,. II. 20 ,. .. III. Section A. 20 .. .. III. .. B.

Of these the following numbers were successful:

35 in Part I. 14 .. ,, II. 8 .. ,, III, Section A. 6 ,, ., III, ,, B.

The following are the successful candidates, the names in each class being arranged alphabetically.

#### PART I.

Examiner-Prof. S. L. Loney.

Supervisors-Messes, W. Hughes and G. J. Lidstone.

#### Class I:

J. Bacon.

J. G. Wigner,

#### Class II:

W. R. Ashton.
S. R. Ball.
H. P. Blake.
T. R. Chandler.
A. B. Culley.
S. G. Dunn.

J. Rae. H. T. K. Robinson. L. W. Stewart. E. F. Terry. L. Webb.

P. Gibson.

#### Class III:

W. A. Backett.
S. T. Bennell.
E. W. Bird.
F. E. Blake.
R. W. A. Blake.
F. M. Bowles.
C. Burdett.
H. M. Cook.
H. T. Cross.

W. J. Harriss.

T. M. Harrison.
G. D. Hooper.
T. H. Harding-Newman.
W. Y. B. Pascoe.
C. F. Peters.
G. D. Pigrome.
F. J. Roll.
E. C. Doust-Smith.

C. Sutton. G. M. Wyatt.

#### PART II.

Examiners—Messrs. W. P. Phelps, F. T. Mason Byers, E. R. STRAKER, and A. LEVINE.

Class I:

None.

### Class II:

T. L. Coates. W. P. Elderton. C. E. Howell.

R. A. C. Thomas. T. P. Thompson. C. F. Whigham.

#### Class III:

J. Buchanan. A. J. Doyle.

B. May. J. H. Reeves.

G. Green.

R. P. Smith. A. H. Woolmer.

C. W. Kenchington.

#### PART III.

Examiners-Messrs. T. G. Ackland, F. Schooling, S. G. Warner, H. COCKBURN, E. WOODS, and R. TODHUNTER.

### SECTION A.

Class I:

None.

Class II:

†A. C. R. Cockman.

+F. Marchbank.

#### Class III:

H. T. Adlard. E. H. Brown. C. R. V. Coutts. \*W. H. Hodgson. D. Macphail. †R. G. Salmon.

SECTION B.

Class I:

None.

Class II:

†G. F. Robinson.

†J. Spencer.

Class III:

W. P. Cook.

\*W. H. Hodgson. A. T. Winter.

†C. P. Dawson.

Those marked (\*) passed in both sections, and with those marked (†), have now completed the examination for the Class of Fellow.

In the Colonies the Examination entries numbered 59, as under:

For Part I, 23. ,. II, 25. .. III, Section A, 7. .. III. ,. B, 4.

The results of the Colonial Examinations will be duly announced.\*

A prize of the value of £30, being the first of the three prizes kindly offered by Mr. James Chisholm for the best essays on "The Relation of the Actuarial Profession to the State", has been awarded to Mr. John Nicoll, F.F.A. The essay was read at the sessional meeting in February. No other essay having been adjudged worthy of a prize, Mr. Chisholm has generously allowed the amount of the second and third prizes to remain in the Funds of the Institute for future distribution.

During the past session a series of six lectures on "The London Daily Stock and Share List" have been delivered by Mr. George Clare, before a very full attendance of the members of the Institute, in Staple Inn Hall. These lectures, as well as those delivered last session, on legal subjects. by Mr. Wood Hill, will shortly be published.

In continuation of the educational work of the Institute, the Council will endeavour to make arrangements for the delivery, during the next session, of lectures on other subjects coming within the scope of actuarial training.

The new Mortality Experience Investigation, which is being carried out jointly by the Institute of Actuaries and the Faculty of Actuaries, is progressing very satisfactorily, under the superintendence of Mr. T. G. Ackland, who now has a staff of 30 clerks constantly at work upon the large body of cards constituting the Assurance Experience.

The whole of the data relating to the Experience in respect of Annuitants have been dealt with, and the Tables are now in the press. It was thought better that the original data should be published as soon as possible, for the benefit of Life Assurance and Annuity Companies, and that graduated results and Monetary Tables should appear in a subsequent volume.

The Council have much satisfaction in reporting that, in response to applications made by the Presidents of the Institute and the Faculty, Life Assurance Offices have undertaken to contribute liberally towards the cost of the Investigation, which will necessarily be very heavy. The contributions of the Companies at present promised or received amount to £10,953, which sum, it is hoped, will cover the larger portion of the expense, and thus relieve the Institute and the Faculty from any anxiety as to their ability to carry to a satisfactory conclusion this all-important Investigation.

The second International Congress was held on May 16th. 17th. 18th. 19th, and 20th, under the auspices of the Institute, in Staple Inn Hall. and it is with much gratification the Council record that the gathering was in every way a success, there being a large attendance of members from Foreign Countries and some from the British Colonies. The papers submitted were numerous and covered a wide variety of subjects of the closest interest to the Actuarial profession. A full description of the work of the Congress will doubtless appear in due course in the Journal.

The Council are again much indebted to the Honorary Examiners, who have bestowed great care and labour on the important work entrusted to them.

<sup>\*</sup> These results are given on page 309.

|   |  | 302  |  | The Institute  | of Actuaries   | s.  | [Ост.  |
|---|--|--|--|--|--|---|--|
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| ling 31 March 1898.                           | 1897-98.<br>Journal—<br>Cost of Nos. 185, 186, 187, and 188<br>Clerical Assistance | Library— Binding and Purchases Address to Queen      | General Expenditure— Rent Salaries Salaries Lecturers Examination Charges Meetings Corporation Duty Fire Insurance Stationery and Printing | Postage and Telegrans  | Brown Prize Fund   | Exemined and found correct, 25 April 1898. Geo. R. Bellacoe, H. R. Harbins, C. H. E. Rea, March 1898. | Consols (£2,490, 14s. 9d.), cost Metropolitan Railway 4 per-cent Debenture Stock (£1,050), cost Great Eastern Railway 4 per-cent Debenture Stock (£600), cost Great Northern Railway 4 per-cent Debenture Stock (£600), cost Great Northern Railway 4 per-cent Debenture Stock (£600), cost On Current Account at London and Westminster Bank Arrears of Subscriptions Examined and found correct, 25 April 1898.  H. R. HARDER, C. H. E. REA, C. H. E. REA, C. H. E. REA,   |
| Revenue Account for Year ending 31 March 1898 | s. d. & s. d.<br>15 1<br>16 1<br>1 8 1   | 2 2,284 4 0 I  | 16 0<br>13 0<br>10 0<br>10 0<br>4 0 485 15 0   | 222 12 6<br>37 16 0<br>0 0<br>0 102 18 0<br>236 5 0  | 26 0 27 0 27 0 27 0 27 0 27 0 27 0 27 0  | 19 6 265 3 10<br>   | £ 8, d, £ 8, d, 93 17 8 95 10 5 299 8 1 200 0 0 201 10 9 250 10 9 5 10 5 25 10 9 5 10 5 25 10 9 5 25 10 9 5 25 10 9 5 25 10 9 5 25 10 9 5 25 10 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 5 25 10 9 9 9 25 25 10 9 9 25 25 10 9 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 10 9 25 25 25 10 9 25 25 25 25 25 25 25 25 25 25 25 25 25  |
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#### PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Report of the Council (given on p. 296) having been read,

The President said-In now formally moving the adoption of the report and accounts, I take the opportunity of stating that I am not a believer in the dictum of Montesquieu, that the nation or society is happy whose annals form a blank in the book of history. Fluctuations of experience, the interchange of achievement and failure, occurring in a constancy of effort towards the realization of the supreme end of its formation, constitute the signs of virility and the tokens of success of every genuine institution. And although for a considerable number of years our history has preserved a comparatively uneventful character, it has not, happily, been the monotony of inactivity but the steady uniformity of continuous advance. membership of the Institute still testifies to its vitality and attractiveness; and the only discomfort which I confess I occasionally feel lies in the increasing accumulation of its funds. Opulence in a professional body is desirable but only as an instrument for use in the larger accomplishment of its ends, and I shall indulge the confident hope that the far-seeing ingenuity of our future Councils will be able to devise other suitable modes of liberal expenditure in practical furtherance of our professional aims. In adverting to the accounts for a moment, I might point out that inasmuch as the sum advanced by the Institute towards the cost of the mortality investigation has been discharged, the accounts now presented to you involve no record whatever of the balance of funds which the liberality of companies, in response to our appeal, produced, so that that residue of contributions is kept strictly separate in an independent account, and devoted specifically to the object for which they were collected. I first join with you, gentlemen. in a respectful and earnest tribute to the memory of those of our members who have died since our previous meeting. That list of departed worthies is sadly lengthening with the years, but at the same time, it is peopling our records with remembrances of valued labours, and constitutes a bead-roll of memories which should encourage and animate the energies of ourselves who have entered into the prosecution of the work which our predecessors so successfully pursued. I should like to be permitted for one moment to advert to a subject in which I have always felt, and shall continue to retain, a deep interest, both in respect of the practical and theoretical aspects of the Institute, and especially in view of the interests of the students themselves-I refer to our system of examinations. To a man of my advancing years, gentlemen, examinations always and mainly appeal from the pathetic side, and I mourn, therefore, that so many aspirants have failed in their hopes. I cordially trust, however, that they will find courage to convert their disappointment into a stimulus to renewed and hopeful effort, and I venture also to express the very sincere hope - and I express it with deep conviction that it may be realized—that while a high and honourable standard of efficiency is maintained, as is necessary to be maintained, the scope and character of our examinations can always be rightly and reasonably adjusted to the meagre private opportunities of students whose monotonous daily toil in no degree aids them in their present stage of official life in the acquisition of mathematical and technical knowledge. Amid the events of the year, one grave disappointment has affected me in the fact that only two

members competed for Mr. Chisholm's prize. A generous allowance, I admit, must be made—and I very gladly make it—in the interpretation of this paneity of effort by reason of the wide and somewhat difficult character of the subject itself, demanding, as it undoubtedly did, not merely careful research but some maturity of thought. I would, however, beg my younger brethren to forgive me if, as my last official utterance, I carnestly appeal to them to seize eagerly these opportunities, not simply and primarily, let me observe, for the purpose of securing official and professional distinction, but mainly and essentially with a view to fixing and regulating their course of reading and of concentrating and cultivating their power of consecutive thought upon the elucidation of some specific subject. They will find this labour of permanent service in their future career, for the steady and continuous process of reading and meditation which such an attempt involves is infinitely superior to all isolated and desultory attempts. We have felt sincere pleasure in receiving contributions from three new members during the course of the session, whose careers, thus auspiciously begun, we shall watch with interest, and with the attendant hope, too, that an increasing number of our younger men will take courage by their example, and in a similar manner help usefully to enlarge the boundaries both of our theoretical and practical knowledge. And in connection with this subject, I think the Institute may well be congratulated upon the marked success which has followed on its extension of practical education, in the course of lectures upon finance. The attendance undoubtedly and very clearly showed that this continued effort filled, and most ably filled, a decided want in our system, and it is extremely encouraging to know that the plan is to form a permanent and prominent feature of our future training in completion and extension of that practical educational design which, from my reading of the history of the Institute, appears to have been most definitely and distinctly contemplated by our founders from the outset. With regard to the mortality experience, gentlemen, you will be interested in receiving two or three remarks, perhaps not merely as exemplifying the extreme complexity and difficulty of the subject, but also as accounting for the appreciable time which must necessarily elapse before even the elementary results of that investigation can be published. I am informed by Mr. Ackland, that out of the cards relating to the male whole-life experience only, there are no less than 11,000 which present the name of Smith, and of those, upwards of 1,000 are adornedperhaps that is an improper expression in connection with the term I am about to use—with the plain name of "John Smith." As one example out of many hundreds, it also appears there is one person with an extremely uncommon name, who is insured in seven companies, and in each of them with a different date of birth. Two of them record that he died in 1891, and five report on their cards that he was in existence in 1893. A difficulty of identification with the name of John Smith I can understand, but the unusual spelling of this person's name, both christian and surname, and the fact that the dates of birth appear to oscillate about a common centre, apparently indicate that we cannot be dealing with seven distinct individuals, and yet, on the other hand, it is somewhat difficult to suppose that we are concerned with one person alone. Hence, from the enquiries which have to be instituted among the various companies, you will be able, from these few instances, to form some conception of the immense amount of minute

industry which is required even in the sole department of disentangling these duplicates. The memory of the Congress, gentlemen, is too recentmemory of a double character, both intellectual and sensuous, if I may use the term in its etymological sense—to require any detailed reference. I trust that it reasonably fulfilled its anticipated aims; the extensive range of actuarial interests, and the width of application of actuarial principles, were at all events conspicuously manifest; and the personal intercourse with our genial and courteous foreign colleagues has left with us valued friendships and pleasant memories. I should like, however, to advert to one admirable practical feature involved in the adoption of a universal notation; I refer to the fact that the recommendations included a unanimous decision that the system of notation should be incorporated in the record of the proceedings of the Congress. By this very happy act our system of symbolism was translated from its local position, so to speak, in the Institute publications and confided to the permanent keeping of the journal of the entire profession, and thus acquired a universally accepted authority. And you will permit me, gentlemen, in closing, the indulgence of a few words of a personal character. Those words-hopelessly inadequate as an expression of feelingsimply consist of my grateful acknowledgments of the unceasing courtesy and kindness which, during my tenure of office, I have experienced from every member-a kindness and courtesy, gentlemen, which form at one and the same time both a most pleasant and refreshing record of memory, and an abiding incentive to future effort in the service of the Institute.

Mr. H. Cockburn, in seconding the motion, had a very confident feeling that the work performed by the Institute during the past year had been quite up to the average, alike in quality and in quantity. The President had referred so clearly and fully to the main points which arose on the report, that it was not necessary for him to say much. The report, as usual, and as was desirable, was a statement of simple facts, rather than a eulogy of any past achievements, or prognostications as to the future; but he could not help feeling that all who, like himself, had now been connected with the Institute for a certain number of years, would feel that in this its jubilee year it had lost none of its youthful vigour, but that rather it had blended with that some of the useful experience of age, with a power to accommodate itself to varying circumstances and requirements, a power which was perhaps the best guarantee of lasting usefulness. He should like to say one word in regard to the library. During the past year Mr. Whittall had felt compelled to resign the post of joint hon, librarian, and the Council had already accorded to him a very hearty vote of thanks for the very valuable work he had for many years given to the library, in conjunction with Mr. Schooling. They had been fortunate in securing, as a successor to Mr. Whittall, Mr. Geoffrey Marks, who would prove a very able coadjutor to Mr. Schooling. The sum appearing in the accounts as spent last year on the library might not be perhaps a very large amount, but he could assure the members that the hon, librarians were always ready, and indeed anxious, to spend upon the library any sums which could fittingly be spent upon it. either in the way of obtaining new works, or of duplicating old ones, so that the library should continue to hold its place as a useful factor in their educational system.

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The motion was unanimously adopted.

The ballot for the election of Officers and Council for the ensuing year then took place, Messrs. Faulks and T. B. Macaulay acting as scrutineers, with the result that the following were unanimously elected:—

# President.

## HENRY WILLIAM MANLY.

# Vice-Presidents.

HENRY COCKBURN. WILLIAM HUGHES. GERALD HEMMINGTON RYAN. FRANK BERTRAND WYATT.

## Council.

MARCUS NATHAN ADLER, M.A. THOMAS G. C. BROWNE.
ARTHUR FRANCIS BURRIDGE.
JAMES CHISHOLM.
HENRY COCKBURN.
\*ERNEST COLQUHOUN.
GEORGE STEPHEN CRISFORD.
ROBERT CROSS.
JOHN JAS. WALKER DEUCHAR.
\*JOSEPH ERNEST FAULKS, B.A.
ALEX. JOHN FINLAISON, C.B.
\*GEORGE FRANCIS HARDY.
RALPH PRICE HARDY.
AUGUSTUS HENDRIKS.
\*CHARLES DANIEL HIGHAM.

WILLIAM HUGHES.
GEORGE KING.
HENRY WILLIAM MANLY.
GEOFFREY MARKS.
WILLIE OSCAR NASH.
PHILIP LEWIN NEWMAN.
GERALD HEMMINGTON RYAN.
JAMES SORLEY.
THOS. BOND SPRAGUE, M.A., LL.D.
WILLIAM SUTTON, M.A.
HERBERT CECIL THISELTON.
WM. JOS. HUTCHINGS WHITFALL.
\*ERNEST WOODS.
FRANK BERTRAND WYATT.
THOMAS EMLEY YOUNG, B.A.

\* New Members of the Council.

## Treasurer.

# CHARLES DANIEL HIGHAM.

# Honorary Secretaries.

ARTHUR FRANCIS BURRIDGE.

ERNEST WOODS.

Mr. H. W. Manly, in responding for the Officers and Members of Council who had been elected, tendered the meeting his sincere thanks for the honour they had done him in selecting him to occupy the proud position of President. It would be his earnest endeavour to maintain the honour and dignity of the Institute with the same zeal and devotion as his predecessors, and he felt confident that in such efforts he would have the support of the other officials and members of the Council, for whose election and in whose name he begged to thank the meeting.

Mr. T. B. Macaulay, of Canada, had great pleasure in moving "That Messrs. Harding and Rea be re-elected anditors of the Institute for the ensuing year, and that Mr. Pagden be added to the number as a third auditor." He understood that the first two gentlemen had already served the Institute with satisfaction; and as the auditors worked for the love of the Institute, the least they could do was to avail themselves of their services. This was the first time that he had been present at a meeting of the Institute, and he doubted if the members could realize what a privilege and a pleasure it was to him. If they had been working, as he had been for nearly twenty years, in a city where there was no fellow actuary, where one had to plod along with no help but such assistance as could be obtained from the pages of the Journal of

the Institute and other insurance publications, and where the nearest Fellow of the Institute was 120 miles away, he thought they would begin to realize that it was a privilege of no small magnitude to be allowed to meet, as he had been doing for the last few weeks, with members of the Institutepeople who had the same object in life, whose thoughts could help them in their daily work, and with whom they could associate with business relationship and friendship. It might be said that as there was an Actuarial Society in America this isolation was done away with to a large extent, and within certain limits that was true. But the chance of visiting the Institute, and also of taking part in the International Congress, was one which he very highly appreciated. He told his directors that it was the opportunity of half a generation, if not a whole generation, and that he might come to London many a time to see the different actuaries for a few minutes in their offices without result. But here was a series of meetings which were to last about a week, when business routine was to be thrown aside, and where they were to mingle not merely as individuals, but to come together as in social relationship, and with a chance of getting into touch with the English actuaries such as he had not had for fifteen years. His directors, therefore, thought it was desirable that he should be present.

Mr. Searle, in seconding the motion, said the accounts appeared to give the auditors some trouble, and it was very gratifying to the Institute to have three such gentlemen who would spend their valuable time in auditing them. Referring to the remarks of Mr. Macaulay, although that gentleman might think himself very far away, he was by no means out of touch with the Institute. Mr. Macaulay's name was not strange to the actuaries of this country.

The motion was carried unanimously.

Mr. A. H. BAILEY said that, as one of the oldest members of the Institute, elected in the year 1848, he was going to follow Mr. Macaulay's example and ask a privilege, viz., that of proposing a vote of thanks to the President, Vice-Presidents, Council, Officers and Examiners for their services during the past year, and including in the list the name of Mr. Ackland. The events of the last year were certainly quite exceptional in the history of the Institute. The most remarkable event, and the one that concerned the Institute most closely, was the conclusion of its fifty years' experience. To him the events of fifty years ago were far more vivid than those of five years ago, and consequently he had been thinking a good deal about the incidents of the early life of the Institute. He did not know whether every member was aware of the difficulties that occurred and which were overcome in its creation. He took great interest in what was going on then, and recollected that in 1848 a meeting of actuaries was held in which it was referred to a committee to consider whether it was advisable to form a professional society. The committee consisted of the elder members of the profession at that time; a second meeting was held at which they presented their report to the effect that no such society was necessary, but that they would look after what was going on and call a meeting when it was wanted. He remembered being told by the late Mr. Jellicoe what a scene occurred at that meeting, when the report was presented and not received. Other and younger men were substituted as a committee, and set to work to form the Institute and succeeded in doing so; but there was left behind a very

strong opposition, which lasted for several years. To show what that opposition amounted to be gave a bit of his own personal history. He became a candidate for the actuaryship of a particular insurance society, and with his application sent in his certificate of having passed the examination of the Institute of Actuaries. One of the leaders of the opposition, however, called on each individual director and said he must not be appointed because the Institute meant to dictate to every board of directors in London whom they should appoint as their actuaries. He was credibly informed and believed that he lost the post on that account. The resolution passed forming the Institute stated that it was to be both scientific and practical, and therefore it was a little disappointing to find how few competitors there were for Mr. Chisholm's prize; but he was glad to see that fresh measures were being taken to make the society a practical society, in short, to make it well understood that the occupation of an actuary was not inter sylvas academi quærere rerum. It might be quærere verum, but upon the problems of life, and amid the busy hum of men, not inter sylvas academi. He also referred to the Congress which had just been held, and pointed out that the jubilee and the Congress had increased the work of the President very much during the year, and paid a tribute to the efficiency with which the President had performed all his duties, both social and professional.

Mr. G. F. HARDY, in seconding the motion, said that during the past year the Council and Executive of the Institute had had a very hard and, in some respects, a trying time. They had had social duties to perform, and the not less arduous duties of carrying on the ordinary business of the Institute, and the additional labour involved in the investigation of the new mortality experience. He was pleased that Mr. Bailey should have added the name of Mr. Ackland. As one who had seen a great deal of the working of the mortality experience, he was sure that when they were put in possession of the whole of the results of Mr. Aekland's investigations they would congratulate themselves on having such a man who could take up that work and earry it out. The Institute was admirably served by the Council and Executive, and must be congratulated on having such an excellent President. He would like to add a word on behalf of the Examiners, whose duties were extremely arduous and were invariably performed with very great zeal, and to whom a very great debt of gratitude was due. The labours increased every year, and the Institute was to be congratulated on having men sufficiently patriotic to come forward and help the Council in that important work.

The motion was carried with acclamation, and the President heartily tendered the sincere thanks of all the officials to the members.

On the motion of Mr. A. Barton, seconded by Mr. R. W. Barton, the best thanks of the meeting were given to Messrs. Geo. R. Jellicoe, H. R. Harding, and C. H. E. Rea, for their kindness in auditing the accounts.

Mr. Jellicoe, in returning thanks, and on retiring from the office of auditor, said that the way in which the accounts were presented by Mr. Wiggins (the Assistant Secretary) and his staff was beyond all praise, and to them were really due the thanks which were accorded to the auditors.

### COLONIAL EXAMINATIONS.

Examinations were held on 22, 23, 25, and 26 April, at Sydney, Melbourne, Adelaide, Wellington, Montreal, and Toronto, with the following results:

## PART I.

Twenty-three Candidates sent in their names, of whom seventeen presented themselves, and eleven passed as follows:

```
Class I:

Corbett, E. S. (Melbourne).
Macphail, F. C. (Melbourne).
Papps, P. C. H. (Toronto).
Mower, G. S. (Montreal).

Class III:

Hay, J. D. (Wellington).
```

Ley, James (Melbourne). Thorne, C. M. (Sydney). Tipping, O. (Melbourne).

### Part II.

Twenty-five Candidates sent in their names, of whom eighteen presented themselves, and seven passed as follows:

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Class I:
Wood, A. B. (Montreal).

Class III:
Sutherland, J. (Melbourne).

Class III:
Kaufman, H. N. (Montreal).
Kingsbury, J. W. (Sydney).
Moore, J. P. (Sydney).
Reid. E. E. (Toronto).
Stuckey, E. J. (Adelaide).
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# PART III (SECTION A).

Seven Candidates sent in their names, of whom four presented themselves, and three passed as follows:

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Class III:—

† Elliott. C. A. (Sydney).

* Macaulay, T. B. (Montreal).

Thodey, R. (Sydney).
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# PART III (SECTION B).

Four Candidates sent in their names, of whom two presented themselves, and one passed as under:

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Class II:

* Macaulay, T. B. (Montreal).
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The Candidate marked (\*) passed in both sections, and with the Candidate marked (†) has now completed the examination for the Class of Fellow.

# Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1897:

By whom presented (when not purchased).

Accountants and Auditors, Society of. List of Members, &c. 1898.

The Society.

Actuarial Society of America.
Proceedings of the.

The Society.

Actuarial Society of Edinburgh. Proceedings of the.

The Society.

American Statistical Association. Quarterly Publication. March 1898.

The Association.

Archer (J. A.) F.I.A.

Tables for Repayment of Loans. Third Edition, 1897.

The Author.

Australian Mutual Provident Society. Forty-ninth Annual Report, 1898.

The A. M. P. Society.

Austria.

Die Österr.-Ungar. Privatversicherungs-Gesellschaften im Jahre, 1896. Die Dentschen Privatversicherungs-Gesellschaften im (

Anonymous.

Jahre, 1896. Statuten des Verbandes der Oesterr und Ungar Versicherungs-Techniker.

The Society.

Bagehot (Walter).

Lombard Street. A Description of the Money Market, 1896.

Purchased.

Bastable (C. F.) M.A., LL.D.

The Theory of International Trade with some of its applications to Economic Policy. 2nd Edition, 1897.

Purchased.

Belgium.

Bulletin de l'Association des Actuaires Belges, 1897. L'Association.
Compte Rendu des Operations et de la Situation de la
Caisse Générale d'Epargne et de Retraite, 1897. Belgian Government.

Bimetallism, Sundry Papers on.

H. Erdmann.

Bourne's Insurance Directory for 1898.

Wm. Schooling.

Bourne's Assurance Guide.

Wm. Schooling.

Bourne's Handy Assurance Manual, 1897, 1898.

Wm. Schooling.

Brabrook (Edward William) F.S.A. The Law of Friendly Societies.

Purchased.

Burdett's Official Intelligence of British, American, and Foreign Securities, 1898.

Purchased.

By whom presented (when not purchased).

Carment (D.).

Values of Endowment Assurance Policies, 2nd Edition.

The Author.

Chartered Accountants, Institute of.

List of Members, &c., 1897, 1898.

The Institute.

Civil Engineers, The Institution of.

List of Members, 1897, 1898.

The Institution.

Coglan (T. A.).

The Wealth and Progress of New South Wales, 1895-6 (vol. ii) and 1896-7.

The Author.

Croydon, Corporation of.

The Corporation Act, 1893.

Provident Funds, 1894.

The Corporation and The Corporation.

Darwin (Leonard) Major.

Bimetallism. A Summary and Examination of the Arguments for and against a Bimetallic System of Currency, 1897.

Purchased.

Dawson (Miles M.).

Practical Lessons in Actuarial Science, with Tables.

The Spectator Co.
of N. Y.
The Author.

Del Mar (Alexander) M.E.

History of Monetary Systems, 1895.

Principles of Insurance Legislation.

Purchased.

De Morgan (Augustus).

An Essay on Probabilities, 1841.

Purchased.

Denmark.

Logaritmetabel indeholdende Logaritmer og Antilogaritmer. By V. E. Gamborg.

The Author.

Dibdin (Lewis T.) and Errington (Francis H. L.).

The Acts relating to Estate, Probate, Legacy, and Succession Duties. 4th Edition, 1897.

Purchased.

Edgcumbe (Sir Robert P.).

Popular Fallacies regarding Bimetallism, 1896.

Purchased.

Farrer (The Right Hon. Lord).

Studies in Currency, 1898.

Purchased.

France.

Bulletin de l'Institut des Actuaires Français.

L'Institut.

Geddes (Patrick) F.R.S.E.

The Classification of Statistics and its Results, 1881.

Purchased.

Germany.

Assecuranz-Almanach, 1898.

Das Gesetz der Kleinen Zahlen, von Dr. L. von

Anonymous.

Bortkewitsch. The Author.

By whom presented (when not purchased).

Germany—(continued).

Das Problem vom Risiko in der Lebensversicherung. Von Dr. Karl Wagner.

The Publisher.

Grundriss einer Vorlesung über die Arbeiterversicherung im Deutschen Reich. Von Dr. L. von Bortkewitsch.

The Author.

Zeitschrift des Königlich Preussischen Statistischen Bureaus, 1897 and 1898. The German Government.

Zustand und Fortsbritte der Dentsehen Lebensversicherungs-Austalten im Jahre, 1897.

The Publisher.

Gompertz (Benjamin).

The Application of a Method of Differences to the species of Series whose Sums are obtained by Mr. Landen, by the help of Impossible Quantities, 1806.

Purchased.

Green (Charles H.).
Employers' Liability. Its History, Limitations and Extension.

The Author.

On the Workmen's Compensation Act, 1897. Employer and the Insurance Companies.

Henry (Jardine).

The Handbook for Life Assurers.

The Clergy Mutual Assurance Society.

Holland.

Bibliothèque de l'Utrecht Compagnie d'Assurances. L'Utrecht Compagnie. Catalogue.

Een Bijzondere Klasse van Abelsche Integralen. By W. A. Poort.

The Author. Mr. George King.

Heuvelijk en Sterfte. By. B. Turksma. Jaarboekge van de Vereeniging voor Levensverzekering.

The Algemeene Maatschappij.

1898. Le Nouveau Projet de Loi Hollandais sur les Assurances

The Author.

sur la Vie. "By Dr. J. van Schevichaven. Mededeelingen der Vereeniging voor Levensverzekering. Transactions, 1-5.

The Society.

Toepassing der Waarschijnlijkheids-Rekening op Levensverzekering en Sterfte-Statistick. By J. H. Peek.

The Author.

Indemaur (John) and Thwaites (Charles).

The Student's Guide to the Law of Real and Personal Property and Conveyancing.

Purchased.

Italy.

Bollettino della Associazione Italiana per l'ineremento della Scienza Degli Attuari.

The Associazione.

King (George).

The Theory of Finance. 3rd Edition, 1898. Four copies.

Purchased.

Life Assurance Companies' Returns for 1897.

The Board of Trade.

Life Assurers' Hand-Book-1876, 1877, and 1879.

The Clergy Mutual Assurance

By whom presented (when not purchased),

Life Offices Association.

Reprint of Papers, 1894 to 1898.

The Association.

Metropolitan Life Insurance Company of New York. Souvenir Number of the "Weekly Bulletin."

Anonymous.

Newman (Philip L.).

On Superannuation.

The Author.

New South Wales, Insurance Institute of.

Sessional Proceedings.

The Institute.

Nicholson (J. Shield) M.A., D.Sc.

A Treatise on Money, and Essays on Monetary Problems. 3rd Edition. 1895.

Purchased.

Norwich Union Life Insurance Society.

Statement of Facts showing the Necessity of instituting an Inquiry into the Management and the Appropriation of the Funds of. 1837.

Purchased.

Oss (S. F. van).

Stock Exchange Values. A Decade of Finance. 1885 to 1895.

Purchased.

Parliamentary Papers.

Colonies.

New South Wales.

Statistical Register for 1896 and previous

Gort. Statistician.

Statistics of the Seven Colonies of Australasia. 1861-96.

N.S.W. Government. Vital Statistics for 1896 and previous years. N.S.W. Government. New Zealand.

Friendly Societies. Twentieth and Twentyfirst Annual Reports by the Registrar of. Government Insurance Department. Annual

Report, 1896.

Official Year Book, 1897. Results of a Census taken for the Night of the 12th April 1896.

Statistics of the Colony of, 1896.

South Australia.

Friendly Societies. Second Report of the Proceedings of the Public Actuary relating

Queensland.

Friendly Societies. Third Quinquennial Report of the Registrar on the Valuations

Victoria. Friendly Societies. Nineteenth Annual Report of the Registrar for the Year 1896.

S.A. Government.

N.Z. Government.

N.Z. Government.

N.Z. Government.

N.Z. Government. N.Z. Government.

The Registrar.

The Registrar.

Z

Pearson (Karl) M.A., F.R.S.

The Chances of Death, and other Studies in Evolution. 2 vols. 1897.

Purchased.

| By   | whom  | presented   |
|------|-------|-------------|
| (whe | n not | increhased) |

Pereire (Eugene).

Tables de l'Intérêt Composé des Annuités et des Rentes Viageres. 2nd Edition. 1873.

Purchased.

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Periodicals.

Accountants' Magazine. Institute of Bankers' Journal. The Institute. Insurance Record, 1897. The Editor. Insurance Register, 1898. C. S. E. Layton. London Mathematical Society's Journal. The Society. Official Year Book of the Scientific and Learned Societies of Great Britain and Ireland, 1898. Post Magazine. Post Magazine Almanae. Royal Statistical Society's Journal. Zeitschrift für Versicherungs-Recht Wissenschaft.

Purchased. The Editor. The Editor. The Society. The Editor.

Phelps (Edward Bunnell).

War Risks.

Henry Cockburn.

Porter (James Biggs).

The Laws of Insurance. 3rd Edition, 1898. Three Copies.

Purchased.

Positive Government Security Life Assurance Society. Observations in reply to "Actuary's" Notes, 1873.

Purchased.

Roberts-Jones (M.).

Handbook to the Workmen's Compensation Act, 1897.

The Author.

Russia.

Tableau des Opérations des Sociétés d'Assurance Mutuelle Contre l'incendie. Exercices 1894, 1895. Tableau des Opérations des Compagnies d'Assurance. Exercice 1894.

Russian Government.

Schooling (William) F.R.A.S.

New Life Business and its cost.

The Author.

Scratchley (Arthur) M.A., F.R.A.S.

A Treatise on Life Assurance Societies, Friendly Societies, and Savings Banks, 1856.

Purchased.

Seligman (Edwin R. A.).

Essays in Taxation, 1895.

Purchased.

Smart (William) M.A.

An Introduction to the Theory of Value, 1891.

Purchased.

Smith (Samuel).

The Nationalization of the Land, 1884. Free Trade versus Reciprocity, 1881. Purchased. The Bi-Metallic Question, 1887. Bi-Metallic Money, 1881.

Solvay (Ernest).

Social Comptabilism.

The Author.

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South African Mutual Life Assurance Society.

Mortality Experience, 1845-1895. Two Copies.

Chas. Gordon.

Sprague (T. B.) M.A., LL.D. On Lost Policies.

The Author.

Strachan (T. Y.) F.I.A., A.C.A.

The Midland District Miners' Fatal Accident Relief Society. Actuarial Report.

The Author.

Sweden.

Lifförsäkrings-Aktiebolaget Nordstjernan, Stockholm. Grafiska Tabeller, 1872–1896.

The Nordstjernan.

Underdånigt Betänkande med Förszag till Lagstiftning angäende in-och Utländska Försäkringsanstalters Verksamhet.

Anonymous.

Switzerland.

Rapports du Bureau Fédéral des Assurances sur les Entreprises Privées en Matière d'Assurance en Suisse, 1895, 1896.

The Swiss Government.

Turnbull (A. D. Lindsay).

Life Office Accounts and the Card System. Three Copies.

The Author.

United States of America.

The Economic relation of Life Insurance to Society and State.

The American Academy of Political and Social Science.

Thirty-ninth Annual Report by the Superintendent of Insurance for the State of New York, 1898. New York Life Ins. Co.

Victoria, Insurance Institute of.

Report, 1896.

The Insurance Institute.

Whitaker's Almanac, 1898.

Purchased.

Willich (Charles M.).

Popular Tables for the Value of Life-hold, Leasehold, The Clergy
and Church Property Renewal Fines, &c. 1853.

\*\*Cociety.\*\*

Willis (W. Addington) LL.B.

The Workmen's Compensation Act, 1897.

Purchased.

Yorkshire, Insurance Institute of.

Report, 1896-7.

The Insurance Institute.

The following additional copies of works already in the Library have also been added:

Biden (W. Downing). Practical Rules for Valuers.

By whom presented (when not purchased). The Clergy
Mutual Assurance
Society.

Brown (Samuel) F.I.A.

A few Thoughts on Commission, Divisions of Profits. Selection of Lives, The Mortality in India, and other subjects relating to Life Assurance. 1849.

Purchased.

Decimal Association (Formed June 12th 1854).

Proceedings: With an Introduction by Prof. De Morgan. and Notes.

Purchased.

De Moivre (A.) F.R.S.

The Doctrine of Chances, 1718.

Purchased.

Giffen (Robert).

American Railways as Investments, 1872.

Purchased.

Insurance Register for 1896.

The Clergy
Mutual Assurance
Society.

Jones (David).

On the Value of Annuities and Reversionary Payments, with numerous Tables. (2 vols.). 1844.

Purchased.

Milne (Joshua).

A Treatise on the Valuation of Annuities and Assurance on Lives and Survivorships. Vol. 2. 1815.

The Clergy
Mutual Assurance
Society.

Morgan (William).

The Doctrine of Annuities and Assurances on Lives and Survivorships, stated and explained, 1779.

Purchased.

Parliamentary Papers.

Registrar General's Reports on Births, Deaths, and The Clergy
Marriages in England for 1843 and 1847-48.

\*\*The Clergy Mutual Assurance Society.\*\*

# LIST OF MEMBERS

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FOUNDED 1848.

INCORPORATED BY ROYAL CHARTER 29TH JULY, 1884.

(Corrected to October, 1898.)

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LONDON:

PRINTED BY CHARLES & EDWIN LAYTON, 56, FARRINGDON STREET, E.C.

1898.

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1894 LIEUT.-COL. W. H. OAKES, Stanley House, Granville Road, Sevenoaks,

#### FELLOWS.

Those marked \*\*\* have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

1876\*\*\*Ackland, Thomas Gans, F.S.S., Avalon, 71 Morland-rd, Croydon.

1871\*\*\*Addiscott, Francis, Medical Sickness, Annuity & Life Assur. Soc., 33 Chancery-ln., w.c.

1892 Adlard, Alfred Barton, Law Life Assur. Soc., 187 Fleetstreet, E.C.

1864\*\*\*Adler, Marcus Nathan, M.A., Alliance Assur. Co., Bartholomew-lane, E.C.

1894\*\*\*Aldcroft, William Hancock, Refuge Assur. Co., Oxford-st., Manchester.

1889\*\*\*Allen, Arthur Gregory, 13 Fairfax-road, N.W.

1897\*\*\*\*Allen, John Mayhew, Alliance Assur. Co., Bartholomew-lane, E.C.

1889\*\*\*Anderson, John, Lancashire Insurance Company, 18 Exchange-street, Manchester.

1891\*\*\*Anderson, William Smith, Gresham Life Assurance Nociety, St. Mildred's-house, Poultry, E.C.

1885\*\*\*Andras, Henry Walsingham, University Life Assur. Soc., 25 Pall Mall, s.w.

1885\*\*\*Ansell, Hubert,

London Scottish-American Trust,
75 Lombard-street, E.C.

Date of becoming a Fellow.

1896\*\*\*Archer, Joseph Alfred, Ecclesiastical Commission, 10 Whitehall-place, s.w.

1850\*\*\*Bailey, Arthur Hutcheson, F.S.S. (Ex-President), 26 Mount Ephraim-rd, Streatham, s.w.

1896\*\*\*Baker, Henry James, Metropolitan Life Assur. Soc., 13 Moorgale-street, E.C.

1885\*\*\*Barnes, Joseph Howard, F.S.S., Pelican Life Insurance Co., 70 Lombard-street, E.C.

1895\*\*\*\*Barrand, Arthur Rhys, Prudential Assurance Company, Holborn-bars, E.C.

1890\*\*\*Bearman, Harry, Gresham Life Assur. Soc., St. Mildred's-house, Poultry, E.C.

1889\*\*\*Bell, Frederick,

Imperial Life Insurance Co.,

1 Old Broad-street, E.C.

1867\*\*\* Berridge, George William, Dunton-ldg., The Knoll, Beckenham, Kenl.

1886\*\*\*Berry, Berry Alfred, B.A., London Life Association, Ltd., 81 King William-street, E.C.

1895\*\*\*Besant, Arthur Digby, B.A., Clerical, Medical & General Life Assur. Soc., 15 St. James'ssquare, s.w.

Those marked - have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

- 1879 Besso, Marco, F.S.S., Superior Trade Council, Rome.
- 1894\*\*\*Blackadar, Alfred Kimball, M.A., Mem. Act. Soc. Amer., Government Insur. Department, Ottawa, Canada.
- 1883\*\*\*\*Blakey, James, National Debt Office, 19 Old Jewry, E.C.
- 1897\*\*\* Bradshaw, Thomas, Mem. Act. Soc. Amer., The Imperial Life Assurance Co. of Canada, Toronto, Canada.
- 1875 Browne, Thomas G. C.,

  Guardian Assurance Company,
  11 Lombard-street, E.C.
- 1887 Browne, Willis, India Office, s.w.
- 1866\*\*\*Bumsted, David Alexander, General Reversionary and Inrestment Co., Ltd., 26 Pall-mall, s.w.
- 1894\*\*\*Burn, Joseph,
  Prudential Assurance Company,
  Holborn-bars, E.C.
- 1881\*\*\* Burridge, Arthur Francis (Hon. Secretary),

  Equity and Law Life Ass. Soc.,
  18 Lincoln's-inn-fields, w.c.
- 1887\*\*\*Byers, Frederick Timothy Mason, Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, Westminster, s.w.
- 1888\*\*\*Calderon, Henry Philip, Institute of Actuaries, Stapleinn-hall, Holborn, W.c.
- 1871\*\*\*Carment, David, F.F.A., Mem. Act. Soc. Amer., Australian Mutual Provident Society, Sydney, Australia.
- 1889\*\*\*Chatham, James, F.F.A., F.S.S., Scottish Life Assurance Co., 19 St. Andrew-sq., Edinburgh.
- 1875 Cherriman, J. B., Prof., M.A., c/o The Bank of Montreal, Abchurch-lane, E.C.
- 1883 Chisholm, James, F.F.A., Mem. Act. Soc. Amer., Imperial Life Insurance Co., 1 Old Broad-street, E.C.

Date of becoming a Fellow.

- 1895\*\*\*\*Clarke, Arthur Harold, Clerical, Medical and General Life Assurance Society, 15 St. James's-square, s.w.
- 1863 Clirchugh, William Palin, F.S.S., London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.
- 1879 Cockburn, Henry, F.F.A. (VICE-PRESIDENT),

  North British and Mercantile
  Insurance Co., 61 Threadneedlestreet, E.C.
- 1886 Cockburn, Henry Robertson, F.F.A., Scottish Provident Institution, 6 St. Andrew-sq., Edinburgh.
- 1898\*\*\*Cockman, Arthur Charles Roadnight, Liverpool and London and Globe Insurance Co., 7 Cornhill, E.C.
- 1884\*\*\*\*Colenso, Francis Ernest, M.A., Eagle Insurance Company, 79 Pall-mall, s.w.
- 1864\*\*\*Coles, John, F.S.S., 39 Throgmorton-street, E.C.
- 1882\*\*\*\*Colquhoun, Ernest, Legal and General Life Assur. Society, 10 Fleet-street, E.C.
- 1875\*\*\*\*Cooke, Thomas Homans, Glendower, Teignmouth-road, Torquay.
- 1889\*\*\*Cooper, Walter George, Norwich Union Life Insurance Society, Norwich.
- 1878\*\*\*\*Crisford, George Stephen, Rock Life Assurance Company, 15 New Bridge-street, E.C.
- 1889\*\*\*Cross, Robert, Atlas Assurance Company, 92 Cheapside, E.C.
- 1864 Curtis, Frank Allan, 3 Ennismore-gardens, Salisburyroad, Dover.
- 1864 Cutcliffe, George,

  Coombe-house, Witheridge, North

  Devon.
- Under the Charter. Davies, Griffith, 11 Freeland-road, Ealing, w.
- 1898\*\*\*Dawson, Charles Pearl, Imperial Life Insurance Co., 1 Old Broad-street, E.C.

Those marked \*\*\* have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

- 1855\*\*\*Day, Archibald (Ex-President), Clifton-lodge, St. John's-parkroad, Blackheath, s.E.
- 1885\*\*\* Day, Stanley, Marine and General Mutual Life Assurance Society, 14 Leadenhall-street, E.C.
- 1897\*\*\*Day, William Reginald, Scottish Widows' Fund Life Assurance Society, 35 Blighstreet, Sydney, Australia.
- 1869Deuchar, David, F.F.A., F.R.S.E., Caledonian Insurance Company, 19 George-street, Edinburgh.
- Deuchar, John James Walker, 1883F.F.A., Norwick Union Life Insurance Society, Norwich.
- 1582 Dewey, Thomas Charles, Prudential Assurance Company, Holborn-burs, E.C.
- 1886\*\*\* Dickinson, Arthur Lowes, M.A., F.C.A., 19 Coleman-street, E.C.

Under Docker, Edward, M.A.,

the Charter. Dudley-house, Spring-grove, Isle-

- 1887 Douglas, Gordon, F.F.A., Life Association of Scotland, 82 Princes-street, Edinburgh.
- 1875\*\*\* Dunean, James Heron, Royal Exchange Assurance Corp., Royal Exchange, E.C.
- 1874\*\*\*Dunean, John, Clergy Pensions Institution and Ecclesiastical Insurance Office, 11 Norfolk-street, Strand, W.C.
- 1869Dymond, Joseph John, Friends' Provident Institution, Bradford, Yorkshire.
- 1872Eccles, Yvon Richard, Scottish Amicable Life Assurance Society, 1 Threadneedle-st., E.C.
- 1897\*\*\*Elder, Kenneth William, Atlas Assurance Company, 92 Cheapside, E.C.
- 1881 Elderton, Frederick Fairlie, F.F.A., City of Glasgow Life Assurance Co., 30 Renfield-street, Glasgow.

- Date of becoming a Fellow.
- 1898\*\*\*Elliott, Charles Alfred, Australian Mutual Pravident Society, Sydney, Australia.
- 1889\*\*\* Faulks, Joseph Ernest, B.A., F.S.S., Law Life Assurance Society, 187 Fleet-street, E.C.
- 1897\*\*\*Fellows, Rowland Hill, F.S.S., British Empire Mutual Life Assurance Company, 4 & 5 King William-street, E.C.
- 1864\*\*\*Finlaison, Alexander John, C.B. (Ex-President), Mem. Act. Soc. Amer., National Debt Office, 19 Old Jewry, E.C.
- Under the Charter. Fisher, Richard Charles. 2 Walsingham-rd., W. Brighton.
- 1892\*\*\*Foot, Herbert, B.A., Northern Assurance Company, 1 Moorgate-street, E.C.
- 1884 Frankland, Frederick - William, F.S.S., Mem. Act. Soc. Amer., New York Life Insurance Co., 346 & 348 Broadway, New York.
- 1897\*\*\*Fraser, Duncan Cumming, M.A., Royal Insurance Co., Liverpool.
- 1895\*\*\*Fulford, Frederick Wesley, Prudential Assurance Company, Holborn-bars, E.C.
- 1887 Gillison, John Brotch, F.F.A., National Mutual Life Association of Australasia, Corner of Collins and Queen-streets, Melbourne, Australia.
- Gordon, Charles, F.F.A., 1878 South African Mutual Life Assurance Society, Cape Town.
- 1882\*\*\*Graham, James, F.F.A., Australian Widows' Fund Life Assurance Society, Collins-streetwest, Melbourne, Australia.
- 1886 Gunn, Niel Ballingal, F.F.A., Scottish Amicable Life Assurance Society, 35 St. Vincent. place, Glasgow.
- 1864 Harben, Sir Henry, Prudential Assurance Company, Holborn-bars, E.C.
- 1880\*\*\*Hardy, George Francis, Universal Life Assurance Soc. 1 King William-street, E.C.

Those marked — have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

- 1870\*\*\* Hardy, Ralph Price, 17 Gracechurch-street, E.C.
- 1893\*\*\* Harris, Arnold Stoughton, M.A., Clerical, Medical & General Life Assur. Soc., 15 St. James'ssquare, s.w.
- 1892\*\*\*Hart, James Robert, British Empire Mut. Life Assur. Co., 4 & 5 King William-st., E.C.
- 1879 Harvey, Chas. J., 123 Reid-avenue, Brooklyn, New York.
- 1888\*\*\*Hemming, Arthur George, F.S.S., City of Glasgow Life Assur. Co., 12 King William-street, E.C.
- 1896\*\*\*Henderson, Robert, B.A., Equitable Life Assurance Soc., 120 Broadway, New York.
- 1864 Hendriks, Augustus, F.S.S. (EX-PRESIDENT), Liverpool and London and Globe Insur. Co., 7 Cornhill, E.C.
- Under the Charter. Hendriks, Frederick, F.S.S., 7 Vicarage-gate, Kensington, W.
- 1883 Hewat, Archibald, F.F.A., F.S.S., Edinburgh Life Assurance Co., 22 George-street, Edinburgh.
- 1874\*\*\* Higham, Charles Daniel (TREAS.), London Life Association, Ltd., 81 King William-street, E.C.
- 1898\*\*\*Hodgson, William Horsford, Law Life Assurance Society, 187 Fleet-street, E.C.
- 1888\*\*\* Hopkins, William Raynes, London & Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.
- 1890\*\*\*Hovil, Lewis Frederick, National Provident Institution, 148 Gracechurch-street, E.C.
- 1871\*\*\*\* Hughes, William (VICE-PRES.), Prudential Assurance Company, Holborn-bars, E.C.
- 1894\*\*\* Hutcheson, William Anderson, F.F.A., Scottish Widows' Fund Life Assur. Soc., 28 Cornhill, E.C.
- 1893\*\*\* Hutton, William, F.F.A., Scottish Amicable Life Assur. Soc., 1 Threadneedle-street, E.C.
- 1869\*\*\* Justican, Edwin, F.S.S., Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.

Date of becoming a Fellow,

- 1876\*\*\* Kember, Walter, Scottish Amicable Life Assur. Soc., 60 Castle-street, Liverpool.
- 1897\*\*\*\*Kentish, Owen, Economic Life Assurance Soc., 6 New Bridge-street, E.C.
- 1874\*\*\*King, George, F.F.A., Mem. Act. Soc. Amer., London Assurance Corporation, 7 Royal Exchange, E.C.
- 1887\*\*\*Kyd, Thomas, F.F.A., Northern Assurance Company, 1 Union-terrace, Aberdeen.
- 1876\*\*\*Laing, Francis, F.F.A., Northern Assurance Company, 1 Moorgate-street, E.C.
- 1882 Lancaster, William John, Prudential Assurance Company, Holborn-bars, E.C.
- 1894\*\*\*Laughton, Alexander Millar, F.F.A.,
  National Mutual Life Assoc. of
  Australasia, Limited, Corner of
  Collins and Queen-streets, Melbourne, Australia.
- 1887\*\*\*Lemon, William Kent, Barristerat-Law, 5 Pump-court, E.C.
- 1896\*\*\* Levine, Abraham, M.A., National Mutual Life Assur. Soc., 39 King-st., Cheapside, E.C.
- IS96\*\*\* Lewis, John Norman, F.F.A., Scottish Widows' Fund Life Assur. Soc., 41 Westmortandstreet, Dublin.
- 1892\*\*\*Lidstone, George James, Alliance Assur. Co., Bartholomew-lane, E.C.
- 1898\*\*\*Macaulay, Thomas Bassett, Mem. Act. Soc. Amer., Sun Life Assurance Co. of Canada, Montreal, Canada.
- 1885 Mackenzie, Alexander George, F.F.A., Hillside, Marischal-rd., Lee, S.E.
- 1870\*\*\*Manly, Henry William, Mem. Act. Soc. Amer. (President), Equitable Life Assurance Soc., Mansion-house-street, E.C.
- 1898\*\*\*Marchbank, Frank, A.F.A., Royal Exchange Assur. Corp., 8 Mosley-st., Newcastle-on-Tyne.

Those marked - \* have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

- 1890\*\*\*Marks, Geoffrey (Honorary Librarian), National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.
- 1897\*\*\*May, George Ernest,

  Prudential Assurance Company,

  Holborn-bars, E.C.
- 1875 McCabe, William, LL.B., F.S.S., Mem. Act. Soc. Amer., North American Life Assur. Co., North American Life Building, 112-118 King-st.-west, Toronto, Canada.
- 1874 McClintock, Emory, Mem. Act. Soc. Amer., Mutual Life Insurance Company of New York, New York.
- 1894\*\*\* McDonald, John,
  Prudential Assurance Company,
  Holborn-bars, E.C.
- 1864 McGedy, Frank, 48 Thurlo-square, South Kensington, w.
- 1883\*\*\*\*McGowan, James, B.A., Colonial Secretary's Office, Cape Town.
- 1851\*\*\* Meikle, James, F.F.A., Mem. Act. Soc. Amer., Scottish Provident Institution, 6 St. Andrew's-sq., Edinburgh.
- 1897\*\*\* Miller, Neville, London Assurance Corporation, 7 Royal Exchange, E.C.
- 1864\*\*\*Miller, Thomas, 36 Rock-pk., Rockferry, Cheshire.
- 1893\*\*\*Milner, John William, North British & Mercantile Insur. Co., 61 Threadneedle-street, E.C.
- 1892\*\*\* Milton, Henry, M.A., 37 Threadneedle-street, E.C.
- 1890\*\*\*Molyneux, Arthur Ernest,
  Provident Clerks' Mutual Life
  Assurance Association, 27 y 29
  Moorgate-street, E.C.
- 1897\*\*\*\*Moors, Elphinstone MacMahon, M.A., University of Sydney, Australia.
- 1896\*\*\*Moran, Joseph Flack,
  Marine & General Mutual Life
  Assurance Society, 14 Leadenhall-street, E.C.

Date of becoming a Fellow,

- 1895\*\*\*Muter, Percy.
  - New Zealand Government Life Insurance Department Wellington, New Zealand,
- 1888\*\*\*Nash, Willie Oscar, Law Reversionary Interest Soc., Ltd., 24 Lincoln's-inn-fields, w.c.
- 1883 Neison, Francis G. P., F.S.S., 19 Abingdon-st., Westminster, s.w.
- 1888\*\*\*Newman, Philip Lewin, B.A., Yorkshire Insurance Co., York.
- 1848 Newman, William Lewin, 22 St. Paul's-square, York.
- 1865 Newton, Algernon, M.A., c o London & Westminster Bank, 94 & 96 High-st., Kensington, W.
- 1887\*\*\*Nightingale, Harry Ethelston, Royal Exchange Assurance Corporation, Royal Exchange, E.C.
- 1864 Pearson, Arthur,

  Betchworth-house, The Bank,

  Highgate, N.
- 1891\*\*\* Phelps, William Peyton, M.A., Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields, w.c.
- 1850\*\*\* Porter, Henry William, B.A., 73 Inverness-ter., Hyde-park, w.
- Under the Charter. Priestley, John George, 44 St. German's-road., Foresthill, s.e.
- 1891\*\*\*Pulley, William Pritchard, Norwich Union Life Insur. Soc., 71 & 72 King William-st., E.C.
- 1864 Ramsay, Alexander Gillespie, F.S.S., Mem. Act. Soc. Amer., Canada Life Assurance Company, Hamilton, Canada.
- 1897\*\*\*Rees, Martin, 18 Bersham-road, Wrexham,
- 1898\*\*\*Robinson, George Frederick, Legal and General Life Assur. Society, 10 Fleet-street, E.C.
- 1888\*\*\*Rusher, Edward Arthur, F.S.S., Prodential Assurance Company, Holborn-bars, E.C.
- 1882\*\*\*Ryan, Gerald Hemmington (Vice-President), Mem. Act. Soc. Amer.,

  British Empire Mutual Life

Assurance Co., 4 & 5 King Williamstreet, E.C.

Those marked " have passed the Examination for the Class of Fellow.

Date of becoming a Fellow.

- 1898\*\*\*Salmon, Richard George, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1883 Saunders, Harris Charter Lindon, F.R.A.S., 17 Gracechurch-street, E.C., and 95 Queen's-gate, s.w.
- ISS6\*\*\*Schooling, Frederick (HONORARY LIBRARIAN), Prudential Assurance Company, Holborn-bars, E.C.
- 1896\*\*\*Sim, William Abernethy, F.F.A., Scottish Union and National Insurance Co., 35 St. Andrewsquare, Edinburgh.
- ISS6\*\*\*Simon, Louis Michael,

  Metropolitan Life Assur. Society,
  13 Moorgate-street, E.C.
- 1873 Slater, Duncan McLauchlau,
  Oriental Government Security
  Life Assurance Company, Ltd.,
  Albert-building, Bombay.
- 1875\*\*\*\*Smither, Arthur,

  National Provident Institution,
  48 Gracechuvch-street, E.C.
- 1881\*\*\*Somerville, William Finlay, Liverpool and London and Globe Insur. Co., 1 Dale-st., Liverpool.
- 1877\*\*\*\*Sorley, James, F.F.A., C.A., F.R.S.E., Pelican Life Insurance Company, 70 Lombard-street, E.C.
- 1898\*\*\*Spencer, John, English and Scottish Law Life Assurance Assoc., 12 Waterlooplace, s.w.
- 1894\*\*\*Sprague, Alfred Ernest, B.Sc., M.A., F.F.A., Scottish Equitable Life Assur. Society, 26 St. Andrew-square, Edinburgh.
- 1857 Sprague, Thomas Bond, M.A., LL.D., F.F.A., F.S.S., F.R.S.E. (EX-PRESIDENT), Mem. Act. Soc. Amer., Scottish Equitable Life Assur. Society, 26 St. Andrew-square,
- Edinburgh.

  1896\*\*\*Stahlschmidt, Louis,

  Imperial Life Insurance Co.,

  1 Old Broad-street, E.C.

Under the Charter. Stephenson, John Ware, 186 Clapham-road, s.w.

Date of becoming a Fellow.

Under Stevens, Charles,

- Charter. Provident Life Assurance Co., 50 Regent-street, w.
- 1888 Stewart, John, F.F.A., City of Glasgow Life Assur. Co., 30 Renfield-street, Glasgow.
- 1898 Stirling, Robert, F.F.A.,

  Law Union & Crown Insurance
  Co., 126 Chancery-lane, w.c.
- I868\*\*\*Strachan, Thomas Young, F.C.A., Mowbray-house, Norfolk-st., w.c.
- 1892\*\*\*Straker, Edward Robert, British Empire Mutual Life Assurance Co., 45; 5 King Williamstreet, E.C.
- 1878\*\*\*Straker, Frank Arthur, Legal and General Life Assur. Society, 10 Fleet-street, E.C.
- 1884\*\*\*Stuart, John Moody, F.F.A., 16 St. Helen's-place, E.C.
- 1889\*\*\*Tarn, Arthur Wyndham, Westminster and General Life Assurance Association, 28 Kingstreet, Covent-garden, w.c.
- 1887 Teece, Richard, F.F.A., F.S.S., Mem. Act. Soc. Amer., Australian Mutual Provident Society, Sydney, Australia.
- 1872 Templeton, Col. John M., C.M.G., National Mutual Life Association of Australasia, Melbourne, Australia.
- 1886 Tennant, John Bell, Friends' Provident Institution, Bradford, Yorkshire.
- 1864\*\*\*Terry, James, Hernlee, Lyme Regis, Dorset.
- 1889\*\*\*Thiselton, Herbert Cecil, F.F.A., Mem. Act. Soc. Amer., Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.
- 1893\*\*\*Thomas, Ernest Charles, Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.
- 1895\*\*\*Thompson, Herbert Archer, B.A., *Hand-in-Hand Insurance Soc.*, 26 New Bridge-street, E.C.
- 1880 Thomson, Robert,

  Colonial Mutual Life Assurance
  Society, Collins street west,
  Melbourne, Australia.

These without have present the Esamination for the Class of Fellow.

Date of becoming a Fellow.

- 1876 Thomson, Spencer Campbell, B.A., F.F.A.,
  Standard Life Assurance Co., George-street, Ediaburgh.
- 1893\*\*\*Thorne, Alfred Charles, Equity & Law Life Assur. Soc., 18 Linvolu's-inn-fields, W.C.
- 1891\*\*\*Tilt, Robert Ruthven, General Reversionary & Investment Co., Ltd., 26 Pall-mall, s.w.
- 1881\*\*\*Todd, George, M.A., Economic Life Assurance Society, New Bridge-street, E.C.
- 1894\*\*\*Todhunter, Ralph, M.A., National Matual Life Assurance Society, 39 King-st., Cheapside, E.C.
- 1878 Turnbull, Andrew Hugh, F.F.A., Noottish Widows Fund Life Assur. Soc., 9 St. Andrew square, Edinburgh.
- Under the Charter Tyndall, William Henry, F.S.S., F.R.Met.S.,

  Morlands, Oxford-road, Redhill, Survey.
- 1889 Wallace, Thomas, F.F.A., North British & Mercentile Insurance Co., Edinburgh.
- 1888\*\*\*Warner, Samuel George, Law Union & Crown Insur. Co., 126 Chancery-lane, W.C.
- 1893\*\*\*Watson, Alfred William, Manchester Unity Friendly Soc., Nottingham.

Date of becoming a Fellow.

- 1895\*\*\*Watson, James Douglas, English & Scottish Law Life Assur. Assoc., 12 Waterloo-place, s.w.
- 1897\*\*\* Weedon, Charles Thomas, London Assurance Corporation, 7 Royal Exchange, E.C.
- 1880\*\*\* Whittall, William Joseph Hutchins, Clerical, Medical & General Life Assur. Soc. 15 St. James's-sq., s. W.
- 1864 Wilson, Robert, 44 Talfourd-rd., Camberwell, s.E.
- 1885\*\*\*Wilson, Robert, Jun., General Assurance Company, 103 Cannon-street, E.C.
- Under the tharter. Winser, Thomas Boorman, Sl. Shooter's-hill-road, Black-heath, s.e.
- 1897\*\*\*Wintle, Lancelot Andrewes, Economic Life Assurance Sov., 6 New Bridge-street, E.C.
- 1884\*\*\*Woods. Ernest (HONORARY SECRETARY), Westminster and General Life Assur. Assur., 28 King-street, Covent-garden, w.c.
- 1875\*\*\*Wyatt, Frank Bertrand (Vice-President). Clergy Mulual Assurance Soc., 2 x 3 The Sanctuary, s.w.
- 1874 Young, Thomas Emley, B.A. F.R.A.S. (EX-PRESIDENT).
  Mem. Act. Soc. Amer.,
  Commercial Union Assur. Co.,
  Ltd., 24, 25 & 26 Cornhill. E.C.

Those marked—have passed the first of the three parts of the Exomination for the Class of Fellow. Those marked—have passed two of the three parts of the Exomination for the Class of Fellow; those marked—a or b have also passed either Section A or Section B of Part III of the Examination.

- 1883\*\* Adam, Harold Livingstone, 2 Versailles-road, Anerley, s.E.
- 1894\*\* Adams, Chas. Edward, B.Sc., F.S.S., Lands and Survey Office, Napier, N. Zealand.
- 1869\*\* Adey, Theodore Henry, \*\*Recottish Provident Institution, 17 King William-street, E.C.
- 1894\*\*aAllin, Samuel John Henry Wallis, Northern Assurance Company, 1 Moorgate-street, E.C.
- 1898\*\* Appleton, Frederick, London Life Association, Ltd., 81 King William-street, E.C.
- 1883\*\* Ashley, John Geo., M.A., War Office, Pall Mall, s.w.
- 1881\*\* Ayling, Charles Stephen, Commercial Union Assur. Co., 20 New Bridge-street, E.C.
- 1885 Barton, Arthur, *United Kent Insurance Institution, Maidstone.*
- 1894\*\*aBarton, Robert Whitchurch.

  Clerical, Medical & General Life
  Assurance Society, 15 St. James'square, s.w.
- 1881 Birks, Edmund Alfred, Yorkshire Insurance Co., York.
- 1873\*\* Block, Robert John,

  Essex villa, Chelsham road,

  Clapham, s.w.
- 1869 Bools, William Edward,

  Liverpool and London and Globe
  Insurance Co., 7 Cornhill, E.C.
- 1873\*\* Boon, Gerald Inglis,

  Law Accident Insurance Society,
  215 Strand, London.
- 1861 Bourne, James Pearce, 6 Frederick-pl., Clifton, Bristol.
- 1889 Bremner, Thomas William, F.F.A.,

  Mutual Life Insurance Co. of

  New York, Sydney, Melbourne,

  Australia.
- 1878\*\* Bridgman, Arthur Henry, Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, w.c.
- 1894\*\*aBrown, Edward Harold,
  Prudential Assurance Company,
  Holborn-bars, E.C.

- Date of becoming an Associate.
- 1898 Brown, Hugh Wyllie, F.F.A., Noottish Union & National Insur, Company, 35 St. Andrew-square, Edinburgh.
- 1896 Brown, George Andrew, Clerical, Medical & General Life Assur. Soc. 15 St. James's-sq., s.w.
- 1884 Brownlow, Arthur Chamberlain,
  Sun Life Assurance Society,
  63 Threadneedle-street, E.C.
- 1898\*\* Buchanan, James, M.A., Scottish Widows' Fund Life Assurance Society, 9 St. Andrewsquare, Edinburgh.
- 1886 Buckley, Thomas John Wesley, 4 Wine Office-ct., Fleet-st., E.C.
- 1894\*\*aBull, Ernest James, Atlas Assur. Co., 92 Cheapside, E.C.
- 1882 Burke, David, F.S.S., Royal Victoria Life Insur. Co., Montreal, Canada.
- 1895\*\* Butterfield, William Thomas, A.C.A., c/o Messrs. James and Edwards, 5 Coleman-street, E.C.
- 1876\* Carter, Eric Mackay, 33 Waterloo-street, Birmingham.
- 1888 Churchward, George Gould, Clerical, Medical & General Life Assurance Society, Mansionhouse-buildings, E.C.
- 1898\*\* Coates, Thomas Linnaeus, North British and Mercantile Insur. Co., 61 Threadneedlestreet, E.C.
- 1871 Cook, Arthur James, M.J.1., Victoria Mutual Assur. Society, Farringdon-street, E.C.
- 1878 Cooke, George,

  Commercial Union Assur. Co.,

  Ltd., 24, 25 & 26 Cornhill, E.c.
- 1897\*\* Coop, Charles Rowland,

  United Kingdom Temperance and
  General Provident Institution,
  5 Bennett's-hill, Birmingham.
- 1891\*\* Coote, Ernest Charles,
  Alliance Assurance Company,
  Bartholomew-lane, E.C.

These part of the register that the first the first parts of the Europeant. It is the Class of Fellow, These marked the passed the of the two pasts of the Erom matheting the Class of Fellow; those marked at or the have also passed either Sest on A or section B of Fart III of the Europeanton.

Date of becoming an Associate.

- 1897\*\*\*aContts. Charles Ronald Vawdrey.

  Hand-in-Hand Insur. Society,
  26 New Bridge-street, E.C.
- 1871 Coutts, Edwin Arthur,
  North British and Mercantile
  Insurance Compuny, Victoriastreet, Nottinghum.
- 1884 Craig, Robert Alexander.

  Abstuiners' and General Assur.

  Co., City Buildings, Birmingham.
- 1884 Dell, Vincent John,

  Equitable Life Assur. Society,

  Mansion-house-street, E.C.
- 1855 Dix, James, Huestdale, Wood-lane, Highgate, N.
- 1881 Donaldson, John,

  Australian Widows' Fund Life

  Assurance Society, Collins streetwest, Melbourne, Australia.
- 1881 Dovey, William Roadly, F.F.A., Mem. Act. Soc. Amer., Citizens' Life Assurance Compung, Castlereugh-street, Sydney, Australia.
- 1870\* Dowson, John,
  Royal Insur. Company, Liverpool.
- 1898\*\* Doyle, Arthur James, *Institute of Actuaries, Staple-inn-hall, Holborn*, w.c.
- 1868\* Eaton, Henry William,

  Liverpool and London and Globe
  Insurance Company, Williamstreet, New York, U.S.A.
- 1872\*\* Evans, William, F.F.A., F.R.S.E.. 38 Morningside-park, Edinburgh.
- 1896\*\* Featherstonehaugh, William Irwin, Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.
- 1897\*\* Findlay, Alexander Wynaud, L.L. B., Prodential Assurance Company, Holborn-bars, E.C.
- 1881 Fisher, Frederick,

  Protential Associance Company,

  Holborn-barn, E.C.
- 1886 Fox, Morris, Mem. Act. Soc. Amer..

  New Zealand Government Life
  Insur, Dept., Wellington, N.Z.

Inte of becoming an As-ociate.

- 1800 Fox, Charles Edward, F.F.A., Standard Life Assurance Co., 83 King William-street, E.C.
- 1898 Fraser, Alexander, Jr., F.F.A., Edinburgh Life Assur. Company, 22 George-street, Edinburgh.
- 1894\*\* Fraser, Thomas John, Australian Alliance Assurance Company, Melbourne, Australia.
- 1873\*\* Gage, Uriah Woodard,

  Universal Life Assur. Society,

  1 King William-street, E.C.
- 1897\*\* Galer, Frederic Bertram, B.A., Rock Life Assurance Company, 15 New Bridge-street, E.C.
- 1895\*\* Galwey, Charles Edmund, New Zealand Government Life Insurance Dept., Wellington, New Zealand.
- 1893\*\* Gardiner, Robert Edward, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1885\*\* Gayford, Herbert Stannard, Northern Assurance Co., 15 Victoria-street, Nottinghum.
- 1897\*\* Gillies, George, 6 Goulton-road, Clapton, N.E.
- 1871\*\* Glennie, William Gordon, Scottish Union & National Insur. Co., 3 King William-street, E.C.
- 1805\*\*\*aGlover, Henry Walter, 41 Drayton-park. N.
- 1897\*\* Goggs, Frank Sidney, Scottish Metropolitan Life Assur. Co., 25 St. Andrew-sq., Edinburgh.
- 1882 Goldman, Leopold,
  North American Life Assurance
  Co., North American Life
  Buildings, 112-118 King-streetwest, Toronto, Canada.
- 1897\*\* Goodwyn, John, Jun., Norwich and London Accident Insurance Assoc., 1 Lakenhumterrare, Norwich.
- 1888 Gray, John.
  Scottish Widows Fund Life Assur.
  Society, 55 Corn-street, Bristol.
- 1898\*\* Green, George, B.A.,

  \*\*Institute of Actuaries, Staple-inn-hall, W.C.\*\*

Those marked \* have passed the first of the three parts of the Examination for the Class of Fellow. Those marked \* have passed two of the three parts of the Examination for the Class of Fellow; those marked \*\*a, or \*b, have also passed either Section A or Section B of Part III of the Examination.

- 1868\* Greig, John Andrew, Sun Life Assurance Society, 60 Charing-cross, s.w.
- 1869 Griffith, E. Clifton, 4 Carlton-chambers, s.w.
- 1893\*\* Hall, John Francis Edmund, Eagle Insurance Company, 79 Pall-mall, s.w.
- 1869 Hann, Robert George, Mem. Act. Soc. Amer., The Equitable Life Assur. Soc., 120 Broadway, New York.
- 1894\*\* Hardcastle, Edward Edgington, M.A., Union Central Life Office, Cincinnati, U.S.A.
- 1876\* Harding, Harry Reginald (AUDITOR), Economic Life Assurance Soc., 6 New Bridge-street, E.C.
- 1896\*\* Harris, Frederick Joseph, Australian Mutual Provident Society, Sydney, Australia.
- 1894\*\* Hawkin, Robert Crawford, B.A., 3 Have-court, Temple, E.C.
- 1897\*\* Hayeraft, William Melhuish, Prudential Assurance Company, Holborn-bars, E.C.
- 1897\*\* Hazell, James Stanley, National Provident Institution, 48 Gracechurch-street, E.C.
- 1895\*\* Heness, Leonard Thomas, Prudential Assurance Company, Hotborn-bars, E.C.
- 1878 Henry, Alfred, F.C.A., Throgmorton-house, Copthallacenue, E.C.
- 1884 Higham, William Samuel, Equitable Life Assurance Soc., Mansion-house-street, E.C.
- 1892\*\*aHolliday, John, M.A., Institute of Actuaries, Stapleiun-hall, Holborn, W.C.
- 1894\*\* Hollingworth, Albert Chas., Australian Mutual Provident Society, Sydney, Australia.
- 1883 Holt, Edward Hallett, Law Life Assurance Society, 187 Fleet-street, E.C.

- Date of becoming an Associate.
- 1894\*\* Home, Noel Charles Minchin, LL.B., F.S.S., Westcott, Dorking, Surrey.
- 1881\*\* Hose, Frederick Edward Arthur, Pelican Life Insurance Co., 70 Lombard-street, E.C.
- 1898\*\* Howell, Charles Edward, B.A., LL.D., Standard Life Assurance Compy., 66 Upper Sackville-st., Dublin.
- 1875 Hunt, Richard Aldington, F.S.S., Wesleyan & General Assur. Soc., Corporation-street, Birmingham.
- 1893 Hunter, Arthur, F.F.A., New York Life Insurance Co., 346 & 348 Broadway, New York.
- 1887\*\* Hunter, Samuel,

  Patriotic Assurance Company,
  9 College-green, Dublin.
- 1889 Jacobs, Frederick Job, Australian Mutual Provident Society, Hobart, Tasmania.
- 1876\*\* James, George Trevelyan, 12 Waterloo-place, s.w.
- 1871 Jellicoe, George Rogers, Eagle Insurance Company, 79 Pall-mall, s.w.
- 1883 Jerman, Richard, Commercial Union Assurance Company, Exeter.
- 1896\*\* Jobson, Alexander, Australian Mutual Provident Society, Melbourne.
- 1894\*\* Johannessen, Nikolai Mikal, *Hygea Life Assurance Company, Bergen, Norway.*
- 1894\*\* Johnston, Frederick H., Prudential Life Insurance Co., of America, Newark, N.J., U.S.A.
- 1870 Joyce, Septimus. 24 Bridge-street, Bristol.
- 1898\*\* Kaufman, Henry N., Sun Life Assurance Co. of Canada, Montreal, Canada.
- 1876 Kearry, Joseph, 143 Broomwood-road, Wandsworth-common, s.w.

Those marked \* have passed the first of the three parts of the Examination for the Class of Fellow. Those marked \*\* have passed two of the three parts of the Examination for the Class of Fellow; those marked \*\*a, or \*\*b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming an Associate.

- 1897 Kemp, Julian Ernest Sandford, Eagle Insurance Company, 79 Pall-mall, s.w.
- 1858 Kilford, George William, Rue de Grétry, Paris.
- 1874\* King, Arthur Thomas, National Debt Office, 19 Old Jewry, E.C.
- 1882\*\* King, William Alfred, Northern Assurance Company, 1 Moorgate-street, E.C.
- 1861 Knowles, Richard, 35 Tilson-road, Tottenham, N.
- 1893\*\* Laing, William Claud, Pioneer Life Assurance Company, Ltd., 11 Dale-street, Liverpool.
- 1827\*\* Lane, Arthur Vere, B.A., Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.
- 1885 Ledward, Archibald Prentice, B.Sc., Scottish Provident Institution, 10 Albert-square, Manchester.
- 1879 Leitch, Alexander, Scottish Provident Institution, 17 King William-street, E.C.
- 1897\*\* Le Maitre, Frank William, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1885 Leveaux, Arthur Michael, Registry of Friendly Novietics, Central Office, 28 Abingdonstreet, Westminster, s.w.
- 1885\*\* Lidbury, Isaac Stephen,

  Prudential Assurance Company,

  Holborn-bars, E.C.
- 1868\* Litchfield, Edward, Lancashire Insurance Company, 25 Pine-st., New York, U.S.A.
- 1897\*\* Little, James Fulton, Mutual Life Association of Australasia, Sydney, Australia.
- 1894\*\*aLowndes, Arthur, Royal Exchange Assurance Corporation, Royal Exchange, E.C.
- 1876\*\* Lucey, Herbert,

  General Assurance Company,
  103 Cannon-street, E.C.

- 1890 Lugton, Hugh, F.F.A., North British and Mercantile Insurance Co., 61 Threadneedlestreet, E.C.
- 1898\*\* Lutt, Harold Edward William, Commercial Union Assur. Co., Ltd., 24, 25 & 26 Cornhill, E.C.
- 1867\* Macdonald, William Rae, F.F.A., Scottish Metropolitan Life Assur, Co.,25 St. Andrew-sq., Edinburgh.
- 1884 Mackay, Alexander, Law Union & Crown Fire & Life Insur. Co., 126 Chanvery-lane, W.C.
- 1883\* Mackenzie, Robert Kirkwood, Norwich Union Fire Insurance Office, Norwich.
- 1896\*\* Macmillan, John Campbell, Royal Insurance Co., Liverpool.
- 1895\*\*aMacphail, Donald, F.F.A., North British & Mercantile Insurance Company, Edinburgh.
- 1867 Maepherson, Ronald, Law Union & Crown Insurance Co., 126 Chancery-lane, w.c.
- 1883\*\* Makeham, William Reed,

  Imperial Life Insurance Co.,

  1 Old Broad-street, E.C.
- 1883 Mannering, George Willsher, London & Lancashire Life Assur, Co., 66 and 67 Cornhill, E.C.
- 1880\* Manwaring, Henry, National Debt Office, 19 Old Jewry, E.C.
- 1898 Marr, Vyvyan, F.F.A., Edinburgh Life Assurance Co., 22 George-street, Edinburgh.
- 1878 Marshall, William, South African Mutual Life Assur. Soc., Cape Town, South Africa.
- 1896\*\* Martin, Sidney George, National Mulval Life Assoc, of Australasia, Ltd., Wellington, New Zealand.
- 1897\*\* Mascall, Alfred John,

  Standard Life Assurance Co.,
  83 King William-street, E.C.
- 1898\*\* May, Basil, Gilamont, Maple-road, Surbiton.

Those marked There passed the first of the three parts of the Eaumination for the Class of Fellow. Those marked There passed two of the three parts of the Examination for the Class of Fellow; those marked Ta, or Tb, have also passed either Section A or Section B of Part III of the Examination.

- 1882\*\* McDougald, Alfred, British Empire Mutual Life Assur. Co., Montreal, Canada.
- 1881 McKenzie, Duncan John McGregor, New Zealand Government Life Insur. Department, Wellington, New Zealand.
- 1896\*\* Merfield, Percy Henry, Law Life Assurance Society, 187 Fleet-street, E.C.
- 1874 Miller, John W., Scottish Widows' Fund Life Assur. Soc., 28 Cornhill, E.C.
- 1884 Mills, Daniel Yarnton,
  Scottish Equitable Life Assur.
  Society, 26 St. Andrew-square,
  Edinburgh.
- 1895 Moir, Henry, F.F.A., Scottish Life Assur. Co., 19 St. Andrew-square, Edinburgh.
- 1879\* Monilaws, William MacGeorge, Scotlish Provident Institution, 6 St. Andrew-sq., Edinburgh.
- 1892\*\* Moodie, Peter Boyd, Scottish Office, Whitehall, s.w.
- 1877 Moon, James, Prudential Assurance Company, 30 Dale-street, Liverpool.
- 1877 Moon, John, Prudential Assurance Company, 76 King-street, Manchester.
- 1879\* Moon, Sidney Norman Laming, The Fidelity and Casualty Co., 97-103 Cedar-street, New York.
- 1898\*\* Moore, Joseph Patrick, Citizens' Life Assurance Co., Sydney, Australia.
- 1871\*\* Moore, Roderick Mackenzie,
  United Kingdom Temperance and
  General Provident Institution,
  1 Adelaide-place, London-bridge,
  E.C.
- 1896\*\* Moorhouse, Alfred, Friends' Provident Inst. Bradford.
- 1897\*\*hMorgan, Benjamin Charles, Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.

- Date of becoming an Associate.
- 1893\*\* Munro, Donald Alexander, Africa House, 44-46 Leadenhallstreet, E.C.
- 1897\*\* Newling, Sidney Wallis, B.A.,
  Woodleigh, South Woodford,
  Essex.
- 1881 Nicoll, John, F.F.A., Life Association of Scotland, 82 Princes-street, Edinburgh.
- 1897\*\* Norris, Charles Arthur, National Mutual Life Assoc. of Australasia, Ltd., Melbourne, Australia.
- 1883 Orr, Lewis P., F.F.A., Scottish Life Assur. Co., Ltd., 19 St. Andrew-sq., Edinburgh.
- 1886 Owen, Evan Frederick, F.S.S.,

  Office of the Actuary for Friendly
  Societies, Melbourne, Australia.
- 1895\*\* Pagden, Lionel King (AUDITOR), Union Insurance Society, 81 Cornhill, E.C.
- 1878 Palmer, James William, Actuary of the London Asylums' Committee, Belsize, Sutherlandroad, Ealing, W.
- 1864 Panton, Edward Henry, Standard Life Assurance Co., 83 King William-street, E.C.
- 1895\*\* Paradice, William Henry, Australian Mutual Provident Society, Sydney, Australia.
- 1869\* Park, David Francis, C.A., F.F.A., Crédit Foncier of Mauritius (Limited), 39 Lombard-st., E.C.
- 1884 Park, Leslie John,
  Colonial Mutual Life Assurance
  Society, Melbourne, Australia.
- 1896\*\* Parker, Robert Peter, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1882\*\* Paterson, William Brockie, F.F.A., Mem. Act. Soc. Amer., Norwich Union Life Insurance Society, Norwich.
- 1898 Pearce, Henry John, F.F.A., Scottish Provident Institution, 29 St. Vincent-place, Glasgow.
- 1875 Perratt, William Henry, 4 Finsbury-circus, E.C.

Those marked \* have passed the first of the three parts of the Examination for the Class of Fellow. Those marked have passed two of the three parts of the Examination for the Class of Fellow; those marked "a, or "b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming an Associate. 1895 Pierson, Israel Coriell, Mem. Act.

- Soc. Amer.,
  141 Broadway, New York,
  U.S.A.
- 1883 Pitts, Thomas,

  Commercial Union Assurance
  Company, Exeter.
- 1876\* Pound, Thomas James, Clerical, Medical & General Life Assurance Sov., 15 St. James'ssquare, s.w.
- 1880 Povali, Charles, Lancashire Insur. Co., Exchangest., St. Ann's-square, Manchester.
- 1890\*\* Powell, Alfred,
  Alliance Assurance Company,
  Bartholomer-lane, E.C.
- 1881\* Price, William John,

  Life Association of Scotland, 5

  Lombard-street, E.C.
- 1869\* Pringle, James, C.A., F.F.A., 42 Drumcheugh-gardens, Edinburgh.
- 1884 Pullar, James, F.F.A., Colonial Mutual Life Assurance Society, Melbourne, Australia.
- 1881 Purves, Thomas Peter, New York Life Insurance Company, Sydney, Australia.
- 1867 Rattray, Patrick, C.A., Gresham House, 45 West Nilestreet Glusgow.
- 1874\*\* Ray, Charles Richard,

  Hand-in-Hand Insurance Noc.,
  26 New Bridge-street, E.C.
- 1885\* Rea, Charles Herbert Edmund, F.R.A.S., F.S.S. (AUDITOR), Pearl Life Assurance Company, London-bridge, E.C.
- 1894\*\*aReeve, Charles Ernest, Royal Exchange Assurance Corporation, Royal Exchange, E.C.
- 1898\*\* Reeves, J. Horace, M.A., 85 Eltham-road, Lee, s.E.
- 1898\*\* Reid, Edward E., B.A.,

  London Life Insurance Co.,

  London, Ontario.

- 1887 Richardson, Josephus Hargreaves, F.F.A., Mem. Act. Soc. Amer., New Zealand Government Life Insurance Department, Wellington, New Zealand.
- 1879 Roberts, Thomas B.,

  Australian Alliance Assurance
  Company, Collins-street. Melbourne, Australia.
- 1878 Robertson, William, F.F.A., 54 Queen-street, Edinburgh.
- 1876\* Robinson, Andrew,
  Sunningdale-park, Sunningdale,
  Berks.
- 1885 Ronald, Thomas Robert,

  Law Guarantee and Trust Soc.,

  Ltd., 49 Chancery Lane, w.c.
- 1866 Row, George James, 110 Cannon-street, E.C.
- 1897\*\* Ryley, Edmund,
  Prudential Assurance Company,
  Holborn-bars, E.C.
- 1896\*\* Sanderson, Frank, M.A., Mem. Act. Soc. Amer., Canada Life Assurance Company, Hamilton, Canada.
- 1884 Schooling, John Holt, Fotheringay-house, Montpelierrow, Twickenham.
- 1873 Scott, Ernest Willem, Mem. Act.
  Soc. Amer.,
  Algemeene Maatschappij van
  Levensverzekering en Lijfrente,
  Damrak, 74, Amsterdam.
- 1895\*\* Searle, George Morley, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1861\*\* Searle, Thomas John,

  Mansion-house-chambers, Bucklersbury, E.C.
- 1894\*\*aSheppard, Herbert Norman, B.A., Institute of Actuaries, Staple-innhall, Holborn, W.C.
- 1897\*\* Shimmell, James Edward, Council-chambers, 109 Colmorerow, Birmingham,
- 1896\*\* Shlager, Joseph,

  Mutual Assurance Society, Melbourne, Australia.

Those marked - have passed the first of the three parts of the Examination of the Class of Fellow. Those marked - have passed two of the three parts of the Examination for the Class of Fellow; those marked -a, or \*\*b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming an Associate.

- 1887\*\* Slade, Henry, 24 Durand-gardens, Claphamroad, s.w.
- 1864\* Smith, Howard Samuel, F.F.A., F.S.S., F.C.A., Bank-chambers, 14 Waterloostreet, Birmingham.
- 1898\*\* Smith. Robert Parker, Lancashire Insurance Company, 18 Exchange-street, Manchester.
- 1884 Smithett, Edward Henry, "Hillside," Fitzroy-park, Highgate, N.
- 1863\*\* Smyth, Edward, 52 Wiltshire-road, Brixton, s.w.
- 1871 Spencer, Robert James, F.S.S., 75 King's-road, Southsea.
- 1868 Spens, William George, Scottish Amicable Life Assur, Soc., 35 St. Viacent-pl., Glasgow.
- 1881 Stancliffe, Frederick,

  British Empire Mutual Life
  Assurance Company, Montreal.
- 1860\* Stark, James, F.S.S., Reversionary Interest Society. 30 Coleman-street, E.C.
- 1866 Stark, William Emery, F.S.S., Chapel-walks, Manchester.
- 1870 Stark, Wilson Emery, F.R.G.S., Rydal Lodge, New Park-road, Clapham-park, s.w.
- 1878 Stevenson, Charles, 9 Atbert-square, Manchester.
- 1880 Stock, Edward James, National Mutual Life Assoc. of Australasia, Melbourne, Australia.
- 1896\*\* Stuckey, Jos. James, M.A., Salisbury Chambers, 49a King William-street, Adelaide, South Australia.
- 1895\*\* Strong, William Richard, London Guarantee & Accident Co., 61 Moorgate-street, E.C.
- 1869 Surenne, David John, F.F.A., Caledonian Insurance Company, 19 George-street, Edinburgh.

- 1898\*\* Sutherland, John, M.A.,

  Temperance & General Mutual
  Life Assur. Soc., Swanstonstreet, Melbourne, Australia.
- 1882 Tarn, Walter George, Reversionary Interest Society, 17 King's-arms'-yard, E.C.
- 1893\*\* Taylor, Arthur,
  Westminster and General Life
  Assurance Assoc., 28 King-street,
  Corent-garden, W.C.
- 1875 Taylor, J. Wilford, North British and Mercantile Insur.Co.,61Threadneedle-st.,E.C.
- 1895\*\*aThodey, Robert, Australian Mutual Provident Society, Sydney, Australia.
- 1898\*\* Thompson, Thomas Perey, B.A.,

  Institute of Actuaries, Stapleinn-halt, Holborn, W.C.
- 1883\*\* Titmuss, Walter George, Provident Life Assurance Co., 50 Regent-street, w.
- 1883\* Tregaskis, George Alfred, Hand-in-Hand Assurance Soc., 26 New Bridge-street, E.C.
- 1894\*\* Trenerry, Charles Farley, B.A., 3 North-road, Clapham-pk., s.w.
- 1869\*\* Trew, Edward Bellingham, Law Life Assurance Society, 187 Fleet-street, E.C.
- 1896\*\*b Trouncer, Harold Moltke, B.A., London Life Association, S1 King William-street, E.C.
- 1891\*\* Turnbull, A. D. Lindsay, C.A., F.F.A., Caledonian Insurance Company, 19 George-street, Edinburgh.
- 1877\*\* Turpin, William Gibbs, National Debt Office, 19 Old Jewry, E.C.
- 1884 Vian, William Collett,
  Railway Passengers' Assurance
  Company, 61 Cornhill, E.C.
- 1884 Vincent, Frederick James, F.S.S., London, Edinburgh & Glasgow Assurance Co., Ltd., Insurancebuildings, Farringdon-street, E.C.

Those marked those passed the first of the trace perits of the Examinative for the Class of Fellow. Those marked there passed there of the trace perits of the Eranimative for the Class of Fellow; those marked the, or the have also passed of the Section A or Section B of Part III of the Earnel and the

Date of becoming an Associate.

- 1883\*\* Walker, Davidson, F.F.A., Norwich Union Life Assurance Society, Norwich.
- 1879\* Wall, Walter George,
  Pyrmont, Prenton road west,
  Birkenhead.
- 1878 Walton, William Gandy, F.F.A., Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.
- 1862\* Waterhouse, Edwin, M.A., F.C.A., F.S.S., 44 Gresham-street, E.C.
- 1883\*\* Watson, John Robertson, British Law Fire Insurance Co., 176 West George-st., Glusgow.
- 1887 Watson, Reuben, Manchester Unity Friendly Soc., Nottingham.
- 1894\*\* Watt, George,
  Royal Insurance Co., Liverpool.
- 1883\* Weall, Bertram, City of Glaspor Life Assurance Co., 12 King William-street, E.C.
- 1894 Weeks, Rufus Wells, Mem. Act. Soc. Amer., New York Life Insurance Co., 346 & 348 Broadway, New York.
- 1898\*\* Whigham, Charles Frederick, F.F.A.. Scottish Provident Institution. 6 St. Andrew-square, Edinburgh.
- 1883 Whiting, William Dunbar, M.D., LL.B., Mem. Act. Soc., Amer., 261 Browleau, New York.
- 1884 Whyte, Alexander,
  London Assurance Corporation,
  7 Royal Exchange, E.C.
- 1897\*\* Wickens, Charles H., Registrar-General's Office, Perth, W. Australia.
- 1896\*\* Wilkinson, Edward Berkeley, 12 Highlerer-roud, N. Krusington, W.
- 1870\*\* Wilson, Henry Edward, Northern Ass. Co., 1 Moorgatestreet, E.C.

- 1873\*\* Windett, Charles.

  Legal & General Life Assur.

  Soc., 10 Fleet-street, E.C.
- 1882\* Wingfield, John Tutin, Law Union and Crown Insur. Co., 126 Chancery-lane, W.C.
- 1598\*\* Wood, Arthur Barton, B.A., Asso.
  Act, Soc. Amer.,
  Sun Life Assurance Co. of
  Canada, Montreal, Canada.
- 1883 Woodhouse, Lister, A.C.A.,

  Borough Treasurer, Town-hall,
  Birkenhead.
- 1877\*\* Woods, Arthur Biddle.

  Rock Life Assurance Company,
  15 New Bridge-street, E.C.
- 1866 Woods, Bernard,
  Metropolitan Life Assur. Soc.,
  13 Moorgate-street, E.C.
- 1875 Woods, Edward,
  Victoria Life and General Insur.
  Co., Market-street, Collins-streetWest, Melbourne, Australia.
- 1807\*\* Woolfe, Archibald William, B.A., 4 Gainsborough-road, Leutonstone, E.
- 1898\*\* Woolmer, Alfred Henry, Star Life Assurance Society, 32 Moorpite-street, E.C.
- 1808\*\* Workman, William Arthur, Imperial Life Office, 9 & 10 Kin t-street, Cheapside, E.C.
- 1879\* Wormun, Thornton Selden, Rock Life Assurance Company, 15 New Bridge-street, E.C.
- 1893\*\* Wright, Robert Young Murray, M.A., Royal Insurance Co., Charingeyoss, Birkenheud.
- 1871 Yardley, John.

  Prodential Assurance Company.

  Holborn-bars, E.C.
- 1873 Young, Alexander Hunter, 60 Market-street, Mellourne, Australia.

Those marked \* have passed the first of the three parts of the Examination for the Class of Fellow. Those marked `` have passed two of the three parts of the Examination for the Class of Fellow; those marked \*`a, or ``b, have also passed either Section A or Section B of Part III of the Examination.

| Date of    |  |
|------------|--|
| becoming   |  |
| a Student. |  |

- 1892\* Aaron, David Hyam, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
- 1897 Adams, Frederick Charles Alfred, London Assurance Corporation, 7 Royal Exchange, E.C.
- 1895\*\*\*aAdlard, Howard Tindale, A.K.C., The Equitable Life Assurance Society, Mansion-house-st., E.C.
- 1895\* Adlard, Stanley, A.K.C., London Life Association, Ltd., 81 King William-street, E.C.
- 1894\* Anderson, Adam Thomson, Australian Mutual Provident Society, Sydney, Australia.
- 1895\* Anderson, Thomas Frederic, Royal Exchange Assurance Corporation, Royal Exchange, E.C.
- 1887\* Ansell, George Frederic, National Debt Office, 19 Old Jewry, E.C.
- 1886 Arnold, Thomas, Jun.,

  British Equitable Life Assurance
  Company, Queen-street-place, E.C.
- 1896\* Ashley, Charles Henry, Refuge Assurance Company, Oxford-street, Manchester.
- 1897\* Ashton, William Richard,

  Hand-in-Hand Insurance Soc.,
  26 New Bridge-street, E.C.
- 1893\* Askew, Sydney J., "Oakdene," 49 Park-road, Foresthill, s.e.
- 1896 Auchterlonie, Alexander Douglas, Acme Insurance Co., Limited, Mowbray-house, 14 Norfolk-st., Strand, w.c.
- 1893\*\*aAustin, Herbert Henry,
  Prudential Assurance Company,
  Holborn-bars, E.C.
- 1897\* Backett, William Albert,

  Employers' Liability Assurance
  Corporation, 84 King Williamstreet, E.C.
- 1898\* Bacon, James, Institute of Actuaries, Stapleinn-hall, Holborn, W.C.

#### Date of becoming a Student.

- 1896\* Ball, Sidney Robertson, English and Scottish Law Life Assurance Association, 12 Waterloo-place, s.w.
- 1897 Barfield, Edmund John, Prudential Assurance Company, Holborn-bars, E.C.
- 1891\* Barker, George,

  National Provincial Bank of
  England, 208-9 Piecadilly, w.
- 1896\* Barry, David,
  Office of the Actuary for Friendly
  Societies, Melbourne, Australia.
- 1896 Beavis, Arthur John,

  Hand-in-Hand Insurance Soc.,

  26 New Bridge-street, E.C.
- 1897\* Benjamin. Stanley O.,
  Australian Mutual Provident
  Society, Melbourne.
- 1898\* Bennell, Samuel Thomas, 20 Narford-road, Brooke-road, Clapton, N.E.
- 1898 Bennett, Samuel, 6 Gray-street, Workington.
- 1896 Bevington, Herbert,

  Imperial Life Office, 1 Old

  Broad-street E.C.
- 1895\* Bigby, Robert Frederick
  Mitchell,
  General Assurance Company,
  103 Cannon-street, E.C.
- 1886 Biggar, James, Edinburgh Life Assurance Co., 22 George-street, Edinburgh.
- 1891\* Bird, Edward William,

  Northern Assurance Company,

  1 Moorgale-street, E.C.
- 1898 Bishop, Harold Garfield,

  Northern Assurance Company,

  1 Moorgate-street, E.C.
- 1898\* Blake, Frederick Edward, Prudential Assurance Company, Holborn-bars, E.C.
- 1898\* Blake, Henry Prince,

  Union Assurance Society, 81

  Cornhill, E.C.

Those marked \* have passed the first of the three periods of the Earn, and of othe Cass of Fellow. These marked \* three persod two of its time persod five Ear clant on Forther less of Fellow; the marked \* a, or \* to, have also passed other Section A or Section B of Part III of the Earnahad on.

| Date of                        | marked " $\sigma_i$ or "b, have also possed" ther $s_i$   | Date of<br>becoming<br>a student |   |
|--------------------------------|---|----------------------------------|---|
| becoming<br>a Student<br>1595* | Blake, Robert Walter Austin,  Hund-in-Hund Insurance Soc.,  26 New Bridge-street, E.C.                  | a studeni<br>1594                | i.<br>Callard, Percy Strutt,<br>Standard Life Assurance Co.,<br>83 King William-street, E.C.            |
| 1895                           | Blanch, Frederick William,<br>Mutual Life Insurance Computary<br>of New York, 17 y 18 Cornhill,<br>E.C. | 1898                             | Cambwell, Walter M., North American Life Assur. Co., 112-118 King-street-west, Toronto, Canada.         |
| 1887                           | Blossom, James. 186 South-view-road, Sheffield.   | 1896                             | Carr. Stanley T.,  Australian Mutual Provident Soviety, Spilney, Australia.                             |
| 1892*                          | Boddy, Henry Mitchell,<br>Imperial Life Assurance Co.,<br>Toronto, Canada.                              | 1596*                            | Catchlove, Charles Hamilton<br>Leyland,<br>Australian Mutual Provident                                  |
| 1897                           | Bond, Frederic D.,<br>413 Nouth 44th Street, Phila-<br>delphia, U.S.A.                                  | 1894*                            | Soviety, Adelaide, S. Australia.  Challen, Arthur Gordon,  Mutual Life Insurvince Co. of                |
| 1597*                          | Bowles, Francis Marsh, Pearl Life Assurance Compuny, London Bridge, E.C.                                | 1897*                            | New York, 17, y 18 Cornhill, E.C.<br>Chandler, Thomas Richard,<br>28 Perth-road, Finsbury-park, N.      |
| 1891*                          | Boyd, Henry Norris,   | 1896                             | Chantrey, William Herbert, 57 Moorgate-street, E.C.   |
|                                | City of Glasyon Life Assurance<br>Co., 30 Renfield-street, Glasyon.                                     | 1894                             | Child, Frank Edward, 1 Witton-rd, Aston, Birmingham.  |
| 1897                           | Brielley, William Ernest,<br>Refuge Assurance Company,<br>Oxford-street, Manchester.                    | 1893                             | Chisholm, Daniel Henry, J.P.,<br>7 Arundel Jerrace, Forest-lodge,<br>Sydney, Australia.                 |
| 1893*                          | Briggs, Frederick William,<br>Caxton-villa, Wood-green, N.  | 1894                             | Christmas, Arthur Guy, Pelican Life Insurance Co.,  |
| 1896*                          | Briggs, Harold Netterville,<br>Liverpool and London and Globe<br>Insur. Co., 1 Dale-st., Liverpool.     | 1897                             | 70 Lombard-street, E.C.  Clare, Willis Harry Herbert, Pearl Life Assurance Company, London-bridge, E.C. |
| 1894*                          | Brough, Frank,<br>Federal Life Assurance Company,<br>Hamilton, Ontario.                                 | 1891*                            | Claridge, William, M.A.,  London and Milland Bank-  |
| 1895*                          | Brown, Harold,  City of Glasgow Life Assurance Co., 12 King William - street, Glasgow.                  | 1897                             | chambers, Bradford.  Clark, Beauchamp, Northern Assurance Company, 1 Moorgate-street, E.C.              |
| 1891*                          | Brown, William Heron, Gresham Life Assur. Soc., Ltd., St. Mildred's-house, Poultry, E.C.                | 1593                             | Clarke, Eustace Edgar, British Empire Mutual Life Assurance Company, 4 & 5 King William-street, E.C.    |
| 1889                           | Buckle, Frederick,<br>68 Belleville-road, Wandsworth-<br>common, s.W.                                   | 1897                             | Clinton, George, Prudential Assurance Company, Holborn-bars, E.C.                                       |
| 1896*                          | Burdett, Charles,<br>Institute of Actuaries, Staple-<br>inn-hall, Holborn, W.C.                         | 1896                             | Coates, Alfred Irwin, Friends' Provident Institution, 17 Gracechurch-street, E.C.                       |

Those marked - have passed the first of the three parts of the Examination for the Class of Fellow. Those marked \* have passed two of the three parts of the Examination for the Class of Fellow; those marked \*\*a, or \*\*b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming a Student.

- 1895 Cogar, William Edward, New York Life Insurance Co., Trafalgar-square, W.c.
- 1898 Collier, Charles Aubrey, 66 Josephine-arenue, Brixtonhill, s.w.
- 1895\* Collins, Frank Lakeman, Clerical, Medical & General Life Assurance Soc., 15 St. James'ssquare, s.w.
- 1891 Colvin-Smith, Colvin Arthur Edward, North British and Mercantile Insurance Co., 61 Threadneedlestreet, E.C.
- 1892 Connolly, Edward, Australian Mutual Provident Society, Sydney, Australia.
- 1896\* Cook, Henry Milton,
  Standard Life Assurance Co.,
  83 King William-street, E.C.
- 1889\*\*bCook, William Playfair, Guardian Assurance Company, 11 Lombard-street, E.C.
- 1898\* Corbett, Edwin Somerville, Colonial Mutual Life Assurance Society, Melbourne.
- 1897 Court, Alexander George Dacus, 4 Langdale-road, Greenwich, s.E.
- 1886\*\* Covington, Oliver Henry,

  Prudential Assurance Company,

  Holborn-bars, E.C.
- 1896 Cox, Charles,

  Mutual Life Insurance Co. of

  New York, 17 & 18 Cornhill, E.C.
- 1894 Cox, Edward William, Canada Life Assurance Co., Toronto, Canada.
- 1894 Cox, Herbert Coplin,

  Canada Life Assurance Co.,

  Toronto, Canada.
- 1896 Critchley, George Francis, 18 Handen-road, Lee, s.E.
- 1895 Cross, Eustace Philip,

  Commercial Union Assur. Co.,
  24, 25 & 26 Cornhill, E.C.
- 1887\* Cross, Henry John, 3 Park-rd., Wandsworth-common, s.w.

Date of becoming a Student.

- 1897\* Cross, Howard Turner, Economic Life Assurance Soc., 6 New Bridge-street, E.C.
- 1897\* Crump, Percy C.,

  Prudential Assurance Company,

  Holborn-bars, E.C.
- 1897\* Culley, Alfred Benjamin, Star Life Assurance Society, 32 Moorgate-street, E.C.
- 1897 Curjel, H. W., Royal Insurance Co., Liverpool.
- 1895\* Curtis, William Allen,

  Clerical, Medical & General

  Life Assurance Society, 15 St.

  James's-square, s.w.
- 1897\* Dalton, John,

  London Life Association, Ltd.,
  81 King William-street, E.C.
- 1891 Daniell, Ferrers Aitken, Royal Exchange Assurance Corporation, Royal Exchange, E.C.
- 1896 Daughtrey, William Lamb, Jun., *Life Insurance Co. of Virginia*, *Richmond, Virginia*, U.S.A.
- 1896\* Davey, Clarence,

  Pearl Life Assurance Company,

  London-bridge, E.C.
- 1889\* Davies, Hugh Myddelton, Royal Insurance Co., Liverpool.
- 1891\* Dawson, Frank Aubrey, Ecclesiastical Insurance Office, Ltd., Norfolk-st., Strand, w.c.
- 1896 De Ville, Francis, Clergy Pensions Institution, 11 Norfolk-street, Strand, w.c.
- 1896\*\* Diamond, George Frederick, Australian Mutual Provident Society, Sydney, Australia.
- 1897\* Dick, William Thomas, Newcastle, N.S.W.
- 1895\* Dickinson, Frank Ridley, Peterborough-house, Harrow-onthe-Hill.
- 1890\* Docker, Leslie,
  North British and Mercantile
  Insurance Co., 61 Threadneedlestreet, E.C.

Those marked there passed the first of the three parts of the Englinet in for the Class of Fellow. These marked there passed two of the three parts of the Englination for the class of Fellow; these marked the control of the three parts of the Engline for the class of Fellow; these marked the control of the Engline for the English than the control of the control of the English than the control of the control

| Date of<br>becoming<br>a Student. |  | Date of<br>hecoming<br>a student |  |
|-----------------------------------|--|----------------------------------|--|
| 189 <b>7</b> *                    | Donaldson, James Stuart,<br>Mutual Life Association of Australasia, Ltd., Sydney, Australia. | 1593                             | Finlaison, Dudley Glen.<br>106 Wharton-road, W. Kensing-<br>ton, W.  |
| 1897                              | Dorrian, John Christopher,<br>Citizens' Life Assurance Com-<br>pany, Sydney, Australia.      | 1597                             | Fisher, Harold Arthur, Australian Mutual Provident Society, Melbourne, Australia.                                |
| 1893*                             | Dougharty, Harold,<br>London & Lancushire Life Assur.<br>Company, 66 & 67 Cornhill, E.C.     | 1857                             | Fisher, Hugh Strettell.  1 Ashbrook-terrace, Leeson-park, Dublin.  |
| 1893*                             | Doust-Smith, Ernest Charles, Prudential Assurance Company, Holborn-bars, E.C.                | 1892                             | Fisher, Walter Churchill,<br>Australian Mutual Provident<br>Society, Sydney, Australia.                          |
| 1897*                             | Dunn, Spencer Græme, National Mut. Life Assur. Soc., 39 King-street, Cheapside, E.c.         | 1896*                            | Fisk, George William Victor, Prudential Assurance Company. Holborn-bars. E.C.                                    |
| 1897*                             | Dunn, Walter James, Citizens' Life Assurance Com-  | 1596                             | Fleming-Williams, Clifford, 36 Kenninghall-rd., Clapton, N.E.  |
| 1897                              | pany, Sydney, Australia. Ecroyd, Cuthbert W.,  | 1897                             | Flint, William Charles,<br>Sun Life Office, 63 Threadneedle-   |
| 1891                              | Friends' Provident Institution,<br>Bradford.<br>Edlmann, Herbert Elliot,                     | 1893                             | street, E.C. Forster, Edward, Royal Exchange Assur, Corpora-   |
| 1301                              | Royal Exchange Assurance Cor-<br>poration. Royal Exchange, E.C.                              | 1890                             | tion, Royal Exchange, E.C. Gamman, Robert Ebenezer. London Joint Stock Bank                                      |
| 1892                              | Edwards, Edward Samuel,  |                                  | Princes-street, E.C.   |
| 1001                              | Australian Mutual Provident<br>Society, Sydney, Australia.                                   | 1556                             | Garcke, Emile, F.F.S., M.I.E.E.,<br>Donington-house, Norfolk-street,<br>w.c.                                     |
| 1894                              | Edwards, Frederick Stanley,<br>Sun Life Assurance Society, 63<br>Threadneedle-street, E.C.   | 1895*                            | Gibson, John P. S. R.,<br>St. Cloud, Paris.  |
| 1892*                             | Eedy, Arthur Maleolm,<br>Cilizens' Life Assurance Com-<br>pany, Sydney, Australia.           | 1895*                            | Gill, James Stewart,<br>Australian Widows' Fund Life<br>Assur. Soc., Melbourne, Australia                        |
| 1895**                            | Elderton, William Palin,<br>Guardian Assurance Company,<br>11 Lombard-street, E.C.           | 1893                             | Glasson, George Cornish,<br>Economic Life Assurance Soc.<br>4 St. Stephen's-chbrs., Baldwin-<br>street, Bristol. |
| 1593*                             | Emery, John M  The Security Trust Co., Tenth & Chestnut-streets, Philadelphia.               | 1893*                            | Gledstone, W. L.,<br>Royal Exchange Assur. Corpora-<br>tion, Royal Exchange, E.C.                                |
| 1896                              | Fahle, Richard Julius, Munich Re-Insurance Company, 18 Austin Friars, E.C.                   | 1897                             | Goddard, Egbert, Prudential Assurance Company Holborn-bars, E.C.   |
| 1892*                             | Farrell, John,<br>Citizens' Life Assur. Co., 210<br>Queen-st., Brisbane, Australia.          | 1597                             | Golby, Maurice Edward, 7 Planfair mansions, W. Ken sington, W.   |
|                                   |  |                                  |  |

1894\* Golding, Arthur, 41 Digby-rd., Finsbury-park, N.

Fells, John Manyer, F.S.S., Salt Union, 16 Eastcheap, E.C.

1886

Those marked—have passed the first of the three parts of the Eramination for the Class of Fellow. Those marked—have passed two of the three parts of the Examination for the Class of Fellow: those marked—a, or ¬b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming a Student.

- 1888\* Gooding, Harold John,

  Law Guarantee and Trust Soc.,

  Ltd., 56 Mooryate-street, E.C.
- 1892 Gordon, Alexander, 168 Islington, Liverpool.
- 1896 Gordon, Harry Duncan Lockhart, 221 George-st., Toronto, Canada.
- 1897\*\* Gosset, Thorold, 21 Old-bldgs., Lincoln's-inn, w.c.
- 1886 Gover, Frederick Field, F.S.S., 10 Lee-park, Blackheath, s.E.
- 1893 Gower, Abel Lewes,

  National Mutual Life Assoc. of
  Australasia, Ltd., Melbourne,
  Australia.
- 1894\* Grainger, Wilfred H.,

  Pendential Assurance Company,

  Holborn-bars, E.C.
- 1895 Grant, Kenneth Stuart,
  Alliance Assurance Company,
  1 Bartholomev-lane, E.C.
- 1886 Greening, Herbert Joseph,
  Abstainers & General Insur. Co.,
  City-buildings, Birmingham.
- 1897 Gregory, Edward William, Mercan'ile Mutual Insur. Co., Sydney, Australia.
- 1887 Griffin, William, 18 Patrick-street, Cork.
- 1896\* Hallman, M. S.,
  Ontario Mutual Life Assurance
  Company, Waterloo, Ontario.
- 1898 Hammond, Reginald, Scottish Widows' Fund Life Assur. Soc., 21 Park-row, Leeds.
- 1892 Hancock, Arthur Tom, Clerical, Medical & General Life Assur. Soc., 15 St. James's - sq., s. w.
- 1895\* Harding, Harry Burnard, *Hand-in-Hand Insurance Soc.*, 26 New Bridge-street, E.C.
- 1895\* Harding-Newman, Thomas Harold, Scottish Amicable Life Assur. Soc., 1 Threadneedle-street, E.C.
- 1895\* Harper, Sidney,
  Prudential Assurance Company,
  Holborn-bars, E.C.

becoming a Student.

- 1894 Harpour, Percy Christopher, Royal Exchange Assurance Corposation, Royal Exchange, E.C.
- 1889\* Harris, Henry,
  Friends' Provident Institution,
  Bradford.
- 1897\* Harriss, Walter James,

  Life Association of Scotland,
  5 Lombard-street, E.C.
- 1898 Harrison, Frank, 56 Grafton-street, Preston.
- 1896\* Harrison, Tom Marriott, Life Association of Scotland, 5 Lombard-street, E.C.
- 1896 Haskins, George Frederick, A.C.A., 7 Elmfield-road, Balham, s.w.
- 1894\* Hatten, David Leslie, Standard Life Assurance Co., 3 George-street, Edinburgh.
- 1891\* Haward, Thomas Gilbert, A.C.A., 133 Finsbury-pavement, E.C.
- 1897\* Hay, John Dalziel,
  Government Survey Department,
  Wellington, New Zealand.
- 1892 Hellyer, Arthur Lee, Shannon-court, Bristol.
- 1898 Henry, Norman Augustine, Lancashire Insurance Company, 18 Exchange-street, Manchester.
- 1897 Hepburn, Charles James, 1 Elgin-road, Croydon.
- 1894\* Heslop, Anthony T., 66 New Bank-road, Blackburn, Lancashire.
- 1898 Heyner, Herbert Angustus Otto, Equitable Life Assurance Soc. of the United States, 6 Princesstreet, Bank, E.C.
- 1889\* Hicks, Arthur Joseph,

  Reversionary & General Securities

  Company, Ltd., Northumberlandarenue, W.C.
- 1896\* Hicks, Maxwell, Plaistow-hall, Bromley, Kent.

Thus, metrical these passed the straight three parts of the Emmination for the Class of Fellow. Those marked these passed two of the three parts of the Examination for the third Fellow; those marked  $\exists a, or = b, have also passed either Sect = A or Section B of Part III of the Examination.$ 

| Date of<br>becoming<br>a Student | murkud ("a, or = 5, have 01so passut ether Sc   | Date of<br>becoming<br>a Student. |   |
|----------------------------------|---|-----------------------------------|---|
| 1891                             | Higinbothom, Harry Newburgh,<br>Royal Exchange Assur. Corpora-                              | 1893                              | Hutchins, Alexander Constantine, 109 Farringdon-road, E.C.  |
| 1896*                            | tion, Royal Exchange, E.C.  Hines, Walter Robert, Norwick Union Life Office,                | 1855*                             | Jackson, Edward Ellis,<br>Equity & Law Life Assur. Soc.,<br>18 Lincoln's-inn-fields, w.C.               |
| 1897                             | Norwich.  Hitchins, William Richmond, B.A.,  Manufacturers' Life Insurance                  | 1890**                            | Scottish Widows' Fund Life<br>Assurance Society, Liverpool.   |
| 1896*                            | Company, Toronto.  Hogz. Charles,   | 1898                              | Jackson, William, c o Messen. Barclay & Co., Aylsham.   |
| 1898                             | 10 Whitehall-place, s.w. Holden, Albert, 12 Chester-road, Halifax.                          | 1898                              | Jelly, Frank Edward, M.A.,<br>Felsted School, Essex.  |
| 1894                             | Holdsworth, David Arundell,<br>Star Life Assur. Soc., 1 Mosley-                             | 1895                              | Jenkyn, John,<br>Hearts of Oak Benefit Society,<br>17 Charlotte-st., Fitzroy-sq., W.                    |
| 1898*                            | stree!, Newcastle-on-Tyne.  Hooper, George Duncan.  Prudentia! Assurance Company,           | 1897                              | Jennings, Alfred Wilson,<br>38 Shrewsbury-road, Harlesden,<br>N.W.                                      |
| 1894*                            | Holborn-bars, E.C.  Hopping, Donald McKay, Prudential Assurance Company, Holborn-bars, E.C. | 1896*                             | Jepps, John Blacklee,<br>English and Scottish Law Life<br>Assurance Assoc., 12 Waterloo-<br>place, r.w. |
| 1895*                            | Horn. Ernest Frederick,<br>The Boltons, Sideup.   | 1898                              | Johnston, Arthur Edward,<br>3 Cumnor-road, Sutton.  |
| 1890                             | Howes, John Bennett,  Law Union and Crown Fire and  Life Insurance Company, 126             | 1898                              | Johnston, James O.,<br>Law Union & Crown Insur. Co.,<br>126 Chancery-lane, W.C.                         |
| 1888*                            | Chancery-lane, W.C.   | 1895                              | Jones, Harry Stewart,<br>19 Albert-road, Brockley, s.E.   |
| 1557                             | Northern Assurance Company,<br>1 Moorgate-street, E.C.                                      | 1896*                             | Jones, Richard Foxley,<br>Refuge Assurance Co., Oxford-<br>street, Manchester.                          |
| 1559*                            | Hudson, Frederick Charles,<br>Laurashire Insurance Company,<br>Manchester.                  | 1594*                             | Jupp, Henry Lewis, Guardian Assurance Company, 11 Lombard-street, E.C.                                  |
| 1898                             | Hughes, Arthur J.,  Manufacturers' Life Insurance Company, Toronto, Canada.                 | 1893*                             | Kelham, Cyril Stephen, Prudential Assurance Company,  |
| 1897*                            | Humphrey, Bernard, Prudential Assurance Company, Holborn-bars, E.C.                         | 1897*                             | Holborn-bars, E.C.  Kelly, John Joseph,  Citizens' Life Assurance Co.,                                  |
| 1891                             | Hunt, Arthur Leonard,<br>Wesleyan and General Assur.<br>Society, 18 New Bridge-st., E.C.    | 1896**                            | Prudential Assurance Company,   |
| 1895*                            | Hurst, Henry Alexander,<br>Refuge Assurance Company,<br>Oxford-street, Manchester.          | 1894                              | Holborn-bars, E.C.  Keys, William Jonathan,  York-hill, Loughton, Essex.                                |

Those marked there passed the first of the three parts of the Eremination for the Class of Fellow. Those marked there passed two of the three parts of the Eremination for the Class of Fellow; those marked to, or the have also passed either Section A or Section B of Part III of the Examination.

|  | marked 'e, or b, have also passed either S  |                                  | Section B of Part III of the Examination.   |
|--|---|----------------------------------|---|
| Date of<br>becoming<br>a Student             |   | Pate of<br>becoming<br>a Student |   |
| 1898   | Kidson, Leonard Douglas, 15 Roe-lane, Southport.  | 1894                             | Lucy, Frederick Samuel, F.C.A., 15 George-st., Mansion-house, E.C.  |
| $1896 ^{*}$                                  | Kimber, Richard John, Turney-road, West Dulwich.  | 1897                             | Lumley, Osbert, 81 Avenue-road, Regents-pk., N.W.   |
| 1891**                                       | Kingsbury, James William,<br>Australian Mutual Provident  | 1891                             | Lyon, Thomas Glover, M.D., 1 Victoria-square, s.w.  |
|  | Society, Sydney, Australia.   | 1898                             | Mackenzie, William Alexander,   |
| 1895*  | Knight, Alfred Murray, Bank-ho., Chapel-st., Devonport.   |                                  | North American Life Assur. Co.,<br>112-118 King-stwest, Toronto,<br>Canada.                                       |
| 1897   | Krause, Holger Erasme, Prudential Insurance Company of America, Newark, N.J.                    | 1895**                           | Macuaghten, Stenart Edye,<br>46 Brunswick-road, Brighton.   |
| 1895   | Laing, Oswald George,  North British and Mercantile   | 1898                             | Maples, Ernest Edgar,<br>44 Penn-rdvillas, Holloway, N.   |
| 1890*  | Insurance Co., Park-row, Leeds.  Lawson, Henry Graham Steuart,                                  | 1896                             | Marlow, Thomas Gibbons, A.I.S., 13 St. Ronan's-road, Abbeydale, Shaffeld  |
|  | Scottish Accident Insur. Co., Ltd.,<br>115 George-street, Edinburgh.                            | 1896                             | Sheffield. Marshall, Gerald,  |
| 1893*  | Lawton, George Herbert,<br>Clerical, Medical & General Life                                     |                                  | Imperial Life Office, 22 Pallmall, s.w.   |
| 1891   | Assur. Soc., 15 St. James's-sq., s.w. Layzell, Phillip Cuddington,                              | 1893*                            | Martin, William Anderson, M.A.,<br>Scottish Provident Institution,<br>Dublin.                                     |
|  | Prudential Assurance Company,<br>Holborn-bars, E.C.   | 1894*                            | Maunder, George Harvard,  |
| 1893   | Le Maistre, Charles H.,  Penn Mutval Life Insurance Co.,  | ******                           | Imperial Life Insurance Co., 1 Old Broad-street, E.C.   |
| 1892   | Philadelphia.  Le Maistre, George Harry, Assistant Accountant-General, Public Works Department, | 1895*                            | Mayhew, Percy Craske,<br>Westminster and General Life<br>Assurance Assoc., 28 King-st.,<br>Covent-garden, w.c.    |
|  |   |                                  |   |
|  | Supreme Government, India.  | 1894*                            | McArthur, Harry de C.,<br>Economic Life Assur. Society,   |
| 1 <b>S</b> 93                                | Lennard, Frederick William, 7 Church-lane, Hornsey, N.  | 1891                             | 6 New Bridge-street, E.C.   |
| 1894   | Leonard, Maurice,<br>22 Baalbeek-rd., Highbury, N.  | 1551                             | McCleery, James Carlisle, F.S.S.,<br>Star Life Assur. Soc., Old Bank-<br>chambers, 8 Park-row, Leeds.             |
| 1896*  | Ley, James, Office of the Actuary for Friendly  | 1888*                            | McConway, James Robert, Royal Insurance Co., Liverpool.   |
| 1889*  | Societies, Melbourne, Australia.  Lighton, Harold John, Law Union & Crown Fire & Life           | 1895                             | McLeod, James Stirling, c o Messes. Williams & Kebble, Ltd., Napier, New Zealand.                                 |
| 1005%  | Insur.Co.,126 Chancery-lane, w.c.   | 1897*                            | McPhail, Frederick Charles,<br>Colonial Mutual Life Assurance<br>Soc., Ltd., Melbourne, Australia.                |
|  |   |                                  |   |
| 1890 Love, Robert,<br>Pelican Life Insurance |   | 1893*                            | Meade, Gerald Willoughby,<br>North British & Mercantile In-<br>surance Company, 61 Thread-<br>needle-street, E.C. |
|  | Pelican Life Insurance Company.<br>70 Lombard-street, E.C.                                      |                                  |   |

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|                                  | marked = a, or =b, have also passed either Se   | cti | on A or Se                       | etion B of Part III of the Examination.  |
|----------------------------------|---|-----|----------------------------------|--|
| Date of<br>becoming<br>a Student |   |     | Date of<br>becoming<br>a Student |  |
| 1890*                            | Meikle, Henry George Watson,<br>F.F.A.,<br>Oriental Life Assurance Co.,                                     |     | 1897                             | Mugford, Stanley,<br>New York Life Insurance Co.,<br>Trafalgar-square, W.C.                                      |
| 1897*                            | Bombay.  Melville, Charles Edward, Citizens' Life Assurance Co., Sydney, Australia.                         |     | 1892*                            | Nash, Alfred Charles,<br>Clerical, Medical and General<br>Life Assurance Society, 15 St.<br>James's-square, s.W. |
| 1892*                            | Meyers, Henry Wilson,<br>National Mutual Life Associa-<br>tion of Australasia, Melbourne,<br>Australia.     |     | 1896*                            | Neale, Maurice Baldwin,<br>Alliance Assurance Company,<br>61 New-street, Birmingham.                             |
| 1897                             | Miller, George James,<br>Hanover, Ontario, Canada.  |     | 1895*                            | Neighbour, George Henry, M.A.,<br>LL.B.,   |
| 1896                             | Milligan, Charles Livingstone,<br>Provident Life Office, 3 Cook-  |     |                                  | 29 Selbourne - chambers, Mel-<br>bourne, Australia.  |
| 1894*                            | street, Liverpool. Mills, Thomas Perey,   |     | 1893*                            | Nelson, Charles,<br>148 Inverness-terrace, w.  |
|                                  | Mutual Life Association of Australasia, Sydney, Australia.  |     | 1595*                            | Newnham, Ernest Whiffin,<br>Prudential Assurance Company,  |
| 1897                             | Mirams, Arthur Greyford,<br>AustralianTemperance & General<br>Life Assurance Soc., Melbourne,<br>Australia. |     | 1897                             | Holborn-bars, E.C.  Nicholls, Robert James, Royal Exchange Assurance Cor-  |
| 1897                             | Moore, Arthur James,<br>Citizens' Life Assurance Co.,<br>Melbourne, Australia.                              |     | 1895**                           | poration, Royal Exchange, E.C.<br>Norton, William Ernest,<br>National Provident Institution,                     |
| 1898*                            | Moore, George Ceeil,<br>Imperial Life Insurance Co. of<br>Canada, Toronto, Canada.                          |     | 1895*                            | 48 Gracechurch-street, E.C. Oakley, Henry John Percy,  |
| 1895*                            | Moore, Gerald Leslie, A.C.A., 58 Coleman-street, E.C.   |     |                                  | North British and Mercantile<br>Insurance Company, 61 Thread-<br>needle-street, E.C.                             |
| 1898                             | Moore, Stanley, Prudential Assurance Company, Holborn-bars, E.C.  |     | 1891                             | O'Neill, Harry Duncan,<br>Clerical, Medical & General Life<br>Assur. Soc., 36 Park-row, Leeds.                   |
| 1895*                            | Morgan, George Frederick<br>Hughes,<br>Law Guarantee & Trust Society.<br>49 Chancery-lane, w.c.             |     | 1892*                            | O'Reilly, Anthony James,<br>Government Insurance Depart-<br>ment, Ottawa, Canada.                                |
| 1893*                            | Morland, Alfred,<br>Morland-road, Croydon.  |     | 1897*                            | Osborn, Nathaniel Banner<br>Francis,   |
| 1894                             | Morley, Alfred Lampen,<br>9 <i>Montague-road, Wimbledon</i> .   |     | 1893*                            | 34Lansdowne-road, Tottenham, N. Owen, Edgar Theodore, F.S.S.,  |
| 1896                             | Morrison, Hubert Peter,<br>2 Edmund-street, Birmingham.   |     |                                  | Registrar of Friendly Societies,<br>Perth, Western Australia.  |
| 1897*                            | Mower, George S., B.Se.,<br>Prudential Insurance Company<br>of America, Newark, N.J.                        |     | 1889*                            | Panton, Charles William,<br>25 Comley Bank, Egremont,<br>Liverpool.  |

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|---------------------------------|---|----------------------------------|--|
| 1893*                           | Papps, Percy Charles Herbert, Canada Life Assurance Company, Hamilton, Ontario.                         | 1896*                            | Pipe, Sidney Herbert,  Peart Life Assurance Company,  London-bridge, E.C.                          |
| 1891*                           | Parisot, Oscar, Institute of Actuaries, Staple-inn-hall, Holborn, w.c.                                  | 1898                             | Poort, Willem Anthonie, Phil. Nat.<br>Doct.,<br>Middelberg, Holland.                               |
| 1896*                           | Parry, Joshua Powell, Royal Insurance Company, Liverpool.   | 1897                             | Povah, John Frank,<br>Nelson, British Columbia.  |
| 1895*                           | Pascoe, William Yeoman Bennett, Prudential Assurance Company, Holborn-bars, W.C.                        | 1892*                            | Powell, Harold Charlesworth,<br>Equitable Life Assurance Soc.,<br>Mansion-house-street, E.C.       |
| 1896                            | Pate, Percy Wilfrid,<br>Refuge Assurance Company, 61<br>Manchester-road, Nelson, Lancs.                 | 1893*                            | Pownall, Herbert Wilfred,<br>Australian Mutual Provident<br>Society, Adelaide, Australia.          |
| 1897                            | Paton, Harry Arthur, Royal Exchange Assurance Corporation, Royal Exchange, E.C.                         | 1898                             | Pring, Arnold Lyddon, Prudential Assurance Company, Holhorn-bars, E.C.                             |
| 1897*                           | Patrick, James,<br>95 Carisbrooke-road, Liverpool.  | 1897                             | Proctor, Jr., William, Refuge Assurance Company,   |
| 1892*                           | Paull, Richard Alfred,<br>430 Bourke-street, Melbourne,<br>Australia.                                   | 1895                             | Oxford-street, Manchester.  Quare, Lionel Brain,  Equity & Law Life Assur. Soc.,                   |
| 1892*                           | Pearce, Charles Edward, 3 Birchwood-villas, Clarence- road, Sidcup, Kent.                               | 1886*                            | 18 Lincoln's-inn-fields, w.c.  Quick, John Richard,  Equity & Law Life Assur. Soc.,                |
| 1897                            | Pearce, John Northup,<br>43 Schubert-rd., East Putney, s.w.   |                                  | 18 Lincoln's-inn-fields, w.c.  |
| 1898                            | Peirson, Perey F.,<br>17 Hartford-street, Coventry.   | 1893                             | Quin, Stewart Blacker, A.C.A.,<br>1 Lombard-street, Belfast.                                       |
| 1896*                           | Penman, William, Northern Assurance Company, 1 Moorgate-street, E.C.                                    | 1897*                            | Rae, Joseph, Finance Department, Vestry-hall, Upper-street, N.                                     |
| 1897                            | Pennington, Cyril Burn,<br>23 Trebovir-rd., Earl's-court, s.w.  | 1893**                           | Raisin, Arthur Herbert,<br>Pelican Life Office, 70 Lombard-  |
| 1896**                          | Penny, Charles Augustus, Prudential Assurance Company, Holborn-bars, E.C.                               | 1897*                            | street, E.C. Ramwell, James Murray, Lancashire Insurance Company,                                  |
| 1895*                           | Peters, Charles Furness,<br>L'pool. Victoria Legal Friendly<br>Society, 18 St. Andrew's-street,<br>E.C. | 1898                             | 18 Exchange-street, Manchester. Read, William A., Prudential Assurance Company, Holborn-bars, E.C. |
| 1894*                           | Peele, Thomas,  Refuge Assurance Company, Oxford-street, Manchester.                                    | 1898                             | Reynell, Guy Courtenay,  Lancashire Insurance Company,   |
| 1898*                           | Pigrome, George Davey,<br>Prudential Assurance Company,   | 1894*                            | 14 King William-street, E.C.<br>Richards, Gilbert P. A.,   |

Royston-villa, New Barnet.

Holborn-bars, E.C.

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| Date of<br>becoming<br>a Student |   | Date of<br>becoming<br>a Student |   |
|----------------------------------|---|----------------------------------|---|
| 1897*                            | Richmond, George William,<br>Scottish Widows' Fund Life<br>Assur. Society, 28 Cornhill, E.C.                                | 1594*                            | Sanderson, William, M.A.,<br>"The Economist," 24½ Kiny-<br>street-east, Toronto, Canada.              |
| 1894**                           | Rietschel, Hermann Julius, Sun Life Assurance Society, 63   | 1894*                            | Saunders, Herbert Stewart, M.A., 3 Bolton-gardens, s.w.   |
| 1898                             | Threadneedle-street, E.C.  Robertson, Donglas Gordon, 154 Bethune-rd., Stamford-hill, N.                                    | 1892*                            | Savery, Robert S. B.,<br>Gresham Life Assurance Society,<br>St. Mildred's-house, Poultry, E.C.        |
| 1896*                            | Robinson, Frederick Charles,<br>Royal Exchange Assur. Corpora-<br>tion, Royal Exchange, E.C.                                | 1897                             | Sawtell, John A  Law Accident Insurance Society, 215 Strand, w.c.                                     |
| 1898*                            | Robinson, Hugh Thomas Kay,<br>Clergy Mutual Assurance Society,<br>2.y 3 Sanctuary, Westminster, s.w.                        | 1893*                            | Schonten, Pieter, Algemeene Maatschappij van Lerensverzekering en Lijfrente, Damrak, 74, Amsterdam.   |
| 1897                             | Rogers, Vivian Fydell,<br>Scottish Amicable Life Assur.<br>Society, 1 Threadneedle-st., E.C.                                | 1597                             | Scott, Alexander Lewis,<br>Australian Mutual Provident<br>Society, Melbourne.                         |
| 1893*                            | Roll, Frederick James,  Pearl Life Assurance Company,  London-bridge, E.C.  | 1891*                            | Searls, Edwin Richard,<br>Northern Assurance Company,<br>1 Moorgate-street, E.C.                      |
| 1893*                            | 1893* Roodenburch, Bartholomeus Adrianus, Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 71, Amsterdam. | 1888                             | Sewell, Richard, C.A., F.F.A., 63 Threadneedle-street, E.C.   |
|                                  |   | 1891**                           | Sharman, William Charles, Prudential Assurance Company, Holborn-bars, E.C.                            |
| 1895*                            | Ross, Christopher Watson,<br>c o Messes. M. Moss & Company,<br>Melbourne, Australia.  | 1886                             | Sharp, Joseph Benjamin,<br>Clerical, Medical and General<br>Life Assurance Society, 15 St.            |
| 1895                             | Rowley, James Edward. 7 Waterloo-street, Birmingham.  | tinar                            | James's-square, s.w.  |
| 1895*                            | Rudd, Alfred James,   | 1892*                            | Sharpe, Edgar Cecil Engledue,<br>London Life Association, Ltd.,<br>81 King William-street, E.C.       |
| 1893*                            | Australian Widows' Fund Life<br>Assurance Society, Melbourne,<br>Australia.   | 1896                             | Shawyer, John William,<br>Law Union & Crown Fire & Life<br>Insurance Co., 126 Chancery-<br>lane, w.c. |
|                                  | National Provident Institution,<br>48 Gracechurch-street, E.C.  | 1896%                            | Sherriff, Francis Henry, Provident Clerks' Mutual Life  |
| 1894                             | Salter, George Ferry, Mem. Act.<br>Soc. Amer.,  |                                  | Assurance Association, 27 of 20<br>Moorgate-street, E.C.  |
|                                  | Prudential Life Insurance Co. of America, Newark, N.J.  | 1896*                            | Shute, Oxenham Bent,<br>National Provincial Bank of<br>England, 53 Baker-street, w.                   |
| 1891*                            | Sanderson, Finlay,<br>North British and Mercantile<br>Insurance Company, 61 Thread-   | 1895                             | Simmons, Lancelot, Prudential Assurance Company, Hollown Lare F.C.                                    |

needle-street, E.C.

Holborn-bars, E.C.

Those marked—have passed the first of the three parts of the Examination for the Class of Fellow. Those marked—have passed two of the three parts of the Examination for the Class of Fellow; those marked—a, or ``b, have also passed either Section A or Section B of Part III of the Examination.

Date of becoming a Student. Date of becoming a Student.

| a Student |  | a Student |   |
|-----------|--|-----------|---|
| 1892*     | Simpson, William Murray,<br>North British and Mercantile<br>Insurance Company, 61 Thread-<br>needle-street, E.C.       | 1895*     | Sntton, Cecil Norman Stafford,<br>Marine & General Mutual Life<br>Assurance Society, 14 Leaden-<br>hall-street, E.C.  |
| 1891*     | Sindall, Alfred John,  London and Lancashire Life Assurance Co., 66 Cornhill, E.c.                                     | 1893      | Sutton, Harold Hereward Edgar,<br>British Empire Mutual Life<br>Assurance Company, 4 & 5 King<br>William-street, E.C. |
| 1888**    | Slimon, William James, F.F.A.,<br>23 Grosvenor-street, Edinburgh.  | 1895*     | Symmons, Frank Percy, Prudential Assurance Company,   |
| 1895      | Small, Herbert Charles, Prudential Assurance Company,  | 1892*     | Holborn-bars, E.C.  |
| 1895*     | Holborn-bars, E.C. Smeaton, John Richard, Alliance Assurance Company,  | 1002      | Tappenden, Laurence Barnard,<br>Mutual Life Insurance Co. of<br>New York, 17 & 18 Cornhill, E.C.                      |
| 1.00=0    | Birmingham.  | 1889*     | Taylor, John Theodore,<br>Templemore-park, Londonderry.   |
| 1897*     | Smithert, Charles Ritchie, Trustees, Executors and Agency Company, Limited, 412 Collins- street, Melbourne, Australia. | 1895      | Taylor, Leopold Victor,<br>Prudential Assurance Company,<br>Holborn-bars, E.C.  |
| 1895      | Smith, John White, United Kingdom Temperance & General Provident Institution,  | 1896*     | Terry, Ernest Fairchild,<br>General Reversionary & Invest-<br>ment Co., Ltd., 26 Pall-mall, s.w.                      |
| 1894*     | 1 Adelaide-pl., London-bdge., E.C. Smith, Lionel Gordon,   | 1895*     | Thistlethwaite, William, 4 Warren-terrace, Wakefield.   |
| 1897      | Wyccroft, Bexley.  Soper, Frederick Walter, Prudential Assurance Company, Holborn-bars, E.C.                           | 1894**    | Thomas, Robert Arthur Caradoc,<br>British Empire Mutual Life<br>Insur. Co., 4 & 5 King William-<br>street, E.C.       |
| 1897      | Stamp, Horatio E., Prudential Assurance Company,   | 1897*     | Thorne, Charles McKellar,  Darceyhey, Castle-hill, N.S.W.   |
| 1897      | Holborn-bars, E.C. Stephen, Alfred Hastings, Australian Alliance Assurance Company, Melbourne.                         | 1898*     | Tinner, Thomas, Comptroller's Depart., London County Council, Spring-gardens, s.w.                                    |
| 1898*     | Stewart, Lionel William, Institute of Actuaries, Staple- inn-hall, Holborn, W.C.                                       | 1897*     | Tipping, Oswald,<br>Trustees, Executors and Agency<br>Co., Ltd., 412 Collins-street,<br>Melbourne, Australia.         |
| 1886**    | Stirling, James, Jun., Scottish Imperial Insurance Co., 183 West George-st., Glasgow.                                  | 1897*     | Touzel, Philip Duncan,  Australian Mutual Provident Society, Melbourne, Australia.                                    |
| 1888*     | Stott, Walter, Royal Insurance Co., Liverpool.   | 1892*     | Townsend, Herbert Victor,<br>Colonial Mutual Life Assurance<br>Soc., 419 Collins-st., Melbourne,<br>Australia.        |
| 1893*     | Streeter, Theodore Edward,<br>69 Granville-park, Lewisham.   |           |   |
| 1895**    | Stuckey, Edward Joseph, B.Se.,<br>Australian Mutual Prov. Soc.,<br>Adelaide, South Australia.                          | 1897*     | Townshend, Edward Villiers,<br>North British and Mercantile<br>Insur.Co.,61 Threadneedle-st.,E.C.                     |

These marked there passed the first of the three parts of the Earmination for the Class of Fell w. These marked that class of the three parts of the Eramination for the Class of Fellow; these marked they are the house also passed either Section 4 or section B of Part III of the Earned either.

| becoming<br>a Student |          |            |                |  |
|-----------------------|----------|------------|----------------|--|
| 1895*                 | Treleave | n, Walter, | M.A., B.Sc.,   |  |
|                       | Scots    | College.   | Bellerne-hill, |  |

Date of

- Double Bay, Sydney, N.S.W.
- 1897 Truzzell, Harry, Northern Assurance Company. 15 Victoria-street, Nottingham.
- 1891 Tyler, Edgar Alfred, F.S.S., 1 Queen Victoria-street, E.C.
- 1893 Vine, George Henry Mesban. 23 Grange-road, Canonbury, N.
- 1893\* Vokins, George Alfred, Prudential Assurance Company, Holborn-bars, E.C.
- 1595\* Walker, David Edgar. Australian Mutual Provident Society, Sylney, Australia.
- 1896 Walter, Frederic Percy, A.C.A., 20 Bucklersburn, E.C.
- 1898 Ward, Albert E., Australian Mutual Provident Society, Melbourne, Australia.
- 1891 Waters, Charles Preston, Equitable Life Assur. Society. Mansion-house-street, E.C.
- 1594Watson, Norwood Alexander Reid, 71 Queen Elizabeth's-walk, Stuke Newington, N.
- 1896 Way, Claude Frederic, Scottish Widows Fund Life Assur. Society, 28 Cornhill, E.C.
- Weatherill, Henry, 1893\* National Debt Office, 19 Old Jerry, E.C.
- Webb, Lloyd, 1895\* Hand in Hand Insurance Suc. 26 New Bridge-street, E.C.
- 1893\* Welman, Arthur Joseph, Legal & General Life Assurance Society, 10 Fleet street, E.C.
- 1896 Westlake, William Coventry, 21 Portland-street, Southampton.
- Westland, James Black, Northern Assurance Company, 1858 1 Moorgate-street, E.C.
- Wheatley, George Frederick 1896\* Lavfield, Liverpool and London and Globe Insur. Company, 7 Cornhill, E.C.

- Date of becoming a student. 1597\* Wigner, John Gurney,
- 92 Turwhitt-Road, St. John s. s.E. 1886\* Williams, David,

22 Loftus-rd., Shepherd's-bush, w.

- 1594\* Williams, Frederick Alfred. 34 Sotheby-road, Highbury, N.
- 1595\* Williams, Henry Samuel Walter. The Imperial Insur. Co., Ltd., 410 Collins-street, Melbumrne, Australia.
- 1890\* Wilson, George, Standard Life Assurance Companu, Edinburgh.
- 1897 Wilson, Hammersley Weatherall. Scottish Union & National Insurnace Co., Newcostle-on-Time.
- Wilson, John Sydney,
  Australiaa Widows Fund Life 1506\* Assurance Society, Melburgar, Australia.
- 1894\* Windett, Sydney V., Eagle Insurance Company, 79 P ///-m.///. s.w.
- 1855\* Wingfield, Harry, M.A., A.C.A., 64 Cannon-street, E.C.
- 1593\*\*/Winter, Arthur Thomas, The British Empire Mutual Life Assurance Company, 4 & 5 King William-street, E.C.
- 1895\* Wood, David James, Commercial Caina Assurance Co., 24, 25 3 26 Cornhilt, E.C.
- 1896 Woodhouse, Hubert Allen. Union Assurance Society, 81 Curnhill, E.C.
- 1888 \*\* Worthington, William, Luncushire Insurance Company, Exchange - street, St. Ann's. square, Manchester.
- 1594\* Wyatt, George Matthew, Law Guarantee & Trust Society. 49 Chancery-lane, W.C.
- 1894\* Wylie, Samuel Brown, A.M., Fidelity Mutual Life Association, 914 Walnut-st., Philadelphia.

Those marked—have passed the first of the three parts of the Eramination for the Class of Fellow. Those marked—have passed two of the three parts of the Eramination for the Class of Fellow; those marked \*a, or -b, have also passed either Section A or Section B of Part III of the Eramination.

Date of

| Date of

| a Student |  | a Student. |   |
|-----------|--|------------|---|
| 1886      | Yeatman, Alexander Alfred,<br>2 Gresham-buildings, E.C.                        | 1894*      | Young, Arthur Stanley,<br>Commercial Union Assurance Co.,<br>24, 25 & 26 Cornhill, E.C. |
| 1895*     | Yeldham, William James,<br>Prudential Assurance Company,<br>Holborn-bars, E.C. | 1897*      | Younger, R. Hugh,<br>Lancashire Insurance Co., 18<br>Exchange-street, Manchester.       |

<sup>\*\*</sup> It is requested that any inaccuracy in the foregoing list may be pointed out to the Assistant Secretary.

#### CORRESPONDING MEMBERS.

### Belgium.

#### BRUSSELS.

M. Henri F. G. Adan,

Directeur Général de la Royal Belge Compagnie Anonyme d'Assurances à Forfait sur la Vie, et contre les accidents, Rue Royale (coin impasse du Parc), Membre de la Commission Centrale de Statistique du Royaume de Belgique.

- M. AM. Bégault, Mem. Act. Soc. Amer., Actuary, Compagnie belge d'A'ssurances generales sur la vie.
- M. Léon Hamoir, Directeur Général de la Cie. des Propriétaires Révnis, 16 Rve de Loxum.

M. Omer Lepreux,

Directeur - Général de la Casse Générale d'Epargne et de Retraite de Belgique. Président du Comité Permanent des Congrés Internationaux d'Actuaries. Vice-Président de l'Association des Actuarics Belges. Membre Correspondant de l'Institut des Actuaires Français. Mem. Act. Soc. Amer. Ancien Capitaine du Géine Chargé de Cours à l'Ecole Militaire; 48 Rue du Tassé-aux-Loups.

#### Switzerland.

#### ZURICH.

Herr Gottfried Schaertlin,
Direktor der Schweizerischen Lebensversicherungs-und-Rentenanstalt.

#### France.

#### PARIS.

- M. Edouard Antoine Badon-Pascal,
  Directeur du Journal des Assurances,
  1 Rue Rossini.
- M. Léon Marie, Mem. Act. Soc., Amer., Actuary, Le Phénix Compagnie d'Assucances sur la vie, 32 Rue Jouffroy.
- M. Albert Quiquet,
  Actuary, La Nationale Compagnie
  d'Assurances sur la vie, 13 Rue de
  Grammont.
- M. Paul Guieysse, Mem. Act. Soc., Amer., Président de l'Institut des Actuaires Français, 42 Rue des Ecoles.
- M. Alfred Thomereau, S Rue le Peletier.
- M. Victor Senès, L'Isle Adam (Seine et Oise).

## Holland.

#### THE HAGUE.

M. Henriquez Pimentel, Bezuidenhoul 108.

# RULES

FOR THE

# REGULATION OF THE LIBRARY.

- 1. The Library is open daily, from Ten to Six, except on Saturdays, when it is open from Ten to Three. It is closed for revision during the month of September.
- 2. Members of the Institute are permitted to take out Two Books on making application in person, or by letter addressed to the Assistant Secretary; but no Member may keep any work longer than a Fortnight. When a Book is returned by a Member, it can be borrowed by him again, provided it has not been bespoken in the meantime by another Member.
- 3. Scientific Journals and Periodicals are not circulated until the volumes are completed and bound.
  - 4. Cyclopædias and works of reference are not circulated.
- 5. Any Member damaging a work must pay a fine equivalent to the injury.
- 6. Works taken from the shelves for reference are not to be replaced, but must be laid on the Library table.
- 7. The Honorary Secretaries shall report to the Council any infringement of these Regulations.

Jan. 1899.]

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES.

Opening Address by the President, H. W. Manly, Esq.

[Delivered 28 November 1898.]

WHEN you did me the honour to raise me to this pinnacle of fame, I could not help taking a mental survey of my shortcoming and picturing myself as a pigmy among the giants who have preceded me in the occupation of this chair. The deep historical research, the philosophical rhetoric, and the brilliant rush of oratory which have characterized my predecessor Mr. Young's numerous addresses and speeches, have filled us with profound admiration; and his untiring energy and devotion in the service of this Institute, the consummate art and exquisite tact with which he presided over, not only our own meetings, but the business and social gatherings of the Congress, have elicited universal recognition and earned our deepest gratitude and thanks. I can only hope to follow at a very long distance in his footsteps; and as custom imposes on me the duty of opening the Session with a Presidential Address, I trust you will be merciful in your judgment, and not too critical in your comparisons. will do my best to interest you, if I am unable to instruct you.

The choice of a subject is a difficulty. Not, for reasons frequently advanced, that there is nothing left to talk about, but because the subjects are so numerous it is hard to make a selection. It is true that nearly everything that can be said about the profession of actuary, the character of an actuary, the

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duties of an actuary, the formation of the Institute, the history of the Institute, the work of the Institute, has been said, and said far better than I could say it. Then, again, there are the subjects relating to the conduct and management of a life assurance company: the acquisition of our business; expenses, surrendervalues, the investment of the funds, the valuation of the liabilities, distribution of profits, and the law relating to life assurance; all of which have been more or less discussed, but which, from their ever-changing form and character, have a perennial freshness. And then there are such big questions as the conduct, management, and financial position of those marvels of organization, the industrial assurance companies, and of those estimable institutions which the industrial classes have in their honourable independence and with commendable foresight established and fostered throughout the land, to provide themselves with the necessaries of life in the time of sickness, a support in old age, and a decent burial when they die; to discourse on which requires a knowledge like Sam Weller's knowledge of London, both extensive and Most of the subjects I have enumerated might peculiar. legitimately form the basis of a Presidential Address; but as I have asked you to be merciful to me, I will, on my part, endeavour to be merciful to you. Sufficient has happened during the past year to form an ample text for my Address.

It is with deep sorrow that I have to announce the death of a distinguished and honourable member of our body-one who occupied this chair with credit to himself and advantage to the Institute-Mr. William Sutton. Mr. Sutton was educated at King Edward's Grammar School, Birmingham, whence he proceeded to Cambridge and graduated in high mathematical honours, being elassed 32nd Wrangler in the Tripos of his year. For a short time, I believe, he occupied himself with scholastic work, but eventually entered the actuarial profession as a pupil under His first appointment was as chief clerk under Mr. R. P. Hardy, at the London and Provincial Law Assurance Society, an office which, in its time, had the honour of producing a fair number of actuaries. The Government, being anxious to strengthen the staff at the Registry of Friendly Societies by the addition of a professional actuary, offered Mr. Sutton the appointment, and it is in that capacity that he is best known to the present generation of Institute men. But it is more with his work in connection with the Institute that I should like to speak of him. In 1871, the Council of the Institute resolved to appoint

a tutor to assist the junior members to pass the second year's examination, and Mr. Sutton was selected to form the first class. The excellence of his teaching can be vouched for by many actuaries now holding high positions in the insurance world. A valuable record of his style of work is to be found in three lectures, which were addressed primarily to all the students, but were open to all the members of the Institute, published in the 16th volume of the Journal. The whole ground of study was fully covered in those lectures, and their orderly arrangement and clear, though terse, explanations, showed the scholastic mind. I imagine that those lectures must have been of great service to a long line of students, and that the methods which he originated have been followed by subsequent tutors. During this period he contributed many valuable notes to the Journal. When, subsequently, the Council of the Institute decided to publish a Text-Book, embracing all the subjects contained in the syllabus of the second year's examination, Mr. Sutton was invited to undertake the task of writing it. He completed the first part, dealing with the question of compound interest only, but, unfortunately, was unable to proceed with the second part owing to the pressure of other engagements. If I may be permitted to express my humble opinion upon that work, I should say that it is quite unfitted for the student. Mr. Sutton, no doubt, intended to write a book that should be worthy of the Institute, but with the unfortunate result that it is too erudite for a Text-Book. His principal contributions to our proceedings were the papers "On the Method used by Dr. Price in the construction of the Northampton Table", in which he reconstructed that once famous Table, and "A Comparison of various " Methods of Graduation of a Mortality Table, considered in "reference to the Valuation of the Liability of an Average Life "Office under its Assurance Contracts", both of which show that the author had a keen analytical mind. They will always repay a careful study. But the greatest work he did for the Institute, and for which his name will always be held in the highest esteem by its members, was in relation to the securing of our Charter. I happened to be senior honorary secretary at that time, and I remember well with what energy and determination he threw himself into the work. It was at his instigation that the Institute agreed to petition the Privy Council; it was he who undertook the task of preparing the petition and Charter; and it was he who, with a strong tenacity of purpose, eventually succeeded in obtaining the grant, in spite of the most strenuous opposition from a section of our profession who did not belong to the Institute; and it was upon him that the heavy burden fell of subsequently drafting the Bye-laws.

Mr. Sutton was elected President in 1888, and his Presidential Addresses were characteristic of the man; sharp, crisp, pungent, and full of instruction and advice.

His duty as Actuary to the Registry of Friendly Societies led him to undertake the gigantic task of collecting, arranging and tabulating the mortality and sickness experience of the registered friendly societies in England and Wales. The immensity of the work will be appreciated when you learn that the final results embraced 5,500,000 years of exposure, 8,000,000 weeks of sickness and 61,000 deaths. Happily, the work was completed before he was struck down by his last illness, and there is a pleasurable satisfaction in knowing that he saw it through the press and published before his death. This work is a lasting monument of his industry and skill. He passed away at the early age of 56.

To the actuary engaged in the valuation of friendly societies, this last work of Mr. Sutton's should form a rich field for independent investigation. I hope I am breaking no confidences when I say that Mr. G. F. Hardy, than whom there is no one more competent, has in preparation a collection of tables based upon this experience for use in the valuations of the liabilities of friendly societies, and that we may look forward to their

publication at no very distant date.

The Charter had the effect of altering, to some extent, the constitution and functions of the Institute. We have ceased to be a private body, working solely for the improvement and advancement of our profession, and have become a public body with public duties and responsibilities. I know that some, especially among our elder brethren, lament the days when we had more freedom of action, particularly with regard to the alteration and suspension of our rules-before we were bound in the rigid bonds of a Charter and Bye-laws; but the change, even if it had been delayed for a time, was bound to come. Our vigorous growth in numbers and importance, the necessity of securing a proper official definition of an actuary, the progressive development of social questions requiring the aid of the actuary to investigate and analyze the consequences of adopting any proposed schemes, made our transformation into a public body a natural sequence of our progress.

The very fact, however, of our having become a Chartered body, coupled with the official recognition of the Fellowship as the qualification of a competent actuary, has attracted to us persons who simply want to take advantage of calling themselves members; and thus, by their ingenious discovery that a Student is a member, they disclosed to us a weakness in our qualification for the Studentship which for a time gave rise to some anxiety. The Institute has never attempted to make the profession a Trades Union. It has throughout the fifty years of its existence acted in the most liberal spirit with regard to admissions. It requires no apprenticeship, no certificate of practice, and does not even insist upon an applicant being connected in any way with the business of life assurance. The doors have been practically open to anyone properly introduced, and we have said, "Pass the " examinations and you will then have a certificate of your com-" petency to perform actuarial work." The standard of efficiency which we require our members to reach before they can obtain the coveted Fellowship is high, very high; but it makes the possession of that title a most valuable one. It is in itself, and the public recognize it to be, the sign manual of actuarial ability, and we owe it as a duty to ourselves and to the public to see that that high standard is maintained, and that the good name of the Institute is not abused.

But the high character which the Institute has gained in public estimation, and the ease with which almost anybody was admitted, led to many persons applying for admission as students, who had no intention of passing the examinations, their sole object being to call themselves "Members of the Institute of Actuaries." In fact, one man boldly admitted it, and said that by calling himself a Member of the Institute he expected to have a large number of actuarial cases sent to him for his opinion; but it was his intention to employ a Fellow of the Institute to make the valuations, and he would only sign the opinions. Here was an admittedly fraudulent attempt to deceive the public, and to bring the name of the Institute into disrepute. This, and other cases, having come to the knowledge of the Council, they naturally took a very serious view of the situation. When the Charter was framed, the membership was intentionally divided into three classes-Fellows, Associates, and Students. The Fellows were entitled to use the letters F.I.A. after their names, and the Associates A.I.A.; but no title was assigned to the Students. It was supposed that

no one would publicly advertise himself as a Student of the Institute, but unfortunately there was no prohibition to his calling himself a member. The public are not sufficiently acquainted with us vet to distinguish the niceties of title. remedy for this state of things was urgently required, and we felt that we must either close our doors to all outside the insurance world, or establish a test examination sufficiently severe to prove that the applicant for the Studentship had a serious intention of proceeding to qualify for the Fellowship. I think you will agree with me that the Council have found a happy solution of the difficulty. To obtain the Fellowship, three examinations have to be passed—the third for convenience having in recent years been divided into two sections. To obtain the Associateship, two examinations have to be passed. Then, why should not the Student be made to pass the first of these examinations? By such means we should limit the members to those who seriously desire to become Actuaries, and the Studentship would in time become valuable as the mark of one who possesses a certain amount of mathematical knowledge; while a barrier would be set up against those who have no intention of passing any examination, but merely want to use their membership for advertising purposes.

It has therefore been decided that the first examination, or, to be more accurate, Part I of the syllabus of examinations, shall henceforth be a matriculation examination. At the same time, the Council have been most anxious to afford every encouragement to the young men in our offices to study for the examinations, and a class of Probationers has, consequently, been established, who, although not members of the Institute, will have most of the privileges of the members. They will be allowed to join the classes for Students, to attend the ordinary sessional meetings of the Institute, and to borrow books from the library for the purpose of their studies. The fces for this class have been reduced to half those which the Students pay; that is to say, the admission fee will be 10s. 6d. and the annual subscription 10s. 6d., and when the Probationer has passed Part I of the examinations, and is admitted a Student, the admission fee of 10s. 6d. will be taken as paid on account of the entrance fee of £1. 1s. By this means, the liberal character of the Institute is preserved, its educational advantages are continued, and the fees cheapened for our junior clerks, who are not overburdened with this world's riches.

Concurrently with this change, the Council have revised the syllabus of examinations. Very little alteration has been made in the subjects, but they have been re-arranged with the view of making each part distinctive and complete in itself. When this syllabus comes into force, which it will do piecemeal, Part I in 1900, Part II in 1901, and Parts III and IV in 1902, so as not to disturb the course of reading which the present students have taken up, there will be no longer any doubt in the minds of the candidates as to the nature and extent of the reading required in the various subjects to pass the several examinations.

The most notable event of the past year in the actuarial world was the meeting of the Second International Actuarial Congress in these rooms. The cheers which followed Mr. Young's learned and admirable opening address seem still to echo round these walls; the euphonious dissonance of tongues of all nationalities still lingers in our ear; and the festive and social functions which brought us and our visitors into closer personal harmony and friendship with each other, proved again that in science and learning "all the world's akin," and that we are "brithers a'."

If an International Congress is, as I take it, an assembly from all the world of the principal representatives of the workers and students in the field of some common interest, brought together to extend each other's knowledge and advance the common interest, then, I think, we may fairly claim that the Congress which met here last May was a great success.

The subjects on which papers were invited were in no way limited to purely actuarial matters, but extended to those larger questions which have for long been exercising the minds of statesmen, political economists, and social reformers, and which have taken more or less practical shape in advanced societies; a sphere in which the actuarial mind, with its wide knowledge of economics, its scientific treatment of statistics, and its exact mathematical reasoning, can, and should be, eminently helpful. Thus, the questions of "Compensation to Workmen for Accidents", "Old Age Pensions", "The position of Friendly Societies" and "Legislation in Relation to Life Assurance", were brought prominently forward for discussion. Many valuable papers, on purely actuarial questions, were, of course, contributed, and it is a subject for regret that there was not sufficient time to adequately discuss them.

If I am not boring you, I should like just to run through

the papers in the order in which they were submitted, and make a few flying comments on them. The first day, Monday, the 16th May, was occupied, after the formal business had been settled, and the President, Mr. T. E. Young, had delivered his opening address, with miscellaneous papers which did not come within the range of classified subjects. Dr. Karup, of Berlin, submitted a learned and exhaustive paper on Graduation, which, in the absence of the author, was taken as read. This paper shows a masterly grasp of a difficult subject; discusses in a clear, orderly, and analytical manner the various methods of graduation familiar to us in this room, and is deserving of careful study by all interested in this important question. I have, myself, spent many hours in trying to determine the best method of graduation, and I am inclined to think that it is impossible to find any one formula or any one method that will give entire satisfaction in every case. I think objections, which to most minds would be considered reasonable, may be advanced against the adoption of any one method as a standard method; and, in my opinion, each set of tabulated facts requires separate study and treatment. These remarks are not intended in any way to detract from the value of Dr. Karup's paper; for, I assure you, I think very highly of it, and I commend it to your special attention when you get your copy of the proceedings of the Congress. This was followed by a purely theoretical paper "On the Limits to be held by Life Assurance Companies", by M. Corneille Landré, of Amsterdam. Nearly all that can be said on the practical side of this question has been said by Dr. Sprague in a paper in the 13th vol. of the Journal, to which, curiously, the author makes no reference. A short paper followed, "On the Calculation of Surrender-Values", by M.S. de Savitch, of St. Petersburg, in which he suggests that the deductions from the Reserve-Values should be based on some relation between the numbers of policies surrendered and in force at two different periods. interesting as giving the practice, as to surrender-values, in various countries. Let me digress here a little, in order to say that I think the time is ripe for another discussion on surrender-values, especially with respect to endowment assurances, and to express the hope that some member will undertake to prepare a paper on He will find a most interesting collection of papers in the 1897 Report of the Insurance Commissioner of Massachusetts, in the form of replies to an invitation sent out to American and English actuaries to express their opinions on the

policy of having a legal basis of surrender-values and particularly on the method of valuation adopted by the State of Massachusetts. It is needless to say that nearly all the actuaries who replied condemn the establishment of any legal standard; but I have a modest opinion of my own that the American companies have only themselves to thank for it. The American people are constantly reminded that all their life insurance companies are under State supervision, and have to conform to a certain standard of valuation, and that consequently they must be safe; what more natural, then, when the people became aware of the hard and cruel conditions of the original tontine policies, which were absolutely forfeited if the premiums were not punctually paid, and for which no surrender-values were given, that they should say, "Since State supervision is so "beneficial, we will carry it a little further and have a law " compelling surrender-values to be paid", and then proceed to fix a standard of value. Once let the State gain control, and there is no limit to its interference. Let us hope that we shall continue to conduct and manage our companies on sufficiently sound principles with a proper regard for the interests of the assured. that no demand will ever be successfully made for State supervision in this country.

To return to the Congress. A "Note on the Mortality in the British Navy and Army", by Mr. J. J. McLauchlan, is a collection of tables extracted from the paper read by the author, before the Institute, on the 20th of April last. A very interesting paper was contributed by Mr. D. Carment (Sydney), on "The Rates of Mortality in Australia and New Zealand." In this paper, the author presented some statistics bearing upon the past and present rates of mortality as well as some of the diseases which prove most fatal in the Australasian Colonies. We cannot be indifferent to the rates of mortality prevailing in our colonies, seeing that many of the home companies are extending their operations in that direction, so that Mr. Carment's conclusions will command our respectful attention. After stating that he believes he has been able to show that the death rates in the Colonies are, on the whole, remarkably low, and that there is a tendency towards further improvement, he goes on to say: "We all desire, more or "less, to look into futurity, and are perhaps hardly so anxious to " determine accurately the death rate at the present time, as to " endeavour to estimate what it will be several generations hence, "when a purely Australian type has been fully formed. Doubtless, "Australia possesses, at present, a decided advantage over the "older country, but it may be that if a time should ever arrive "when this vast continent is as thickly peopled as England is now, "most, if not all, of this advantage may disappear, seeing that "the comparative sparseness of population is one of the greatest "factors conducing to our present favourable position." There we have the analytic and synthetic mind of the true inquirer into causes and effects. We, of this generation, however, may rest content with the present condition of the death rate, without troubling ourselves to consider what premiums should be charged and reserves made in that far-off distant time.

From a paper by M. Hankar (Belgium), on "Life Assurance by the Caisse Générale d'Epargne et de Retraite, of Belgium", we learn that, as part of the scheme for encouraging thrift, the Belgian Government helps workmen to become owners of their own dwellings, by advancing as much as 90 per-cent of the value, with, usually, an endowment assurance on the life of the borrower as collateral security. There is also a scheme by which friendly societies can re-assure their contracts bodily with the Government. Provision is also made for a distribution of profits every five years. These are very interesting experiments, but they have not been in operation long enough to afford any trustworthy experience.

"The Limitations of the System of Net Valuations", was a subject chosen by our old and revered friend, the late Mr. Sheppard Homans, for a paper he intended to write for the Congress, but owing to his lamented death, his friend Mr. Walter S. Nichols (New York), undertook the task of introducing it, which he did in a most ably written paper. The subject is familiar to all of us, principally through the writings of Dr. Sprague; but the paper is to be considered as another strong and well-reasoned protest against the adoption of a net premium valuation as a legal standard of solvency: a conclusion which we, in this happily free country, have no hesitation in endorsing.

We shall not, however, be so unanimous in agreeing with Mr. E. W. Scott (Amsterdam), who, in a paper on "The Influence" on the Prosperity of Life Offices, of Valuations, taking account "of Expenses of Management", advances the proposition that, in valuations for ascertaining the divisible surplus, the net premium method should be the exception, and that, as a rule, the great expenditure incurred by the offices in obtaining new business, justifies some scheme of gross premium valuation, in order to

distribute the bonuses equally over the whole period of the policy. We can, however, give him our unqualified support in his protest against the introduction of a legal standard of surrender-value. Grandmotherly legislation has an enfeebling effect on every industry affected by it.

Tuesday's proceedings were limited to the reading of, and discussion on, papers on one comprehensive subject, namely: "Legislation in relation to Life Assurance, including the general " principles of the laws which regulate Life Assurance Contracts, "and the constitution of Assurance Companies", and in this department, if I may so term it, we had a fairly complete set of papers. Thus we had an account of the laws in Australasia by Mr. Richard Teece; papers on "State Life Insurance Legislation in New Zealand", and "On the law with respect to Life Assurance "in New Zealand, outside that regulating the Government "Insurance Department", both from the pen of Mr. J. H. Richardson; and also a paper on the law in Cape Colony, by Mr. James McGowan. We also had a resumé of the laws in France (by M. L. Massé, in the German Empire by Dr. Karl Samwer), in Holland (by M. J. F. L. Blankenberg), in Russia by M. M. Ostrogradsky', in Spain (by Dr. J. Maluquer v Salvador, in Switzerland (by Dr. Ceresole), and in the United Kingdom (by Mr. A. R. Barrand). There were also two learned and instructive papers by our Belgian friends, M. H. Adan ("On various Laws " relating to the Life Assurance Contract from the International " point of view") and M. Charles Le Jeune ("On Legislation in "its Commercial and Economic bearing on Life Assurance, from "the National and International points of view"]. The reading of these papers led to a very interesting discussion. I do not propose to give you a dissertation on them; first, because it would be wearisome; and second, because Messieurs Adan and Le Jeune, have, in their able essays, expressed nearly all that there is to say on the subject, and in a far better manner than I could.

To have a summary of the laws of different nations and States brought together in one volume, is a distinct convenience to the political student and the progressive manager, and, in this respect, the volume of Proceedings will become a useful work of reference.

The third day, Wednesday, the 18th May, was taken up by the reading and discussion of papers on "The Position of Friendly Societies in Various Countries." The subject is one which must interest all who are concerned in the welfare and well-being of

the community; and the collection of full information from every country and State where these organizations exist must add materially to our knowledge, and give rise to intelligent inquiry and investigation, and lead to a full consideration of the constitution and proper management of these societies. The papers contributed dealt exhaustively with the condition of friendly societies in Belgium (M. L. Duboisdenghien), France (M. J. Cohen), Germany (Herr Unger), Holland (M. J. van Schevichaven), Spain (Dr. J. Maluquer y Salvador), United Kingdom (Mr. E. W. Brabrook), Cape Colony (Mr. J. McGowan), and New Zealand (Mr. George Leslie); and Mr. P. L. Newman contributed a paper "On the Relation which should exist between the State and Non-collecting Friendly Societies." An exceedingly valuable discussion followed the reading of these papers, the fullest on any subject before the Congress, showing the widespread interest taken in the welfare of these institutions. I cannot trust myself to keep within reasonable bounds if I once start on a review of the papers; but I hope that someone here to-night will take the question up and present the Institute with a paper on the position of friendly societies in Europe.

On Thursday, the 19th, the Congress adopted resolutions of the most vital importance, which will have the immense advantage of promoting friendly relations and an exchange of ideas between actuaries throughout the world. A universal system of notation, which should be comprehensive in character as well as flexible and symmetrical in adaptation, was required to represent all our mathematical expressions in one consistent analytical language; and the members of the Institute had the gratification of finding that the Institute Notation, which had been tentatively approved by the First International Congress, held in Brussels, was unanimously adopted by this Second Congress. A flattering compliment has thus been paid to this Institute; and we can never forget the gratitude we, and all the actuaries throughout the world, owe to Dr. Sprague, in addition to the many valuable improvements he has made in our science, for having taken the initiative more than thirty years ago in framing that system of Notation. The submission of the resolution was preceded by the reading of explanatory papers by M. Bégault and Mr. George King, which were extremely valuable in disposing of some minor criticisms which had been advanced in a discussion at a meeting of the Institute of French Actuaries on 18 June 1896. neighbours across the Channel had generously spoken in high

praise of the system, but there was still a lingering feeling that many of the symbols were capable of improvement—and we certainly could not claim to have arrived at perfection—so it was agreed "That such expansions and improvements thereof, as the continually extending sphere of actuarial science may render desirable, shall be considered at future Congresses." Notation is to be printed in the Proceedings of the Congress, omitting such symbols as are required for transactions peculiar to any one nation. On the motion of Dr. Moser (Switzerland) the question of preparing a subsidiary Notation (for other kinds of insurance) was referred to the Permanent Committee in Brussels, in order that they might appoint a sub-committee to consider the suitable extension of the Notation to interests somewhat different from those with which the Congress was principally concerned, with a view, if possible, of submitting a report on the subject to the next Congress.

The next action of the Congress, in order "to facilitate among "actuaries of different countries the establishment of fraternal "relations", as the Rules of the First Congress at Brussels felicitously described one of its functions, was to approve a proposal by M. Albert Quiquet (France), that an "International Dictionary of Actuarial Terms" should be compiled; a proposal which I warmly supported, for the difficulty I had experienced in translating the technical phrases in the numerous letters I had received from foreign correspondents was fresh in my mind. M. Marie (France), Dr. Schaertlin (Switzerland), and M. Lepreux (Belgium), having warmly supported the idea, it was agreed to refer the matter to the Permanent Committee in Brussels to carry out the proposal.

The remainder of the day was taken up with papers on the question of compensation to workmen for accidents. Valuable contributions were submitted by Herr Karl Kögler (Austria), M. Louis Maingie (Belgium), M. Louis Weber (France), Herr Unger (Germany), M. Gioberti Luzzatti (Italy), Dr. J. Maluquer y Salvador (Spain), M. A. Pokotiloff (Russia), Mr. Jas. McGowan (Cape Colony), and Mr. Stanley Brown (United Kingdom). To have made the list complete, we ought to have had contributions from Norway, where a law came into force in July 1895; and from Denmark, where a law was passed which will take effect in January next.

In all civilized countries, the Civil Code or Common Law has recognized that an injured workman is entitled to compensation,

from his employer, for damages which occur during his work and do not arise from his own negligence or acts; but the law in time became surrounded with so many subtle doctrines, and its uncertainty was so strongly accentuated as regards its results, that the workman's remedy became a fiction. Germany was the first State to formulate a scale of compensations (in 1885), followed two years later by Austria (1887), ten years later (1895) by Norway, and by our own country last year. Denmark followed this year. These laws run more or less on the same lines, of basing the scale of compensation for death or disablement on the average weekly wages, confining the total amount within certain defined limits. They have the distinct advantage of removing all doubt as to what compensation an injured workman or his relatives are entitled to receive, and define the liability of the employer. In no country, however, does the Act extend to all industries. The funds for providing compensation have to be found by the employer in all countries except Austria, where the employed have to find 10 per-cent. In Germany, the first 13 weeks of illness arising from accident are provided by the sick insurance associations to which the employed contribute two-thirds of the

Apart from the social aspect of the question, about which there can be no two opinions, we, as insurance men, must look with favour upon a law which partakes of the nature of compulsory insurance. Only the very largest employers of labour can feel safe in being their own insurers. To distribute the risks equally, the employers must pool their funds, and for many reasons they have generally sought the aid of those companies which have been willing to undertake the insurance.

The premium has, in the first instance, to be paid by the employer, whether he is his own insurer or not. Who will eventually pay it is another matter. Of this we may be certain, that capital, as a rule, will not. The burden will fall either upon the consumer or the workman. The consumer, in the first place, will be asked to pay in higher prices, so as not to disturb the wages market; but in cases where he will not pay, as in those products in which foreign competition is very keen, then the workman will have to pay in some form or other. One of the first effects will be that the dull and stupid, and those past vigorous manhood, will either get no employment or have to accept greatly reduced wages. Instead of the measure relieving the Poor Rates, as was hopefully prophesied when the Bill was

passing through Parliament, I am afraid it is going to increase them.

I ventured to state at the Congress that so far as the Actuary was concerned, the question was only now in its experimental stage; and the variety of opinions as to the proper rates to be charged for the insurance of these risks confirms my views. I was amused, as no doubt you all were, with the headlines which appeared in some of the daily papers, just before the Act came into force, over articles exposing the great difference in the premiums which were being asked by different companies for the insurance of these risks. "Actuaries at fault". "Actuaries at Sea", stared one in the face every morning and evening. If they had said "Actuaries in the dark" they would have been much nearer the mark. To ask the Actuary to make a rate without reliable statistics to work upon, is like the old task set the Israelites in their captivity, of making bricks without straw. To determine the premiums for fatal risks is easy; but how is it possible to determine the cost of an annuity, in cases of total or partial disablement, when you have no statistics of the length of time for which such annuity will have to be paid. Even if you had statistics of a kind, they would hardly be applicable; because an accident which has compensation attaching to it, is very different to one without that golden fringe. Then, again, it is not easy to calculate the extra allowance which will have to be paid when there is a sharp lawver making a claim on one side and a company fearful of losing a reputation, by going into Court, on the other. Judging from the experience in third party risks, I certainly back the lawver. Whether the premiums charged have been too high or too low only time will show.

On Friday, the 20th May, we had some interesting and instructive papers on the subject of Old Age Pensions, from M. Lepreux (Belgium), M. Duplaix (France), Dr. Unger (Germany), Dr. O. Sistilli and MM. F. Ruinaldi and F. Benedetti (Italy), Dr. J. Maluquer y Salvador (Spain), and M. de Savitch (Russia). No one contributed a paper on the position in Great Britain, presumably because the Commission on Old Age Pensions was sitting at the time. These papers did not touch on the broad question of old age pensions for the working classes, but treated of existing pension funds. It appears that most of the States have, like our own, arrangements for providing Government servants, and sometimes their widows and orphans, with pensions; that municipal bodies and public institutions have schemes for securing pensions to their

servants; and that large commercial companies, such as railways, mining companies, &c., have their funds often established and regulated by law. Only Germany has ventured to adopt a compulsory pension insurance scheme for the industrial classes; and all Governments are watching with close attention the working of the measure. As in all cases where funds are accumulated to provide for deferred benefits, the real test will not come until the Act has been in operation for at least 30 years. The last paper read was a production of my own, "On the Solution of some Problems which frequently arise out of the Rules of Pension Funds and Friendly Societies", to which I may refer presently, to point a moral and adorn a tale.

M. Léon Marie invited the Congress to meet in Paris in 1900, when a great International Exposition will be opened there, and the invitation was accepted with acclamation.

Cordial votes of thanks to the Honorary Vice-Presidents, to the contributors of Papers, and to the President and officers, terminated the official proceedings of a very successful Congress.

It may be as well for me here to place on record that this "feast of reason" was accompanied with the "flow of soul." On the opening day, the Lord Mayor, who was one of the Honorary Vice-Presidents, with that generous hospitality for which the first Magistrate of the City of London is famous, and the Lady Mayoress invited all the members of the Congress and their ladies to a reception at the Mansion House. On the Tuesday, the Directors of the Prudential courteously entertained the delegates and officers and all the foreign visitors to lunch, and afterwards showed them over that vast building wherein is centred the most marvellous organization in the world. On Wednesday, the delegates and their ladies were entertained at Richmond by the Institute of Actuaries' Club. On Thursday, the President and Vice-Presidents of the Institute of Actuaries held a reception at the rooms of the Royal Institute of Painters in Water-Colours; and the whole culminated on Friday, in a grand dinner, in celebration of the Jubilee of the Institute, in the King's Hall at the Holborn Restaurant. Some of those who were staying in London over the Saturday, paid a visit to Windsor, where they were shown over the State apartments, and after lunch had a drive through the park and forest.

Before finally disposing of the Congress, I should like to make a few remarks on the labours of the Organizing Committee. They will serve at least to assist and encourage subsequent committees when they take up the heavy burden.

In June 1897, a circular was addressed to all actuarial organizations and institutions throughout the world, informing them of the date of the meeting of the Congress, inviting them to send not less than two delegates, and to induce their members to contribute papers, which should be sent in not later than the end of February. That useful organization, the Permanent Committee of Actuarial Congresses in Brussels, set itself in active motion, translated the circular into French, and undertook its distribution over the Continent. Towards Christmas the correspondence had increased to such an extent that the Honorary Secretaries of the Institute were unable to cope with it in addition to their Institute duties, which alone are very heavy, and eventually a small committee, which could meet in the City, was formed of the President (Mr. Young), Ex-President Finlaison. Mr. George King, and myself; Mr. King being appointed Foreign Secretary and I Home Secretary and Treasurer, with charge of the entertainment department. We worked most industriously; but February came and departed without our getting much more than promises. Affairs seemed to be getting desperate; and anxiety was stamped on all our countenances. Mr. King was anxious to get the papers in, I was anxious to get the subscriptions in, and between us, the Permanent Committee in Brussels had what the schoolboy would eall a warm time. To avoid the inconvenience of requiring adherents to send their subscriptions direct to London, I appointed local treasurers in the various countries, with the excellent result that we had 183 adherents in the United Kingdom, and 201 from the Colonies and Foreign States, a total of 384 in all. Early in March we issued another circular, headed "Regulations," again inviting Colonial and Foreign Governments and Actuarial Institutions to nominate delegates, and urging that all papers should be sent in at once. Under a sub-heading of "Subscriptions" we gave the equivalent of £1 in the money of every Foreign country, and the names of the local treasurers. Towards the end of March we were able to issue a "Preliminary Programme." Our efforts were at last bearing fruit. Papers began to drop in, slowly at first, but faster as the end of April approached, and at last they came with such a rush that we in turn became overwhelmed. Mr. King might be seen every day dragging about a huge portmanteau containing ponderous papers and a vast correspondence; I, sometimes, with a smaller bag, also containing correspondence, but, in addition, remittances in every conceivable form. In our straits, we invited

Mr. Burridge and Mr. Levine to join us, and they both threw themselves heartily into the work. The greatest efforts were made to obtain translations of the foreign papers, but it was impossible to accomplish the task before the Congress met. Mr. Levine rendered most valuable assistance in superintending the printing of the papers, and getting all the printed matter ready for the meetings each day. To Mr. Burridge I was indebted for his excellent help and advice in arranging the entertainments, and I desire to say that to him the whole credit is due for the perfect arrangements made for the grand Jubilee dinner at the Holborn Restaurant. My colleagues on the Organizing Committee, I am sure, will join with me in saying that it is impossible to find words sufficiently expressive of our indebtedness to Mr. King for his really Herculean labours in connection with this Congress work. His labours are not vet finished, for he is still editing the volume of Proceedings, and he has earned, what, I am sure, you will accord with enthusiasm, our heartiest and warmest thanks.

I must not omit to mention the great help we received from the Permanent Committee in Brussels. Their experience and advice were most valuable, and they certainly contributed very largely to the ultimate success.

Gentlemen, there is one subject on which I should like to address a few words to you—a subject which has been exercising the minds of politicians, economists, philanthropists, social reformers, and actuaries for some time, and a satisfactory solution of which seems almost as far off as ever; I refer to the subject of Old Age Pensions. I know I am treading on dangerous ground, because your President is expected to avoid questions of a controversial nature, and there is no question of recent years on which so many dogmatic opinions have been expressed. We are also very careful here to avoid all reference to politics, but it is agreed that this is not, in its present stage, at least, a political question. It is true, as Sir William Harcourt, in his robust, blunt manner, declared one night in the House, "We are all socialists now", and we should all like to see the lot of the toiler in his old age made more pleasant, if it can be done without destroying the energy, independence, and manly self-reliance of the working population. In his elegant and masterly speech at the Jubilee Dinner, Mr. Courtney said: "In the midst of their comfort and "luxury, they could not help but feel some sympathy with "those members of the proletariat, and with those who were a

"rank above them in the social scale, who contemplated the "dreary round of ill-paid toil, old age coming with limited " powers, with no resources, and with years of want coming as a " necessary sequel to a manhood of fully occupied industry. "they could in any way alter the dismal features of that picture, "the Legislature would be most proud to accomplish the task. "They could not do it without the assistance of the actuaries. "Whether it was possible or not, he was very slow to say, " because they had to reconcile what appeared to be two incon-"sistent laws. They wanted to maintain and develop the " strength of individual character, and unless the penalty of "want followed upon the sin of improvidence, they lost one of " the best incentives to thrift." The problem is undoubtedly one of very great difficulty; and the Report of Lord Rothschild's Committee does not aid us very much. Their reference was "To consider "any schemes that may be submitted to them, for encouraging "the industrial population, by State aid or otherwise, to make "provision for old age"; and the Committee, at the end of their report, say: "We approached our task with a deep sense of the "importance of the question into which we were charged to "inquire, and of the benefit which would be conferred upon the "community if a scheme could be elaborated giving encourage-"ment to the industrial classes, by the exercise of thrift and " self-denial, to make provision for old age, while it fulfilled the "moral conditions prescribed by the terms of our reference. It "is only very slowly, and with very great reluctance, that we "have been forced to the conclusion that none of the schemes " submitted to us would attain the objects which the Government "had in view, and that we ourselves are unable, after repeated "attempts, to devise any proposal free from grave inherent "disadvantages." This, coming from a very strong committee, on which three actuaries and the Registrar of Friendly Societies sat, practically decides that the question cannot be solved from the point of view of the reference; and, after reading the evidence, I do not see how it was possible to come to any other conclusion.

But even if the Committee could have found their way to recommend any scheme, it would only have affected the *élite* of the working classes—skilled craftsmen, who receive good wages and are in constant employment, probably about one in three of the male members of the industrial population. If that were the only class which had to be dealt with, I do not think it would be

impossible to find some means of helping them if it were proved necessary; but the problem which we are anxious to solve is a far more important and difficult one than that. How are we to help the helpless and thriftless; that large army of unskilled labourers, whose occupations are often intermittent, and whose wages barely suffice for their daily needs, and those who have not got the selfcontrol to save, even when they are in receipt of good wages? How to provide for them in their old age without discouraging thrift; for most assuredly if State pensions were given as a right, without any discredit attaching to the recipient, the praiseworthy efforts that are now being made by the better portion of the workers would immediately cease, and the nobility of the man, his self-reliance and strength of character, would disappear. It is very well for Mr. Courtney to say that unless the penalty of want followed upon the sin of improvidence they lost one of the best incentives to thrift. It is a proposition to which we should all agree, and which we should gladly act upon, if everyone had the means wherewith to make provision. But every man has not, and certainly not every woman.

Again, it must not be forgotten that the best, and, indeed, the first duty of a citizen to the State, is to provide for himself and family, and educate his children in skilled craftsmanship, and so make them good and useful citizens, rather than spend his money in purchasing a deferred annuity for himself. I agree with Sir Spencer Walpole in his memorandum, \* that "an elaborate scheme "for bribing the working classes to insure against old age, in " preference to other forms of saving, is, in reality, a scheme for "nurturing the egoistic and restraining the altruistic qualities " of mankind." One of the most pleasing and hopeful paragraphs in the report of the Commission is the one on the question of Thrift, †: "No evidence given before the Royal "Commission on the Aged Poor was more conclusive than that "which related to the enormous and rapidly-increasing amount " of the accumulated savings of the working classes. It showed "that to industry, intelligence, and self-denial, the way to an "independent position lies open, and that a constant, increasing " number of individuals among the working classes are finding it." Do not then, I say, let us do anything to check it.

What, then, have been the remedies proposed. There are two

<sup>\*</sup> Report of Committee on Old Age Pensions. Page 21.

<sup>†</sup> Ibid. Sec. 61. Page 15.

prominent suggestions of a diametrically opposite character. On the one hand, Canon Blacklev proposes that every man and woman on attaining the age of 18 shall be made to subscribe a minimum sum of £10 within three years, which sum, with its accumulations, is expected to provide sick pay up to age 70 and a pension afterwards. Setting aside the question of the adequacy of the proposed payments, or even the possibility of every person being able to pay, I am satisfied that the people of this country would never submit to compulsion of that kind. Such a scheme must be ruled out as quite impossible of realization. On the other hand, there is Mr. Charles Booth's proposal, that every individual, on attaining a certain age, say 65, shall, by right, have a fixed pension from the State, irrespective of his health, antecedents, character, or private means. If we are prepared to make light of the enormous cost of such a scheme, and the evil consequences it would have on the self-reliant, virile character of the masses, followed by an early deterioration of the race; it must still be rejected, because there is no finality in it. Let the people once get a pension from the State for nothing, and with no condition but that they shall live to 65, an agitation would be immediately started to lower the age to 60. And if 60, why not 55 or 50, and in fact, why should we not all have at once 10s. a week from the State, by Act of Parliament. That, at least, would put an end to all pauperism. It is heroic, but it is not statesmanship.

Between these two extremes, innumerable proposals have been suggested of subsidizing, by State-aided pensions, a class who have belonged to some society, or anyone who has managed by saving to provide himself with an income, however small. All, or most of these schemes have been exhaustively examined by the Old Age Pensions Committee, and their report is a clear judicial verdict against them. They say: "From what has been " said above, it follows that any pension scheme coming within "the terms of our reference, would be limited to a comparatively " small section of the community, and we are thus face to face with " a very serious difficulty. We can hardly, for the benefit of so " limited a section of the community, recommend the Government " to establish a pension system which must be extremely difficult " and eostly to administer, which excludes the really destitute, " and those who, owing to broken health or misfortune, or want " of employment, or a lower rate of wage-earning, can make no con-"tribution, and which would be open to innumerable fraudulent "claims, difficult, and often impossible, to detect." I am convinced that every one with an unbiassed mind, after reading that report, and the evidence, will endorse the conclusion.

Are we then to say that the object aimed at—the amelioration of the condition of the aged poor—is impossible of attainment? Certainly not. We actuaries are used to looking at a problem from different points of view, and if we cannot arrive at a satisfactory solution one way, we try another, and occasionally turn it upside down to see how it looks that way. I believe that all these worthy enthusiastic social reformers have been following the wrong road. The ideal socialist, who dreams of an ideal, but impossible, state of society, has formulated an ideal theory that everybody is to cease work at a certain age, and thereafter be supported by the State, as an inheritance of his birth; and this is the trail which everyone has unconsciously been following.

Most of you will remember the story of the early attempts to reach the summit of the Matterhorn. For a long time the attempt was made on that side which appeared to be the easiest and direct way; but at last one man, Mr. Whymper, studied the formation of the mountain, and discovered that it was a stratified rock, with the strata tilted downwards towards the side they had all been trying to scale, with the result that they had all been endeavouring to get up overhanging steps. If that were so, then the other side, which looked impossible, ought to prove easy, for it should present just a series of steps like a staircase; and behold! when it was tried, the monarch was conquered.

So, I believe, it will prove with regard to this giant of a problem. Try as much as you will to get up the apparently direct and easy way, and you will ever be met with overhanging and insuperable difficulties. Anyhow we might at least study the formation of the rock, and see if there is not another way to conquer it.

Our western civilization is the result of a slow but progressive social transformation, which has been in process of development since the beginning of the Christian era, commencing with the gradual breaking down of the military organization of society when the people were divided into two distinct classes—citizens and slaves—and ending in the emancipation and enfranchisement of the people. "The enormous expansion of the past century", says Benjamin Kidd, in his Social Evolution, p. 55, "has been "accompanied by two well marked features in all lands affected by it. The advance towards more equal conditions of life has

"been so great, that amongst the more progressive nations such "terms as lower orders, common people, and working classes are "losing much of their old meaning, the masses of the people are "being slowly raised, and the barriers of birth, class and privilege " are everywhere being broken down. But on the other hand, the "pulses of life have not slackened amongst us; the rivalry is " keener, the stress severer, the pace quicker, than ever before." One of the vital changes in progress in our civilization has been the great development of the humanitarian feelings amongst us. "This is to be marked more particularly in a widespread interest " in the welfare of others, which exhibits itself in a variety of less "obtrusive forms. There may be noticed in particular the " extraordinary sensitiveness of the public mind amongst the " advanced peoples to wrong or suffering of any kind. One of "the strongest influences prompting the efforts which the " British nation has persistently (although quite thanklessly and " unobtrusively) made towards the suppression of the slave trade, " has been the impression produced by accounts of the eruelties "and degradation imposed upon the slaves. . . . . No one "can closely follow from day to day that living record (of our " daily life which the press furnishes) without becoming profoundly "conscious of the strength and importance of the altruistic "feelings of the present time. Appeals in respect of injury, "outrage, or wrong, suffered by any particular class, have " become one of the strongest political forces, and may sometimes "be observed to be more effective than even direct appeals to "private selfishness. . . . . The record in the press of "a case of death from starvation sends a tremor which may "almost be felt through the community. It is not that the " sensitiveness of the public mind in such cases is shown by "noisy denunciation; it is those hesitating, heart-searching "comments—frequently so pathetically misdirected—which the "circumstance oftenest evokes, that are so eloquent and so " significant." \*

This demand for old age pensions is the strengthening and deepening of this humanitarianism which cannot tolerate the thought of the honest worker, after long years of toil, being reduced in his old age, and when no longer able to work, to the degradation of the workhouse. It is no answer to say that it should not be looked upon as a degradation; that the State has

<sup>\*</sup> Benjamin Kidd's Social Evolution, p. 159.

provided a system of relief out of the rates, and that every destitute person is entitled to relief. For some reason or other the thought of the workhouse, or of workhouse relief, sends a thrill of horror through every respectable man and woman who has endeavoured, during the working years of life, to support himself or herself and family; and we know that many of them would sooner face starvation, than seek the merciless pity, as they view it, of the Poor Law. Perhaps it arises from the necessity of being required by the Law to go into the house, the breaking up of their little homes, the removal from free intercourse with their friends, the separation of the sexes, the mingling with the worthless and degraded classes, the wearing of the paupers' dress;-regulations which have undergone a wonderful change for the better in recent years ;-but whatever it is, the fact remains that the name of the workhouse is repulsive. "What's in a name"? asks Shakespeare. Well, there seems to be a good deal in it. These people are not averse to charity. Most of them will take it freely if it comes by way of any of the numerous channels through which private charity is (not always wisely) dispensed.

Yet I feel satisfied that it is from this side that the mountain of difficulty must be climbed. The industrious, honest, hardworking man has no desire to keep an able-bodied pauper in idleness, no matter what his age. He certainly would not submit to a man or woman capable of performing some light labour, being placed in a position to underbid him in the wages market, by the receipt of a State pension. And he largely recognizes that relief, in whatever form it comes, whether of State pension or otherwise, still carries with it some disgrace.

Obviously then, if we are to carry our humanitarianism so far as to provide for the worn-out aged poor, without hurting their feelings too much, we must change the name of, and the mode of administering, the relief. In fact, it appears to me that we want a revised Poor Law, providing better regulations for classifying and separating the inmates of our institutions for the destitute. Thus, we want "Workhouses" for the able-bodied paupers, "Infirmaries" for the sick and infirm who need that special care which they cannot obtain outside, and "Asylums" for the imbeciles. Then, there must be a well-conceived and carefully drafted outdoor "Old Age Pension Relief" scheme. It would be unfair to put the whole charge for this on the rate-payers, so there would have to be a large State subsidy; but only

for a portion of the expense,—a large portion if you like,—for it would be unwise to withdraw altogether the salutary restraining influence of the ratepayer. A new Board would have to be created in each parish for the administration of this relief. My idea is, that a Board consisting of the Justices of the Peace and the Guardians, would be best. Then, as the State would contribute a large portion of the expense, there would have to be official inspectors or assessors appointed by the Government to secure uniformity of practice throughout the country. would have to be laid down as to who should be entitled to relief, and the amount. A person, no matter what his age, if he could do some light work, should not receive full pay. On the other hand, every encouragement should be given to those who make a provision for themselves, however small; and if a person has an assured income, only a portion of it, say a half, should be taken into account in fixing the relief. All relief should be subject to revision at any time; but there is one regulation which I consider absolutely necessary to provide against fraud and deception, which is, that the names and addresses of all the recipients should be published yearly. The names of the pensioners on the Civil List are published, and there is no valid reason why the same rule should not be followed in this case. publicity as the greatest safeguard that can be established.

Different interests have been relieved by grants in aid from specific sources of income; and in my idea, the most appropriate fund from which to subsidize this enlarged outdoor relief, would be the Beer Tax.

I have no intention of drafting a Bill; but I have ventured to inflict these few observations on your attention, because I have studied this question deeply for a long time, and I have been forced to the conclusion that it is not a question for actuaries, but for statesmen.

There is still one little matter I should like to refer to before I leave this subject. Great efforts have been made to find a way by which the Government could help those who are willing to help themselves. If this is to be done, the assistance must not be confined to any class of persons, but must be open to everyone. I think the enormous number of poorly paid clerks are deserving of as much sympathy as anybody. Now it is admitted on all hands that the ordinary deferred annuities are unpopular, not only with the working classes, but with every section of the community, yet it is to this form of investment that we naturally turn if we

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want to assist anyone to provide for his old age. How would it do to offer that form of annuity which is described in Problems III and V in the paper which I contributed to the International Congress; that is to say, a deferred annuity purchased by a single or annual payment, with the condition that the payment shall be returned without interest in the event of death before the annuity commences, and even if, after the annuity commences, death occurs before the annuity payments amount to the premiums paid, the balance shall be returned to the relatives or next-of-kin. This, to most minds, would partake of the nature of a saving's bank where no interest is allowed; and I see no reason why it should not be like a savings bank, where the money can be withdrawn, if each payment is treated as a single premium to purchase a certain amount of deferred annuity, and the contracts allowed to be cancelled.\* Then if a State subsidy for old age pensions is to become an accomplished fact, the State might offer to grant these annuities, to commence at the age of 60 or 65, on a 4 per-cent basis, provided that no person be allowed to purchase a larger total annuity than £26 a year. It is probable that the withdrawals would reduce the effective guaranteed rate of interest to 3 per-cent. If the scheme became popular, the subsidy for the annuity scheme would in time diminish the subsidy for the old age relief scheme. Anyhow, I think it is worth considering.

Before I close, I shall be expected to address a few words to my younger brethren. You have received wise counsels from my predecessors, and I hope you have taken them to heart. I cannot equal that beautiful and eloquent advice of Mr. Young, that you should not consider the passing of the examinations the end of your studies, but rather the commencement of a wider mental training, and the step to an independent search after knowledge. I would urge you to read it over and over again.

But I so often hear the remark, "I cannot think of

<sup>\*</sup> The Post Office has a scheme of Deferred Annuities, where the purchasemoney will be returned on application, or on the death of the Nominee, if an
instalment of the Annuity shall not have become due. What is wanted is that
the purchaser should be allowed to deposit a shilling, or any number of complete
shillings, at any time, that he should have a deposit book, that once a year the
amount of deferred annuity which his year's deposits will purchase as a single
premium should be credited to him, just as interest is now credited to him in his
Savings Bank book, and that the annuity should be payable by monthly
or even weekly instalments. I think that it is more than probable that large
employers of labour would then encourage and help their men to adopt this form
of saving, and the friendly societies might also induce their members to adopt
the scheme.

anything new; everything that is worth doing seems to have been done already." That is a great mistake. There is plenty yet to find out, and I am now going to tell you how it is done. You remember the story of Isaac Newton and the apple: (It may not be literally true, but it is a good story.) How he was sitting one day under an apple tree, and an apple fell on his head, which incident led to the discovery of the theory of gravitation. Why? Because he asked himself the question, "Why did that apple fall"? Hundreds of apples had previously fallen on people's heads, but they never awakened a single thought. the grandest discovery in mathematical science had its origin in that simple incident. No discovery ever comes by accidental inspiration, although accident often does lead to discovery. It is the constant direction of the mind on some subject or object that leads to discovery and invention. It is thinking, ever thinking of the solution of a difficulty that induces inspiration. Let me illustrate this by a little incident in my own life.

It was about a year and a half ago that an officer in the Indian Service was introduced to me, who wanted to devise some scheme for inducing his brother officers to save something out of their pay to enable them to supplement their pensions on retirement by an additional income. Of course, there was the usual objection to deferred annuities. The banking system by which an officer should deposit a certain sum every year and allow it to accumulate until retirement, when he could buy an annuity with his accumulations, was discussed, but it was thought that sinking money in an annuity would be objected to on the usual ground that you might not live to get it all back again. Then came the question, Would it not be possible, if a man bought an annuity, to arrange that if he died before he received all his money back, the balance should be paid to his widow or relatives. I saw no reason why it should not be done, and I worked out the solution. I gave it to one of my juniors, who had passed his second examination, and he worked it out. I put it to another student who had also passed his "second", and he likewise worked it out. The problem, once stated, was not difficult of solution; but it is not the sort of problem that you would casually evolve out of your own head. It was a problem which came to me just in ordinary conversation. Well! I thought something might be made out of that problem, but I had other things to think about, so I just noted it down in my scrap book and set it aside.

Talking to some of our young brethren afterwards about the desirability of writing a paper for the Institute, I offered this problem to them as a seed which might be made to grow into a tree. But the inspiration never came to them. Then Congress began to approach, and as the English actuaries did not seem to be contributing many papers, I bethought me of my little friend which had been put away, and, after studying its possibilities for a short time, I produced that paper "On the Solution of "some Problems which frequently arise out of the Rules of "Pension Funds and Friendly Societies." Now comes the interesting part of my story. I was just a little proud of having evolved something new and, I hoped, useful; and I was relating afterwards to the family circle that I had just produced an interesting paper out of a problem which I had generously offered to one or two young actuaries, who could make nothing of it, when one of my children said, "Why, father, it is like that parlour game of 'I shine a light'; you see, you shone a light." You no doubt know the game. The party sit round a room and someone is chosen to think of a word. It is generally a word with a double meaning. The one who has chosen the word sits in the middle of the room, and the others ask in rotation various questions with the view of eliciting what the word is, and if anyone thinks he has discovered the word, he or she says "I shine a light." Well, I did shine a light indeed. It was the simplest and most perfect explanation of "how it is done." When Newton was pondering over the reason of why the apple fell, see what a brilliant light he shone. Take Makeham and his "Law of Mortality." The germ of that law is to be found in a small discovery, probably arising out of some simple incident, that three terms forming an irregular series, when increased by a certain constant, can be made to follow a geometrical progression; and when he subsequently applied it to modify Gompertz's Law, see what a light he shone. And so it has been with nearly all great discoveries—the growth of a tiny seed set in a fruitful soil; a little incident set working in a thoughtful brain.

Now I ask you, for your own sakes, to try and shine a light. In your daily work, in conversation with your friends, in your reading, it may be of an ordinary novel, some problem, some remark, some incident, will arrest attention and set in motion a train of thought which will throw a new light upon an old question, or lead, perhaps, to the solution or elucidation of some abstruse point in one of the numerous subjects which form

part of the actuary's education. When the apple falls upon your head, just stop to think why it fell. Think of it, and think of it, until you shine a light: and when you have shone a light, don't hide it under a bushel. Bring your light out; make yourselves known. Enrich our knowledge, and you will gain advancement for yourselves. Let your light so shine before men, that they may see your good works; then will you redeem that debt to your profession, which Baeon speaks of, "from the which as you of "course do seek to receive countenance and profit, so ought you "of duty to endeavour yourselves by way of amends to be a help "and ornament thereunto."

Do not imagine that it is necessary to wait until you have produced some ponderous essay, or made some wonderful discovery, before you stand up at that reading desk. assemble here to discuss any question that has any connection with the profession of actuary, with the laudable object of imparting and receiving knowledge and instruction; and if you watch the proceedings at our meetings, as I have for now over 35 years, you will find, as I have done, that it is generally the short papers which lead to the best discussions. Further, I would urge you not to hesitate to take part in the discussions. I know perfectly well that you are often burning to ask a question or express an opinion, and vet you let the occasion slip by for fear of making a fool of yourself. I know the feeling perfectly well, for I have gone through it all myself. You must get over that feeling, and the best way is to make a start. It is the first plunge which is so terrifying. When you have once taken a header, you will go in with much greater confidence a second time. And if you should say something which does not appear afterwards to be over wise, do not be discouraged. Always remember that incident in the life of Benjamin Disraeli, afterwards Lord Beaconsfield, when he made his first speech in the House of Commons, and they all roared with laughter at him. He turned round and said "You will not hear me now but some day you will listen to me"; and he rose to the greatest position in the State, in highest favour with his Sovereign, and the whole of Europe listened intently to every word he uttered.

Gentlemen, I thank you for the very patient attention with which you have listened to my remarks this evening. I hope I have not wearied you. If my remarks do not seem over wise, I rely upon that great generosity and kindness which you always show to the poorest speaker in this room. This Institute has had

many great lights which have passed away; we have amongst us lights which have shone brilliantly in their time, but are now growing a little dim; new lights are appearing which will, I hope, shine out with a brightness exceeding even those of the past; and while this succession of lights continues, so long will this Institute flourish and shine a light in the world.

Some Notes on Valuation Methods with special reference to Mr. Chatham's Paper on this Subject in the J.I.A., rol. xxxii. Being Presidential Address delivered to the Actuarial Society of New South Wales on the 28th February 1898, by W. R. Dovey, F.F.A., the First President of the Society.

In considering a subject on which to address you to-night, it occurred to me that I might invite your attention to some practical matters of office work, especially in connection with valuation books and forms.

Allow me, however, at the outset to express my thanks for the honour you have conferred on me by my election to the office of President. As a duty of my office an address is expected from me, and I must ask your indulgence in attempting to fulfil this obligation towards the close of what is to me the busiest season of the year.

It is, I think, a matter for congratulation that Australian legislation has been framed mainly on the lines of the great British Act, and not on those of the United States. The returns to the Board of Trade disclose pretty clearly the financial position of each office. Sudden collapses are not known, and are not at all likely to occur. I believe that for stability of management British offices occupy the very first rank amongst financial institutions of the world. During the past 20 years there has been a slight tendency, but not a marked one, to shorten the intervals of valuations. The great majority of offices have, however, contented themselves with declaring intermediate bonuses, while retaining their practice as to the periods between the investigations. During the past two or three years it has been shown that very little, if any, more trouble is involved in

making five annual valuations (when a set of valuation records is kept) than in making a single quinquennial one when the particulars are worked up de novo from the office records. It appears that an evolutionary process is going on which will lead to the general adoption by offices of annual valuations, whether they declare profits annually or not.

Recent valuations in Australia have shown that 4 per-cent will not long hold its place as a suitable rate of interest for valuing with-profit ordinary policies. The question naturally arises, Should not the rates of premium also be altered? Unfortunately there is an increased amount of competition now which renders it difficult to raise rates. I do not know whether any general increase of premiums by the offices could be brought about, but there can be little doubt that, in view of the probable interest earnings of the future, Australian rates of premium are low.

Mr. Chatham, in his most useful essay "On the Books and " Forms to be used in Scheduling the Particulars of the Risks " of Life Assurance Offices", proposes to make a net-premium valuation without recording the net premiums. He has suggested that the value of the total net premiums may be obtained by knocking off a percentage of the value of the gross premiums for each class of policies. Nothing could be more simple if we knew what percentage to use. In nearly every office, however, an enormous amount of labour would be required to find out what percentage should be deducted. And, moreover, the percentage would change from year to year. When once obtained it could not be used again. A fresh investigation would be necessary to obtain it, owing to changes in the proportion of policies under yearly, halfyearly and quarterly premiums, and other causes. For these reasons, my study of the paper leads me to acquiesce entirely with Mr. Manly's comments, in the discussion following the reading of the essay, when he said: "The author had been exceedingly "bold in suggesting the omission of the net premium column in "the valuation class book, although he had, as stated, made "some exceedingly close approximations." And again, "It "might be, that by averaging the loading upon the premiums " from 30 to 40, they would be able to arrive at the correct net " premium. Still, he thought they would be, like Mr. Chatham, " inclined to check the assumption by putting the net premiums "in. At any rate, until they were satisfied by a cheek of that

"kind that the assumption was correct, he did not think they "would feel confident in simply taking a percentage off the gross "values." Mr. Manly also thought that more details should have been given as to the balancing of the valuation books with the other office records.

I purpose to briefly sketch the main points of a system which is analogous in several respects to that devised by Mr. Chatham, yet gives the net premiums their natural place in the valuation records. The difficulty referred to by Mr. Chatham, of checking the net premiums, is got over by a method which, while fulfilling this important purpose, is of great use in another way.

Before coming to particulars as to the valuation system, I would remark that it presupposes that all the work of the office is adjusted to and works in harmony with the valuation processes. This does not imply any extra elaboration of office work, but is, on the contrary, I think, a characteristic of a well-organized system. The records of one department should be so arranged as to act as a check on those of others without going out of the way to be so utilized.

Another most important matter is that the check on the accuracy of the respective records should be made once a month. It is, in a very large number of life offices, usual to make an Annual Reconcilement of the books, with a view of detecting any errors that may have occurred during the past 12 months in either the accounts or the other principal records of the business. In lieu of this an efficient check can be made on both the main accounts and the main record books once a month. By this plan errors are more easily found, as the period during which they can have occurred is limited to a month. The chief matter in the verification of the accounts of an office lies in connection with the Ledger Account, which is variously termed "Assurance Account" or "Premium Account", &c., to which all premiums are debited as they fall due, and credited as they are paid or lapsed. There is always a large balance in this account consisting of premiums due but not paid. This account is the key to the book-keeping system as regards all receipts from premiums, and if it is kept with complete accuracy furnishes a thorough check on the intronissions of the cashiers and agents all through the ramifications of the office. The balance of this account, containing, as it does, those premiums which are due and unpaid, gives the amount of premiums outstanding. By having this account kept also in full detail, by clerks independent of the

book-keeping staff, we are able to check the ledger balances month by month, and to show exactly the items of which it is composed. As the premiums for a particular month fall due, lists are made of them, checked with the totals of the receipts prepared for the month and debited in detail by adding them to those already outstanding. As premiums are paid they are credited on the lists by entries made opposite to those debiting the amounts. Policies lapsed or surrendered, or which have become claims, are struck out of the lists and the amount of premium deducted. In the ledger the receipts from premiums are given in their final stage of condensation; in the lists, on the other hand, the amounts due and received are displayed individually, the entries being drawn from the most primary sources. By giving effect to the new premiums on policies and the alterations, we complete the dealings with the lists for the month. This amounts to a thorough duplicate system to verify monthly the ledger account. Thus, let us suppose the policy register shows the annual premium on a certain case to be £8. 6s. 10d., and the agent who collected the sum paid in a penny less, or £8. 6s. 9d., and it was so entered in the accounts. The detailed lists having in the correct sum would bring the mistake to light, as they would not balance with the ledger account by that penny. The lists would, of course, be kept for each colony separately. Without such a continuous and prompt check on the accounts, any system of valuation of the nature of the one proposed would lose most of its value, as it would not be possible to make a satisfactory Valuation Balance Sheet at the end of any month, unless the Assurance Fund for that date was verified and free from error.

Coming now to the plan of valuation, the first and fundamental step is the writing on cards the particulars of the policies. An outline card is shown (Appendix A) which, with slight pen and ink alterations, may be made to answer for any description of policy. It will, however, be more convenient to have separate colours, as is usual, for male and female lives, also for other divisions of the business as may best suit office requirements. As framed it is adapted for valuing rated-up lives at the assumed age, but if this course is not to be adopted the entries can, of course, be struck out. On the reverse side is a column for bonuses, which, it is assumed, will be declared annually. The total bonus in force would be entered in, either from the Certificates before they were despatched, or otherwise as may be

convenient. The column "Valuation Age" has no special connection with the bonus, but will be filled in afresh each year, at the same time as the bonus, and is useful in dealing with the card, if the policy is discontinued or altered in any way. When written, the cards are kept separated in the following order .-First, the main divisions according to tables, these subdivided into colonies, then again into valuation groups, and finally into The valuation groups of the Whole of Life numerical order. policies are the years of birth, for Endowment Assurances the years of maturity, for Joint Life policies the year of birth of the equivalent equal ages, and for Limited Payments the year up to which the payments run. The cards are always kept in the valuation groups, all necessary alterations being given effect to as will be described. The point I would now specially direct attention to is that the cards are kept continuously in order. As soon as possible after the close of any month, the various business transactions for the month are given effect to on the cards, every step in this process being independently checked. The particulars thus exist for making a valuation at the end of any month, either of individual policies or of groups, on any basis that may be desired.

In this system of valuation the particulars are not obtained direct from the cards, but from a set of books worked in conjunction with the cards. These books consist—

- (1) Of a set of Valuation Policies in Force Books.
- (2) Of subsidiary books designed to facilitate the keeping of the first set up to date, and
- (3) Of books condensing the figures in the Valuation Policies in Force books into the usual forms of the Valuation Statement and Balance Sheet.

The Policies in Force books are thus the centre of the system. One book is used for each of the main tables, minor tables being included in one volume. They show from month to month the number of policies, sums assured, bonuses, gross premiums, and net premiums in force in each of the valuation groups. In the case of Endowment Assurances and Limited Payments a column is added giving the aggregate valuation ages. Specimens of the ruling are given for the Whole of Life policies and Endowment Assurances. (Appendices B and C). A glance at the page will show that the policies in force are obtained month by month for

each valuation group by making the necessary additions for new business, &c., and deducting discontinuances, &c. When a valuation has to be made, the appropriate annuities and reversions are inserted in the columns for the purpose, and the multiplications made by the arithmometer or otherwise. The Cost of Bonus of £1 has connection with the plan of uniform reversionary bonuses, and is inserted once a year when a bonus is to be declared. By ascertaining the cost of £1 bonus all through the tables, it is only a matter of proportion to find what bonus any surplus to be divided will yield.

In Mr. Chatham's method the policies are entered in detail, but in the system now being reviewed only the total figures pertaining to each valuation group appear. A considerable amount of unnecessary writing is thus saved. Practical experience shows that the particulars can be readily summed from the cards direct, and the unnecessary process of writing down the details saved. The system now being described is essentially one of totals. Should the details, however, of the individual policies in any group be required, they can at once be obtained from the cards. In Mr. Chatham's books the policies in a valuation group would, through alterations, &c., be thrown considerably out of their numerical order. In the card system this is never done. They are always in order, and any one can be easily found. Another but a minor point may be mentioned. In Mr. Chatham's Class Valuation Register, as the particulars of the policies are detailed, and the number entering into different groups must differ considerably, the spacing of the books for the different groups must be a matter of difficulty. The card system prevents any inconvenience of this sort. The space occupied by the new business or discontinuances of a month is always the same, and the same for each group of any particular table. The sheets submitted to-night are not arranged fully for all the points included in Mr. Chatham's system, but they could be included if desired.

As in the case of the cards, the valuation groups for Whole of Life policies are according to the year of birth, the year running from the 1st July of one year to the 30th June inclusive of the next. The valuation is assumed to be made at the 31st December. Endowment Assurances are grouped according to year of maturity. The age at which the annuity is taken is the average age of the group, obtained by summing up the valuation ages and dividing by the number of lives, or preferably by multiplying the ages

by the sum assured and dividing the total by the amount assured.

Reference has been made in papers in the Journal of the Institute of Actuaries, and elsewhere, to the grouping of Endowment Assurances in this manner, a frequent comment being that the method gives very close results. It is well to know that in some circumstances it will not do so. When the policies are not long in force, and there is included in the group one or more where the age at entry is widely different from the others, and also the past duration different, the class valuation is thrown out. For example, if we take the following policies:—

#### ENDOWMENT ASSURANCES.

### $\mathbf{H}^{\mathbf{M}}$ $3\frac{1}{2}$ PER-CENT.

### FUTURE DURATION 18 YEARS.

| Age at Entry | Duration     | Val. Age   | Net Prem.     | Policy-values       |
|--------------|--------------|------------|---------------|---------------------|
| 19           | 3            | 22         | 3.598         | 9.746               |
| 46           | 1            | 47         | 4.705         | 3.622               |
|              |              |            |               |                     |
|              | Total        | 69         | 8.303         | 13.368              |
| The clas     | s value, ta  | king valua | tion age at 3 | $4^{1}_{2} = 9.562$ |
| Making       | a difference | of         | •••           | 3.806               |

or a divergence of 28.5 per-cent below the true value.

Mr. McLauchlan, in his paper, "On Some Formulas for Use in Life Office Valuations", states:—"For instance, the difference between an annuity for 10 years at age 10, calculated at H<sup>M</sup> 3½ per-cent, and a similar annuity at age 40, is only '318. We see from this that, when the number of payments of the annuity is 10, Endowment Assurance policies on lives aged between 10 and 40 at valuation may, with sufficient correctness, be included in the same group and valued at the common age of say 35." It is evident this statement requires qualification. The point, I am aware, is not new to some gentlemen here, but having come across it independently, and failing to meet with any special reference to it in actuarial literature, it seems worth drawing attention to.\*

Joint Lives are treated on the equivalent equal ages plan, referred to in Part II of the *Institute Text-Book*, Chapter XII, Article 26. This arrangement enables them to be treated like

<sup>\*</sup> The above was written before the papers on this subject by Messrs. H. A. Thomson and G. J. Lidstone appeared in the J.I.A,

single-life policies, the only difference being, of course, the employment of the appropriate annuities and reversions. Mr. Schooling's remarks on this method in his paper "On the Grouping of Policies for Valuation (J.I.A., vol. xxxii, p. 307), are of much interest. The valuation of Limited Payment policies is modelled on that of Endowment Assurances, the grouping being according to the duration of the future payments.

In the case of Endowments, the particulars on the cards can be suited either to the simple method of valuation by accumulating the gross premiums or to valuing the reversions and net premiums, in which case the procedure followed would be analogous to that for Endowment Assurances.

We now come to the question of the Net Premiums. It will be noticed they are entered in a column in the Valuation Policies in Force books. A slight digression is here necessary. In this system of valuation the net premiums are multiplied by the annuity-due, and the liability increased by reserving the proportion of premiums for the period between the date of valuation and the next due date. A convenient formula for valuing Endowment Assurances on this plan is given in Mr. McLauchlan's paper already referred to (page 381).

It seemed advisable to devise a system of recording the net premiums, which would enable the proportion of premiums to be arrived at with accuracy, and at the same time furnish a means for thoroughly checking the net premiums in the valuation books. On the valuation cards the premiums are assumed to fall due on either the first or fifteenth of a month, and if not falling due on these dates, then put in the nearer date. The Net Premiums in Force book contains the net premiums on the policies in force grouped (1) according to tables, (2) according to whether they are yearly, half-yearly, or quarterly, and (3) according to their due dates. Each folio of the book is ruled according to the specimen given (Appendix D). The annual premiums are entered in their respective dates, the half-yearly ones on the dates in the first half of the year only, and the quarterly ones on the dates they fall due in the first quarter. Having the net premiums thus arranged for the different tables, there is no difficulty in calculating the proportion of premiums in advance at the date of a valuation. The net premiums in force are kept up to date by successively adding month by month the net premiums for new business, under their proper headings, and deducting the discontinuances. The total net premiums for each table in this

book, under the respective due dates, should tally with the aggregates of the net premiums in the Policies in Force books for that table in the various valuation groups. The actual balance of the net premiums for any table is not, however, made with the corresponding Valuation Policies in Force book, as the net premiums in the latter for the different valuation groups are not summed together, but with another book to be referred to later, called the Valuation Class Summary book, which collects into small compass the figures from the Valuation Policies in Force books.

Turning now to the subsidiary books, their object is to facilitate the keeping up of the Valuation Policies in Force books. They present the new business, lapses, surrenders, deaths, &c., for each month, arranged in valuation groups, in a form in which they can, on the one hand, be easily checked with independent office records, and on the other be readily posted into the respective Policies in Force books. There is one of these books for each of the main tables, part of the book being devoted to new business and a second part to discontinuances. Minor tables are included in one book. The following rulings relating to new business give the groupings for Whole of Life and Endowment Assurance policies (Appendices E and F). Joint Lives are similarly treated to Whole of Life, as already explained, and Limited Payments follow the grouping plan of Endowment Assurances.

At the end of each month cards are written for each policy issued, sorted according to tables and valuation groups, and the aggregate number, sums assured, and other particulars entered in their proper places in the Valuation New Business book. is a space in each page for the summations of the various groups which give the totals of the new business for the month. this is being done, in another department of the office the new business entered in the Policy Register is analyzed on specially ruled sheets according to its character, Whole of Life, Endowment Assurance, &c., and the totals must agree with the corresponding summations of the new business in the subsidiary Valuation book just described. After the cards have been used to post up the Valuation New Business books, those for each table are taken and re-sorted according to the due dates of the premiums, and whether payable yearly, half-yearly, or quarterly. The totals for the net premiums for each date are then entered in the Net Premium New Business book, and the totals for all dates

brought together in a column at the side. Specimen ruling of this book is shown (Appendix G.) The totals of the net premiums, yearly, half-yearly, and quarterly, for any table, are then balanced with the total net premium in the Valuation New Business book for that table. The cards are then re-sorted into valuation groups and numerical order and put into the proper boxes.

In dealing with the discontinuances of a month, an analogous process is followed. The cards in connection with the policies are taken from their boxes, the cause of exit marked on them, and those of each table classified into (1) lapses (less revivals), (2) surrendered cases, and (3) deaths. The figures are entered in their respective columns in the discontinuance portion of the several subsidiary books. The rulings for Whole of Life and Endowment Assurances are shown (Appendices H and I). will be noticed that on each folio the discontinuances for each valuation group are brought together in summary columns at the side, and the summations of these at the foot give the total figures for the table for the month. In the case of alterations a new card is written giving effect to the change, treated as a revival, and the old one lapsed. Simultaneously with the performance of this work, another department of the office is tabulating the lapses, surrenders, deaths, and alterations of the month from the general office records. The results are brought out for the various divisions of the business, separately certified, and when correctness is assured they are compared with the corresponding totals derived from the valuation books. the gross premiums are concerned, an exact monthly balance is also made between the Policy Register and the books from which the Renewal Premium Receipts are written. The chief forms for these purposes are as follows (Appendices J and K).

In connection with the net premiums, the cards for the discontinued policies of each month are taken, classified according to tables, and whether yearly, half-yearly, or quarterly. The summation of the net premiums for each due date of payment is obtained and entered in the net premium subsidiary book. The totals are then obtained and balanced with the aggregate of the discontinued net premiums shown in the valuation subsidiary book for the same table.

When the entries for each month in the respective subsidiary books for new business and discontinuances are completed and

balanced, they are posted into the respective Valuation Policies in Force books, the necessary additions and deductions are made, and the policies in force brought out at the end of another month in each valuation group throughout the tables.

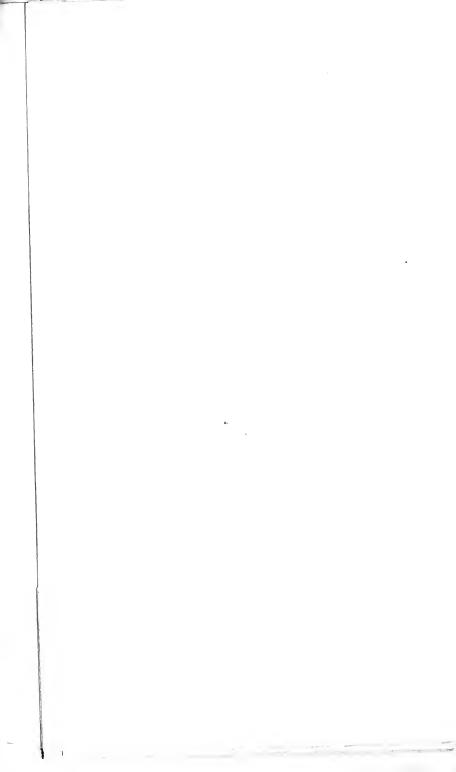
We have now to refer briefly to the third set of books which bring the figures from the Valuation Policies in Force books into the shape required by the usual Valuation Statements and Balance Sheets. The particulars of the policies in each group for each table are carried from the Policies in Force books into what is termed the Valuation Class Summary, a specimen page of which is given (Appendix L). In the Valuation Policies in Force books the valuation of one group only is shown on each page, but in the Summary book the results of the valuation of each table are condensed on one or at the most two pages. All the figures for each table, as to number of policies, sums assured, bonuses, gross premiums, net premiums, and the values of sums assured. and bonuses, and net premiums are brought together. The total particulars in force for each table being here collected, it is from this book that the figures are taken when balancing the policies in force with those shown in the general office records.

The totals of the respective tables in this book are copied into what is termed the Valuation Final Summary, each folio of which is a blank form of the usual Valuation Statements and Balance Sheets as published (Appendix M). This, of course, completes the process as far as the valuation.

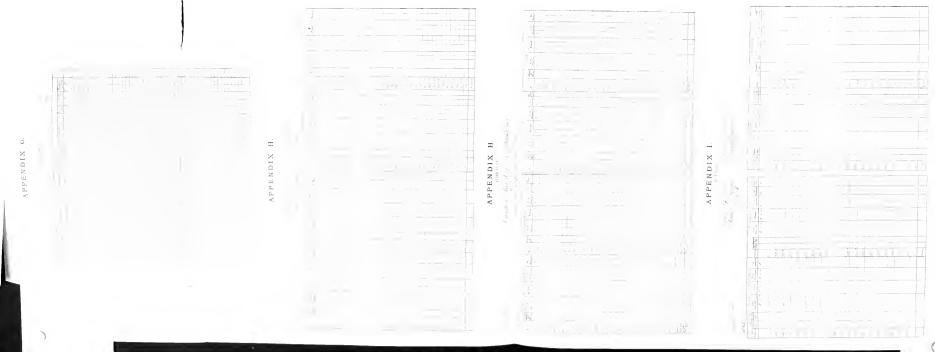
The system is best adapted to the plan of uniform reversionary bonuses. It can, however, be adapted to the contribution method of bonus distribution by putting on the back of the eard spaces for the bonus, for interest, and loading and total. The particulars on the face of the cards, together with small specially prepared reference tables, would enable these to be entered without difficulty.

The system in which the cards are kept renders it easy, when necessary, to take out on sheets the particulars of the policies in force in each valuation group for any particular colony, and thus the liabilities for any division of Australasia may be obtained if desired. As regards the amount of labour, three clerks could with ease maintain the system for a business of 40,000 policies in force, and make two valuations in the year.

The work of the actuary requires the exercise of various faculties of the mind. It would be possible, with the greatest



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|--|--|--------------------------|---|---|
| 1899   18 | APPEND   |                          |   | APPEN   |
| NDIX A.  |  |                          | DIX D.                                  |   |
|  |  |                          | APPEN                                   |   |
| TVISITE ORDINARY GENERAL   | В В И О Г<br>В Т В В В В В В В В В В В В В В В В В В |                          |   | E N D I X D |
| P  | A P P  |                          |   | A P P   |
|  |  |                          | 1 |   |





fidelity to theoretical accuracy, to be a very poor adviser. To grasp the trend of the business of the office in its various aspects must be to the actuary a primary and ever-present duty. The more thoroughly he knows how the present basis and methods of the office will affect its financial future the better fitted he will be to forecast the effects of any proposed change. There is required a certain sympathetic relation to the requirements of competition and progress in business, without the infringement of sound principles. Further, the insight into the true meaning of statistics is of high importance, and this presupposes an accurate perception of the circumstances which give rise to the figures themselves. The duty, however, which lies first to his hand is the framing of his valuation methods so that the outgoings for claims may, on the average, fall within the provision made for them, and leave the office none the weaker for their occurrence. Age must not impair its strength, nor growth lessen its acuteness. Dealing as it does with the inevitable frailty of human life, the well-conducted Life Office vet contains within itself the secret of perennial vonth.

The researches of many minds have laid bare, and made casy of acquisition, those principles and data by which an assurance institution may possess a practical immortality. From one generation to another it may pursue its career of usefulness, gathering more and more prestige and honour as time passes. As actuaries and actuarial students, you are devoting your lives to the attainment of these most worthy ends.

The Actuarial Society has been framed to assist the younger members of the profession. Permit me, at the close of its first session, to wish all such that their aspirations may materialize in lucrative positions and a long term of successful service. On the Assurance Risks incident to Professional Military and Naval Lives; and the Rates of Extra Premiums which should be charged for such Risks.\* Being Extracts from a Joint Report, made in May 1890, to the Board of Directors of the Gresham Life Assurance Society. By A. H. SMEE, M.R.C.S., and Thomas G. Ackland, F.I.A.

In assessing the risks to be provided against in the case of professional military and naval lives, consideration has to be given to the following distinct elements:

- (1) The risk of death at home stations, or the normal mortality.
- (2) The risk of death at foreign stations in time of peace, or the *climate risk*.
- (3) The risk of death in active military service in the constantly recurring "small wars."
- (4) The risk of death in active military operations in "national wars."
- (5) The liability, in time of peace, to foreign service.
- (6) The liability to active service in the field in "small wars."
- (7) The liability to active service in the field in "national wars."

### (1.) As to the Home or Normal Mortality of the British Army and Navy.

We have examined the official returns of mortality, during the six years 1882-87, among the commissioned officers serving in the United Kingdom, and also those during the 20 years 1868-87 among the non-commissioned officers and men serving in the United Kingdom.

The results respectively are given in Tables (1) and (2) appended to this Report, and indicate that the average annual rate of home mortality has been about 5 per 1,000 for commissioned officers, and between 7 and 8 per 1,000 for non-commissioned

<sup>\*</sup> In connection with the paper of Mr. McLauchlan on page 251, it has been thought that it would be useful to place on permanent record, and for easy reference, the main portions of this valuable Report of Messrs. Smee and Ackland. The Report was originally printed by the Directors of the Gresham Life Assurance Society, to whom thanks are due for permission to reproduce these extracts here.—ED. J.I.A.

officers and men. These results indicate a light rate of mortality as compared with the general male population of the United Kingdom, in which the annual rate of mortality between ages 20 and 50 averages 12 per 1,000. The mortality of assured lives, as shown in the Healthy Male Table of the Institute of Actuaries' Experience, gives an average annual rate between ages 20 and 50 of about 10 per 1,000. The "select" mortality table on the same basis, representing the mortality during the first year of assurance, gives an average annual rate, between ages 20 and 50, of somewhat over 5 per 1,000; and it will be seen that this latter rate accords closely with the experience of commissioned officers in the British army, and does not differ materially from that of the army generally. We had anticipated this agreement between the "Select HM Tables" and the mortality of the army at home, as it may be considered that the careful selection as to physical fitness of the soldier in great measure finds a parallel in the medical selection of lives proposed for assurance. It is, however, to be remarked that the mortality of the army, as a class, would seem to be generally somewhat less than that of assured lives as a class; and this would probably arise in part from the undoubtedly detrimental effect exercised, in the case of assured lives, by the lapse or surrender of assurances, which tends, by taking off a superior class of lives, to deteriorate the experience as a whole.

We have not been able to obtain access to any very detailed particulars as to the mortality in the army and navy at successive ages. We append however, in Table (3), the rates of mortality experienced during the 10 years 1877 to 1886, and also during the year 1887, in quinquennial groups of age from 20 to 40, and at ages above 40, by officers and troops in the United Kingdom; and for purposes of comparison we have set out in the same table the mortality of the general male population, of the HM Table (assured lives), and of the Select HM Table (assured lives during the first year of assurance. It will be seen that the mortality in the army at home, as indicated by these figures, is in early manhood considerably more favourable than that of the general population, or even than that of assured lives; but that as age progresses the difference diminishes, until ultimately the mortality of the soldier is less favourable than that of assured lives, or even of the population generally.

In general terms it may be stated that the mortality of the soldier exceeds that of the entire population from about age 35;

exceeds that of assured lives  $(H^M)$  from about age 30; and exceeds that of select assured lives (first year of assurance) from about age 25.

The light mortality in the army at home at early ages may doubtless be attributed to the careful selection of recruits. The heavy mortality at ages above 35 arises, no doubt, in great measure from a considerable proportion of soldiers of advancing years being included in the experience, which is also swelled by the heavy death rate in regimental depots and the departmental corps.

In Table (4) appended to this Report we give the distribution according to age of the non-commissioned officers and men at home on 1 January 1890. It will be seen that out of a total home strength closely approximating to 100,000, 26 per-cent were under age 20; 45 per-cent between ages 20 and 25; 17 percent between ages 25 and 30; 6 per-cent between ages 30 and 35; 4 per-cent between ages 35 and 40; and 2 per-cent over age 40. 88 per-cent of the whole strength were thus under age 30.

Reviewing the above statistics and conclusions as to the mortality of the army at home, there would seem to be no doubt that on the whole it may be safely assumed that the death rate will not exceed that of assured lives, and that the Healthy Male Table of the Institute of Actuaries may be considered as a fair representation of the risk in this respect.

## (2.) As to the Mortality at Foreign Stations in Time of Peace (Climate Risk).

We give in Table (5) the average mortality per 1,000 among the troops at foreign stations during the ten years 1876 to 1885, and also during the two years 1886 and 1887.

It will be observed that the average mortality in time of peace at foreign stations was  $16\frac{1}{5}$  per 1,000 during the year 1886, (including 408 deaths from disease and climate in Egypt) and 13 per 1,000 during the year 1887, and that the average rate of mortality over the 10 years 1877 to 1886 approximated to  $15\frac{1}{5}$  per 1,000 (excluding the Experience in Egypt during the four years of military occupation, 1883–6, and in South Africa during the campaign of 1879–81).

Upon reference to Table (1) it will be seen that the mortality among commissioned officers in the three Indian Presidencies during each of the six years 1882 to 1887, over which period there appear to have been no active military operations of any magnitude in India, averaged 19 per 1,000 in Bengal, 11 per 1,000 in Madras, and 12 per 1,000 in Bombay, or 14 per 1,000 over the three Indian Presidencies.

In Table (6) is shown the experience amongst British troops in India during the year 1887, the rate of mortality being exhibited according to quinquennial groups of age. It will be seen that the average death rate per 1,000 has been  $14\frac{4}{5}$  in Bengal,  $13\frac{1}{10}$  in Madras, and  $14\frac{1}{5}$  in Bombay, or  $14\frac{2}{5}$  per 1,000 over the three Presidencies. In Table (7) the experience of British troops in India is shown during the year 1887, classified according to duration of service. The death rate, in each of the Presidencies, shows a tendency to diminish, after the first year's service, to a minimum between three and five year's service, after which it shows a rising tendency. The average rate for all durations of service is, as above stated,  $14\frac{2}{5}$  per 1,000.

As regards the mortality among the officers and men of the British navy, we append in Table (8) the mortality per 1,000 from disease and from violence during each of the 17 years 1871 to 1887. It will be seen that the mortality from disease shows an average rate of  $5\frac{1}{2}$  per 1,000, and that from violence of  $3\frac{1}{2}$  per 1,000, giving a total average annual mortality of 9 per 1,000.

We have carefully weighed the indications given by the above statistics as to the rate of mortality in the army and navy abroad in time of peace, and consider that this risk may fairly be represented by an average annual rate of 15 per 1,000.

As the home or normal mortality, indicated in Table (2), may be taken upon the average at  $7\frac{1}{2}$  per 1,000, the climate risk in time of peace represents upon these assumptions an addition of  $7\frac{1}{2}$  per 1,000 to the normal home rate.

### (3.) As to the Mortality from Active Military Operations (War Risk). Small Wars.

We give in Table (9) some statistics as to the whole of the military operations in which the British army has been engaged during the 15 years 1872 to 1886 inclusive; showing the locality of the active service, the number of troops engaged, the number of deaths from casualties in action, and the rate of mortality per 1,000. It will be seen that the British army has been engaged in "small wars" during 12 out of these 15 years, in which the total number engaged has been 92,650, and the deaths in

action, 1,396. The death rate has varied from 64 per 1,000 (in South Africa in 1879) to 1 in 10,000 (in Egypt in 1885), and has averaged over the whole almost precisely 15 per 1,000.

We propose accordingly to adopt this rate for the mortality of the British army in "small wars."

# (4.) As to the Mortality from Active Military Operations (War Risk). National Wars.

As regards "national wars", the question of the estimated mortality has been very fully dealt with in our previous Report of May 1888, to which we beg leave to refer.\* The general conclusion at which we then arrived was that the extra mortality arising from military operations may be taken at not less than 5 per-cent or 50 per 1,000 for a national war of a duration not exceeding one year. We may add that we have, since the date of our last Report, had an opportunity of inspecting the very voluminous details officially published at Washington as to the American Civil war (1861-66). The general results are shown in Table (10), appended to our present Report, which shows, in each year of the campaign, the mean strength, the number of actual deaths from classified diseases, wounds in battle, and other causes; and the annual ratio of such deaths to a mean strength of 1,000. The average annual death rate during the whole term of the war (5½ years) was exactly 7 per-cent of the mean force engaged.

As regards the duration and intensity of national wars in which the British Army may be expected to be engaged, we propose, after careful consideration, to assume that 20 per-cent of the force called out on active service will die from wounds in battle, or from the hardships of the campaign. This is equivalent to a campaign of about  $2\frac{1}{2}$  years' duration on the basis of the mortality obtaining in the Franco-German war (5 per-cent of the force engaged during 7 months), or to a campaign of about 3 years' duration, on the basis of the mortality obtaining during the American Civil war (7 per-cent per annum of the force engaged).

<sup>\*</sup> See appendix to these extracts, being that portion of the Report of May 1888, here referred to.—Ep. J.I.A.

### (5.) As to the Liability, in Time of Peace, to Foreign Service.

It appears, from an official return, recently issued from the War Office, that the average strength of the army abroad during the 20 years 1870 to 1889 inclusive closely approximated to one-half of the total army strength, and this proportion equally held with regard to the officers and to the non-commissioned officers and men. This will be seen from Table (11) extracted from the official return, from which it appears that during the above period there were upon the average 4,115 officers at home, and 3,799 abroad, also that there were upon the average 93,655 non-commissioned officers and men at home and 90,979 abroad, and that these proportions were generally maintained over the 20 years under observation.

### (6.) As to the Estimated Liability to Active Service Abroad. Small Wars.

In Table (9) of the Appendix is shown the average number of officers and men engaged in military operations abroad during the 15 years 1872 to 1886, the total strength on foreign service in each of these years, and the annual proportion engaged in active military operations. It will be found that the proportion of the troops stationed abroad, who were annually engaged in active military operations, varied from 2 to 22 per-cent, and that on the average somewhat more than 6 per-cent of the total strength abroad was annually engaged in small wars. We propose to adopt this ratio of 6 per-cent as the basis for the assessment of the risk of active service abroad in small wars.

## (7.) As to the Estimated Liability to Active Service Abroad. National Wars.

The estimate of the liability to active service in a national war must necessarily be largely a question of judgment, and we have given careful consideration to the available data and the whole of the information bearing upon this point to which we have been able to obtain access.

The national wars in which the British army has been engaged during the last century include the Napoleonic wars, which extended from 1793 to 1815, and comprised 20½ years of

actual warfare; and the Crimean war, which extended over the two years from March 1854 to March 1856.

In the appended Table (12) will be found a statement of the number of non-commissioned officers and men serving at home and abroad severally in each of the 15 years 1800-1814 inclusive, during the Napoleonic campaign. It will be seen that the proportion of the whole force serving abroad during those years varied from 33 to 75 per-cent, with an average over the 15 years of about 56 per-cent.

We have been unable to obtain precise information as to the number of officers and troops engaged at home and abroad respectively during the period of the Crimean war, but it would appear that the total number of all ranks on active service in that campaign did not at any time exceed 60,000 officers and men.

The conditions of national warfare have so materially varied of recent years, with the increased precision and efficiency of offensive weapons both on sea and land, that it is probable that considerations of the available data in respect of past campaigns would be practically of little or no value in forming an estimate as to the extent, duration, and mortality of future national wars. It is, however, to be remarked that, while the efficiency of warlike methods and weapons has wonderfully developed of late years, the mortality from disease and climate risks has very materially diminished.

After carefully weighing the available data, and giving consideration to the conditions of modern warfare and the present strength and efficiency of European armies and navies, and also to the diminished mortality in respect of climate risks, we propose, for the purposes of the present Report, to assume that the British army and navy will be engaged once in 40 years in a national war, involving the calling out on active military service, for a term not exceeding three years, of two-thirds of the whole force of officers and men in both branches of the service.

### SUMMARY OF ESTIMATES AND CONCLUSIONS.

We now proceed to summarize the several estimates and propositions which we have discussed, and upon which we base the conclusions of our present Report.

(1) That the home or normal mortality of the officers and men of the British army and navy may be taken on the basis of the Healthy Male Table of the Institute of Actuaries.

- (2) That the mortality of the army and navy on foreign service in time of peace may be considered to be equal, on the average, to an additional death rate of three-quarters per-eent per annum.
- (3) That the annual mortality from "small wars" may be estimated at  $1\frac{1}{2}$  per-cent of the total force engaged on active service.
- (4) That the mortality from "national wars" may be estimated at 20 per-cent of the total force engaged in the campaign.\*
- (5) That it may be estimated that one-half of the British army and navy will be engaged on foreign service at any particular time.
- (6) That 6 per-eent of the force on foreign service will, on the average, be engaged annually in small wars.
- (7) That it may, for the purposes of this Report, be assumed that the British army and navy will be engaged once in 40 years in a national war, involving the service of two-thirds of the whole force in active military operations, for a term not exceeding three years.\*

### COMBINATION OF ELEMENTARY PROBABILITIES.

Upon the basis of the several liabilities above deduced, we may compute, with sufficient accuracy for our present purpose, the probabilities of the different events affecting the risk, from the point of view of life assurance, of professional lives in the British army and navy.

Thus, in respect of such a life at present subject to the normal or home mortality, the annual probabilities of death from the effects of climate or war risk may, upon the assumptions made in this Report, be deduced as follows:

#### I. Climate Risks.

- (a) The probability of serving at a foreign station  $= \frac{1}{2}$ .
- (b) The increased probability, when so serving abroad, of death in time of peace  $=\frac{7.5}{1.00000}$ .

<sup>\*</sup> The assumptions numbered (4) and (7) above are, as will be seen, purely matters of judgment; and it is probable that individual conclusions, based upon the available data, would show materially different results upon these points. We consider, however, that in combining the different assumptions thus made, in order to arrive at the total estimated probability of death in active service in a national war, the final result would probably not differ materially from that arrived at in this Report.

(c) The combined or total probability for a life now stationed at home, of death abroad in time of peace

$$=\frac{1}{2}\times\frac{7.5}{1.0.000}=00375.$$

- II. War Risks-" Small Wars."
  - (d) The probability, when abroad, of active service in a small war= $\frac{6}{100}$ .
  - (e) The probability of death during such active service  $=\frac{15}{1000}$ .
  - (f) The combined or total probability, for a life now stationed at home, of death in active service abroad in a small war

$$=\frac{1}{2} \times \frac{6}{100} \times \frac{15}{1000} = 00045.$$

- III. War Risks-" National Wars."
  - (g) The probability of the occurrence of a national  $war = \frac{1}{40}$ .
  - (h) The probability of active service in a national  $war = \frac{9}{3}$ .
  - (i) The probability of death during such active service  $=\frac{20}{1000}=\frac{1}{5}$ .
  - (j) The combined or total probability, in respect of a life stationed at home, of death in active service in a national war

$$=\frac{1}{4.0} \times \frac{2}{3} \times \frac{1}{5} = .00333.$$

The total additional probability of death from war and climate risks (exclusive of the home or normal mortality) may be deduced, with sufficient accuracy for our present purpose, by treating the several combined probabilities set out above under the headings (c), (f), and (j), as cumulative, and summing their values, and we thus arrive at—

(k) The total annual probability (in respect of a life now subject to the home or normal mortality) of death from the combined effects of climate and war risks

$$= .00375 + .00045 + .00333$$
  
= .00753.

This represents an annual addition to the normal intensity of mortality, arising from military and naval risks of war and climate, amounting to about  $7\frac{1}{2}$  per 1000, of which one-half, or  $3\frac{3}{4}$  per 1000, represents the addition in respect of foreign service in time of peace (or climate risk), and one-half, or  $3\frac{3}{4}$  per 1,000, the addition in respect of active military operations (or war risk).

As regards the age at which retirement from military service may be expected to take effect, we have referred to the conditions of the army regulations, which are summarized in a memorandum received from the War Office in reply to our enquiries. The conditions as to voluntary and compulsory retirement are somewhat complicated, being governed mainly by rank, age, and length of service. After careful consideration of the practical effect of these regulations, we have, in our calculations, assumed that an officer in the British army or navy may, upon the average, be expected to retire from active service between 50 and 55 years of age; when the special risks incident to their profession may be considered as no longer operative.

#### Computation of Premium Rates.

Upon the above bases we have computed, at each age at which military service arises, the total rate of mortality and the net premiums required to cover the combined risks. The results are set out in Table (13), which shows (1) the normal or  $H^M$  mortality rate at each age: (2) the estimated additional rate in respect of (a) climate, (b) war risk, constant up to age 50 and gradually decreasing to zero at age 55; (3) the total mortality rate at each age, or probability of dying in a year; (4) the probability of living a year at each age.

In Table (14) are shown (1) the numbers living at each age out of 123,660 living at age 20; (2) the numbers dying at each age (a) from normal or home risk, (b) from climate risk, (c) from war risk. The numbers living and dying, at age 55 and upwards, agree with those of the H<sup>M</sup> Table.

We have also computed, at  $3\frac{1}{2}$  per-cent interest, the values of life annuities and of net premiums at each age, on the basis of the combined rate of mortality.

In Table (15) are shown the net premiums for whole-life assurances, endowment assurances payable at 60 or previous death, and whole-life assurances with 20 premium payments.

Table (1).—Commissioned Officers in the British Army. Death Rate per 1,000 in the United Kingdom, and at Indian Stations, 1882–1887 inclusive.

| (Extracted from | the | Army | Medical | Reports | s.) |
|-----------------|-----|------|---------|---------|-----|
|-----------------|-----|------|---------|---------|-----|

| Calendar | Officers in<br>the United | Officers serving in India |        |        |  |  |  |
|----------|---------------------------|---------------------------|--------|--------|--|--|--|
| Year     | Kingdom                   | Bengal                    | Madras | Bombay |  |  |  |
| 1882     | 4.64                      | 14.02                     | 9.50   | 19.61  |  |  |  |
| 1883     | 3.44                      | 23.62                     | 9.58   | 5.90   |  |  |  |
| 1884     | 4.08                      | 24.39                     | 13.70  | 14.45  |  |  |  |
| 1885     | 7.60                      | 15.79                     | 10.17  | 16.44  |  |  |  |
| 1886     | 6.31                      | 26.13                     | 10.49  | 8.29   |  |  |  |
| 1887     | 4.39                      | 10.24                     | 10.16  | 8.42   |  |  |  |

The commissioned officers are numerically about  $35~{
m per}$  1,000 of the total forces.

Table (2).—Non-Commissioned Officers and Rank and File. Death Rate per 1,000 in the United Kingdom, and at home and abroad, 1868–1887 inclusive.

(Extracted from the Army Medical Reports.)

| Calendar Year | Troops serving<br>in the United<br>Kingdom |               |
|---------------|--|---------------|
| 1868          | 10.90                                      |               |
| 1869          | 9.41                                       |               |
| 1870          | 9.48                                       |               |
| 1871          | 8.62                                       |               |
| 1872          | 7.95                                       |               |
| 1873          | 8.26                                       |               |
| 1874          | 8.79                                       |               |
| 1875          | 9.36                                       | 12.47         |
| 1876          | 8.43                                       | 11.03         |
| 1877          | 7.20                                       | 9.55          |
| 1878          | 6.53                                       | $12 \cdot 17$ |
| 1879          | 7.55                                       | 20.00         |
| 1880          | 6.83                                       | 13.34         |
| 1881          | 7.45                                       | 12.85         |
| 1882          | 6.94                                       | 12.06         |
| 1883          | 6.28                                       | 9.57          |
| 1884          | 5.33                                       | 8.42          |
| 1885          | 6.68                                       | 11.12         |
| 1886          | 6.68                                       | 11.48         |
| 1887          | 5.13                                       | 8.84          |

The mortality "at home and abroad" during the years 1879, 1880, and 1881, shows a high ratio on account of the Zulu war. In 1879 occurred also the Afghan campaign.

Table (3).—Comparative Mortality of British Troops in the United Kingdom (1877—1887), of the General Population,

and of Assured Lives.

(Classified according to Grouped Ages).

|  | Ages                              |                               | Under 20 | 20-25   | 25-30   | 30-35   | 35-40  | 40 & upwards |
|--|-----------------------------------|-------------------------------|----------|---------|---------|---------|--------|--------------|
| Mortality  | General<br>Population,<br>English | Life Table<br>No. 2           | 92.9     | 8:39    | 9.18    | 10-19   | 11.57  | :            |
| y AMONG<br>LIVES                                   | Select 113                        | Assurance                     | 4.10     | 1.1.1   | 97-4    | 19.4    | 21.5   | :            |
| Mortality among<br>Assured Lives                   | Healthy                           | Male (Table (H <sup>M</sup> ) | 4-10     | 99-9    | 96.9    | s.10    | £1-6.  |              |
|  |                                   | Mortality<br>per 1,000        | 99.71    | 3.95    | 5.48    | s.<br>S | 16:31  | 27:37        |
| nd kinebon   | 1887<br>1887                      | Deaths                        | 51       | 180     | S       | 99      | ŝ      | 92           |
| S IN THE UNIT                                      |                                   | Home                          | 27,701   | 45,911  | 15,495  | 6,352   | 5,588  | 910.2        |
| EXPERIENCE OF BRITISH TROOPS IN THE UNITED KINGDOM |                                   | Mortality<br>per 1,000        | 3.5      | 4.81    | 6.25    | 80.0    | 1.4.55 | 23.48        |
| NPERRIENCE OF                                      | 1877—1886                         | Deaths                        | 589      | 1,656   | 1,037   | 919     | 1,142  | 615          |
| 25.  |                                   | Home                          | 181,303  | 344,243 | 165,744 | 810,29  | 78,457 | 27,460       |
|  | Ages                              |                               | Under 20 | 20-25   | 25-30   | 30-35   | 35-10  | to & upwards |

Table (4).—Ages of the Non-Commissioned Officers and Men at Home on 1st January 1890.

| Ages          | Home Strength<br>Non-Commis-<br>sioned Officers<br>and Men | Grouped Totals | Percentage of<br>Total Home<br>Strength |
|---------------|--|----------------|---|
| Under 18      | 2,491  |                |   |
| 18-19         | 9,956  |                |   |
| 19-20         | 13,300   | 05 545         | 95.0                                    |
| 20-21         | 11,230   | 25,747         | 25.8                                    |
| 21-22         | 9,932  |                |   |
| 22-23         | 8,962  |                |   |
| 23-24         | 7,671  |                |   |
| 24-25         | 6,838  |                |   |
| ~1 ±0         | 0,000  | 44,633         | 44.8                                    |
| 25-26         | 5,449  | 11,000         | 1.0                                     |
| 26-27         | 4,279  |                |   |
| 27-28         | 3,135  |                |   |
| 28-29         | 2,303  |                |   |
| 29-30         | 2,000  |                |   |
|               |  | 17,466         | 17.5                                    |
| 30-31         | 1,623  | ,              |   |
| 31 - 32       | 1,354  |                |   |
| 32-33         | 1,126  |                |   |
| 33-34         | 1,013  |                |   |
| 34-35         | 862  |                |   |
|               |  | 5,978          | 6.0                                     |
| 35-36         | 864  |                |   |
| 36 - 37       | 818  |                |   |
| 37-38         | 908  |                |   |
| 38-39         | 853  |                |   |
| 39-40         | 698  |                |   |
| 40.50         | 1 700  | 4,141          | 4.2                                     |
| 40-50         | 1,566  |                |   |
| Upwards of 50 | $\frac{62}{54}$  |                |   |
| Not reported  | 54   | 1 000          | 1.7                                     |
| TOTAL         | 99,647   | 1,682          | 1.7                                     |

Table (5).—Death Rate per 1,000 of Troops serving at Foreign Stations, 1876-1887.

|                          |                  | 1887   |                   |                  | 1886           |                   | 1876 to<br>1885   |
|--------------------------|------------------|--------|-------------------|------------------|----------------|-------------------|-------------------|
| Foreign Station          | Mean<br>Strength | Deaths | Rate<br>per 1,000 | Mean<br>Strength | Deaths         | Rate<br>per 1,000 | Rate<br>per 1,000 |
| Gibraltar                | 4,649            | 29     | 6.29              | 4,307            | 41             | 9.51              | 7·08              |
| Malta                    | 5,499            | 36     | 6.24              | 4,736            | 34             | 7.17              | 9.94              |
| Artillery)               | 360              | 1      | 2.78              | 359              |                |                   | 5.79              |
| Cyprus (1)               | 472              | 3      | 6.36              | 636              | 7              | 11.00             | 17:31             |
| Canada                   | 1,282            | 7      | 5.16              | 1,283            | 3              | 2.34              | 5.37              |
| Bermuda                  | 1,183            | 15     | 12.68             | 1,227            | 15             | 12.22             | 7.90              |
| West Indies (White)      | 1,121            | 13     | 11.60             | 1,020            | 15             | 14.70             | 16.68             |
| " (Black)                | 1,205            | 14     | 11.62             | 1.120            | 22             | 19.64             | 15.21             |
| West Africa (2)          | 570              | 11     | 24.56             | 503              | 10             | 19.88             | 19.31             |
| South , (3) .            | 3,490            | 28     | 8.02              | 3.971            | 17             | 4.28              | (35.03)           |
| Mauritius                | 400              | 8      | 20.00             | 414              | 9              | 21.73             | 19.15             |
| Ceylon                   | 1,077            | 16     | 14.86             | 949              | 10             |                   | 14.63             |
| Hong Kong, &c India—     | 2,266            | 13     | 5.74              | 2,269            | $\frac{1}{26}$ | 11.45             | 9.64              |
| T) I                     | 40,921           | 614    | 15.00             | 39,555           | 625            | 15.80             | 17:90             |
| Bengal<br>Madras         | 11,738           | 162    | 13.80             | 11,199           | 156            |                   | 14 02             |
| Bombay                   | 11,283           | 163    | 14.45             | 11,000           | 147            | 13.36             | 17.13             |
| Total India              | 63,942           | 939    | 14.68             | 61,757           | 958            | 15.51             | 17:02             |
| Egypt (4) On board Ship— | 5,272            | 81     | 15.36             | 11,062           | 408            | 36.88             | (29.58)           |
| Going out                | 1,068            | -4     | 3.74              | 1,442            | 1              | 0.69              | 4.03              |
| Coming home (5)          | 772              | 10     | 12.95             | 741              | 12             | 16.16             |                   |
| Transferring .           | 465              | 1      | 2.15              | 324              | 1              | 3.09              | 6.77              |
| On board Ship—<br>Total  | 2,305            | 15     | 6.50              | -<br>2,507       | 14             | 5.28              | -<br>7:03         |
| TOTALS .                 | 95,093           | 1,232  | 13.00             | 98,120           | 1,589          | 16:20             | 15.20             |

<sup>(1)</sup> Cyprus. The great decrease in the death rate arises from a comparison which includes the early years of our occupation, when the mortality was very high.

<sup>(2)</sup> West Africa. These are black troops entirely, with the exception of about 10 non-commissioned officers from the West Indies.

<sup>(3)</sup> South Africa. The high mortality from 1877 to 1886 includes the Zulu campaign (1879-81).

<sup>(4)</sup> Egypt. The ratio in the last column is the average of 4 years only (1883-6), a period of military occupation.

<sup>(5)</sup> On board ship—Coming home. The average death rate over the period 1877—1886 "differs but slightly" from that of 1886-7.

Table (6).—Comparative Mortality of British Troops at Indian Stations, 1887.

(Classified according to Grouped Ages.)

|             | Rate<br>per 1,000  | 8.1         | 14.7   | 15.1   | 13.3  | 17:4   | 10.5         | 14.4   |
|-------------|--------------------|-------------|--------|--------|-------|--------|--------------|--------|
| TOTAL INDIA | Deaths             | 72          | 430    | 284    | 96    | 55     | 10           | 905    |
| T           | Actual<br>Strength | 3,332       | 29,166 | 18,774 | 7,209 | 3,161  | 952          | 62,594 |
|             | Rate<br>per 1,000  | <b>₹9.6</b> | 15.07  | 14·19  | 10.56 | 19-74  | 8.62         | 14.50  |
| Bombay      | Deaths             | 4           | 81     | 42     | 11    | 6      | 7            | 148    |
|             | Actual<br>Strength | 415         | 5,376  | 5,960  | 1,042 | 456    | 116          | 10,365 |
|             | Rate<br>per 1,000  | 1.87        | 10.95  | 17.01  | 17.88 | 14.21  | 12.34        | 13.10  |
| Madras      | Deaths             | 7           | 64     | 99     | 21    | œ      | 81           | 152    |
|             | Actual<br>Strength | 531         | 5,844  | 3,292  | 1,174 | 563    | 162          | 11,569 |
|             | Rate<br>per 1,000  | 9-23        | 15.88  | 14.85  | 12.82 | 17.74  | 10.39        | 14.80  |
| BENGAL      | Deaths             | 22          | 285    | 186    | 64    | 38     | 1.           | 209    |
|             | Actual<br>Strength | 2,383       | 17,946 | 12,522 | 4,993 | 2,1.42 | 674          | 40,660 |
| -           | Ages               | Under 20    | 20-25  | 25-30  | 30-35 | 35-40  | 10 & upwards | TOTALS |

Table (7).—Comparative Mortality of British Troops at Indian Stations, 1887.

(Classified according to years of service.)

|              | Rafe<br>per 1,000   | 9.81    | 16.8           | 12.8     | 12.4  | 10.2   | 12.8   | 163          | 1991      |
|--------------|---------------------|---------|----------------|----------|-------|--------|--------|--------------|-----------|
| Fotal, India | Deaths              | 186     | 0 77           | 131      | 901   | 19     | 18:    | 40           | 706       |
| 150          | Aetnal<br>Strength  | 986'6   | 12,468         | 9.488    | 8,040 | 5,726  | 14,432 | 2,454        | 62,594    |
|              | Rafe<br>per 1,600   | 26-21   | 15.60          | 9.17     | 13-93 | 16.7.1 | 10.68  | 15.08        | 14.20     |
| Вомвау       | Deaths              | -16     | 98             | 27       | 61    | 23     | 61     | ÷            | <u>\$</u> |
|              | Actual<br>Strength  | 2,560   | 2,307          | 1,307    | 1,361 | 717    | 1,779  | 333          | 10,365    |
|              | Rafe<br>per 1,000   | 15:97   | 13:01          | 6.18     | 12.68 | 8:32   | 19.00  | 8:33         | 13:10     |
| Madras       | Dea(hs              | ?}      | 6 <del>8</del> | G.       | 17    | s.     | 51     | ÷            | 152       |
|              | Actual<br>Strength  | 2,001   | 2,998          | 1,455    | 1,340 | 1,082  | 9.210  | 480          | 11,569    |
|              | Rate<br>per 1,000   | 19-92   | 18.85          | 1.1.87   | 66-11 | 10.18  | 82.11  | 81-61        | 14:80     |
| BENGAL       | Deaths              | 108     | 135            | 100      | 19    | 0.1    | 123    | 33           | 602       |
|              | Actual<br>Strength  | 5,422   | 7,163          | 6,726    | 5,336 | 3,927  | 10,443 | 1,643        | 40,660    |
| Years of     | Military<br>Service | Under 1 | 1:1            | 61<br>62 | 3- 4  | ÷      | 5-10   | 10 & upwards | Totals    |

Table (8).—British Navy. Death Rate per 1,000, 1871-1887 inclusive.

(Extracted from the Navy Medical Reports.)

| Calendar <b>Y</b> ear | Deaths from<br>Disease | Deaths from<br>Violence | Total Deaths |
|-----------------------|------------------------|-------------------------|--------------|
| 1871                  | 6:30                   | 2.10                    | 8.40         |
| 1872                  | 5.65                   | 2.55                    | 8.20         |
| 1873                  | 6.05                   | 2.25                    | 8.30         |
| 1874                  | 6.70                   | 2.70                    | 9.40         |
| 1875                  | 6.90                   | 1.90                    | 8.80         |
| 1876                  | 5.99                   | 3.24                    | 9.23         |
| 1877                  | 4.92                   | 2.13                    | 7.05         |
| 1878                  | 5.32                   | 9.09                    | 14.41        |
| 1879                  | 6.23                   | 2.34                    | 8.57         |
| 1880                  | 4.62                   | 7.95                    | 12.57        |
| 1881                  | 5.27                   | 5.67                    | 10.94        |
| 1882                  | 6.87                   | 2.62                    | 9.49         |
| 1883                  | 4.05                   | 1.82                    | 5.87         |
| 1884                  | 5.83                   | 3.16                    | 8.99         |
| 1885                  | 4.77                   | 2.27                    | 7.04         |
| 1886                  | 5.04                   | 1.83                    | 6.87         |
| 1887                  | 4.89                   | 3.42                    | 8.31         |
| Annual )<br>Average ) | 5.60                   | 3:40                    | 9.00         |

The high mortality from disease in 1882 arose from service in Egypt.

The mortality from violent deaths in 1878, 1880, 1881, and 1887 was occasioned by the loss of, or accidents on board, the ships Eurydice, Atalanta, Doterel, and Wasp respectively,

Eurydice-training ship, lost off the Isle of Wight, 311 lives.

Atalanta—training ship, lost on the Australian station, 279 lives.

Doterel-blown up in the Straits of Magellan, 144 lives.

Wasp-wreeked in the China Seas-80 lives.

Table (9).—"Small Wars."—Total Strength at all Foreign Stations, Numbers in Command, and Deaths in Action, 1872-1886.

#### (Extracted from the Army Medical Reports.)

| Calendar              | Total<br>Strength<br>serving |                             |                   | IN COM-                           |               | THS 1N<br>TION                |
|-----------------------|------------------------------|-----------------------------|-------------------|-----------------------------------|---------------|-------------------------------|
| Year                  | at all Foreign Stations.     | Seat of War                 | Number<br>Engaged | Per 1,000<br>of Total<br>Strength | Num-          | Per 1,000<br>of No.<br>Engage |
| 1872                  | 87,853                       | West Indies (Black Troops)  |                   |                                   |               | 1.36                          |
| 1873                  | 87,796                       | West Africa ,, ,,           |                   |                                   |               | 9.02                          |
| 1874                  | 88,754                       | 22 22 22 22                 |                   |                                   |               | 1.26                          |
| 1875                  | 87,476                       | China Station               | 1,861             | 21.3                              | 2             | 1.07                          |
| 1876                  | 87,287                       | 4                           |                   |                                   |               |                               |
| 1877                  | 88,598                       |                             |                   |                                   |               |                               |
| 1878                  | 91,557                       | Cape and St. Helena Station | 5,629             | 61.5 7                            | 5             | 0.89                          |
| ,,                    |                              | Bengal Field Service .      | 1,857             | 20.3                              | 7             | 3.76                          |
| 1879                  | 103,075                      | Cape and St. Helena Station | 12,564            | 122.0 )                           | 807           | 64.23                         |
| ,,                    | ,                            | Bengal Field Service .      | 9,613             | 93.3                              | 65            | 6.76                          |
| 1880                  | 97,099                       | Cape and St. Helena Station | 5,552             | 57.2 7                            | 68            | 12.25                         |
| ,,                    | 0.,000                       | Bengal Field Service .      | 9,271             | 95.5                              | 51            | 5.20                          |
| 1881                  | 96,852                       | Cape and St. Helena Station | 10,636            | 110.0 7                           | 268           | 25.20                         |
|                       | 00,00                        | Bengal Field Service .      | 1,429             | 14.7 )                            | 1             | 0.70                          |
| 1882                  | 94,725                       | Egypt                       | 6.198             | 65.5                              | 113           | 18.23                         |
| 1883                  | 91,686                       |                             |                   | 000                               | 110           |                               |
| 1884                  | 92,790                       | Egypt                       | 6.468             | 69.7                              | <br>5         | 0.77                          |
|                       |                              | Madras (W. District)        | 917               | 9.9                               | ı             | 1.09                          |
| 1885                  | 106,218                      | Egypt                       | 9,593             | . 90.5                            | Î             | 0.10                          |
| 1886                  | 105,507                      | CLA                         | 11,062            | 104.0                             | $\frac{1}{2}$ | 0.18                          |
|                       | 100,007                      | ,,                          | 11,002            |                                   |               |                               |
| Totals                | 1,407,273                    |                             | 92,650            |                                   | 1,396         |                               |
| Annual )<br>Average ) | 93,818                       |                             | 6,177             | 65.8                              | 93            | 15.10                         |

In the Abyssinian Campaign (1868) the deaths were 12:83 per 1,000, none of them from wounds in action.

Table (10).—American Civil War (1861–1866).—Classification of Deaths, and Ratios of Mortality in the United States Army (White Troops). (Compiled from the Official Returns.)

| Year ending  | 30 June 1861<br>(2 months)              | 1861<br>(hs)                                       | 30 June 1862  | 1862   | 30 Jun  | 30 June 1563                                       | 30 June 1864  | 0 1864   | 30 June 1865   | e 1865   | 30 June 1866                                      | 2 1866   | Total   | =  |
|--|---|--|---|--|---|--|---|--|--|--|---|--|---|--|
| Mean Strength in Field and<br>Garrison<br>Do, in General Hospitals .   | 41,556                                  | 99   | 279,371<br>9,54                                     | 9,548  | 614   | 514,325<br>45,630                                  | 619,<br>55,   | 619,703<br>55,710                                  | 574  | 574,022<br>71,484                                  | 00<br>0 8   | 99,080<br>2,817                                    | 2,193,427<br>185,189                                      | 127<br>189   |
| Total Mean Strength .  | 41,556                                  | 95   | 288,919   | 919  | 629   | 659,955  | 675,  | 675,413  | 615  | 615,506  | 101,  | 101,897  | 2,378,616   | 919  |
| Class of Discusos  | Actual 1. Deaths 3                      | Annual<br>Ratio to<br>1,000 of<br>Mean<br>Strength | Actual<br>Deaths                                    | Annual<br>Ratio to<br>1,000 of<br>Mean<br>Strength | Actual<br>Deaths  | Annual<br>Ratio to<br>1,000 of<br>Menn<br>Strength | Actual<br>Deaths  | Annual<br>Ratio to<br>1,000 of<br>Mean<br>Strength | Actual<br>Deaths   | Annual<br>Ratio to<br>1,000 of<br>Mean<br>Strength | Actual  | Annual<br>Ratio to<br>1,000 of<br>Mean<br>Strength | Total<br>Deaths   | Aumual<br>Ratio to<br>1,000 of<br>Mean<br>Strength |
| 1. Zymotic Diseases. 11. Constitutional Diseases 111. Parasitic Diseases 1 V. Local Diseases 1 V. Wounds in Battle Accidents and Injuries 1 Annicide 1 Suicide 1 Execution of Sentence Unclassified Diseases 1 | 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 | n : : 4 a : : : : 1 i su                           | 9,394<br>671<br>671<br>3,750<br>4,634<br>230<br>.:: | 3 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 = 1 =            | 29,794<br>2,561<br>1<br>9,359<br>1,225<br>7,295<br>26<br>26<br>13 | 충속 :즉즉 · : : : :                                   | 23,384<br>1,552<br>3<br>7,639<br>9,357<br>886<br>43<br>59<br>50 | 85 H H H H H H H H H H H H H H H H H H H           | 27,470<br>1,616<br>4<br>7,358<br>10,310<br>819<br>45<br>76<br>41 | £6. :114 : : : :                                   | 3,360<br>212<br><br>717<br>210<br>174<br>30<br>31 | <sup>™</sup> a iraa i a i :                        | 93,436<br>6,644<br>8,22,811<br>2,911<br>104<br>104<br>449 | 8 1 2 4 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1          |
| Total Deaths   | 118                                     | 17   | 19,159  | 99   | 51,870  | 2/8  | 42,969  | 8  | 47,772   | 7.4  | 4,735   | 94   | 166,623   | 20   |

Table (11).—Average Strength of the Army at Home and Abroad, during each year from 1870 to 1889, distinguishing the number of Officers and Men.

| Calendar |            | Officers |       | Non-Commiss | SIONED OFFICER | RS AND ME |
|----------|------------|----------|-------|-------------|----------------|-----------|
| Year     | At<br>Home | Abroad   | Total | At<br>Home  | Abroad         | Total     |
| 1870     | 4,822      | 4,805    | 9,627 | 84,848      | 85,969         | 170,817   |
| 1871     | 4,880      | 4.314    | 9,194 | 100,877     | 82,594         | 183,471   |
| 1872     | 4,848      | 4,114    | 8.992 | 99.306      | 83,709         | 183,015   |
| 1873     | 4,815      | 3,877    | 8,692 | 95,768      | 83,919         | 179,687   |
| 1874     | 4,521      | 3,763    | 8,254 | 93,114      | 84,991         | 178,105   |
| 1875     | 4,391      | 3.691    | 8,082 | 92,802      | 83,785         | 176,587   |
| 1876     | 4,365      | 3.776    | 8,141 | 92,781      | 83,511         | 176,292   |
| 1877     | 4,294      | 3.766    | 8,060 | 97,610      | 84,832         | 182,442   |
| 1878     | 4,195      | 3.750    | 7,945 | 105,006     | 87,807         | 192,813   |
| 1879     | 3,835      | 4,007    | 7,842 | 84,350      | 99,068         | 183,448   |
| 1880     | 4,044      | 3,773    | 7,817 | \$7,543     | 93,326         | 181,169   |
| 1881     | 3,954      | 3,658    | 7,612 | 87.992      | 93,194         | 151.186   |
| 1882     | 3,818      | 3,518    | 7,336 | 90,075      | 91,207         | 181,282   |
| 1883     | 3,714      | 3,479    | 7,193 | \$5,960     | 88,207         | 174,167   |
| 1884     | 3,578      | 3,519    | 7,097 | 56,013      | 89,171         | 175,284   |
| 1885     | 3,248      | 3,827    | 7,075 | 57,927      | 102.391        | 190.318   |
| 1886     | 3,556      | 3,662    | 7.218 | 94.060      | 101,845        | 195,905   |
| 1887     | 3,758      | 3,513    | 7.271 | 102,560     | 99,061         | 201,621   |
| 1888     | 3,773      | 3,549    | 7,322 | 102,688     | 100,413        | 203,101   |
| 1889     | 3,599      | 3.559    | 7,488 | 101,498     | 100,456        | 201,95    |
| Annual / | 4,115      | 3,799    | 7,914 | 93.655      | 90,979         | 184,635   |

Table (12).—Non-Commissioned Officers and Men serving at Home and Abroad during the Napoleonic Campaign, 1800—1814.

(Extracted from the Journal of the Institute of Actuaries, vii, 278.)

| Calendar<br>Year      | Troops<br>serving at<br>Home | Percentage<br>of Total<br>Strength | Troops<br>serving<br>Abroad | Percentage<br>of Total<br>Strength | Total<br>Strength |
|-----------------------|------------------------------|------------------------------------|-----------------------------|------------------------------------|-------------------|
| 1800                  | 92,600                       | 58.3                               | 66,300                      | 41.7                               | 158,900           |
| 1801                  | 81,000                       | 49.4                               | 82,600                      | 50.6                               | 163,600           |
| 1802                  | 67,400                       | 48.8                               | 70,300                      | 51.2                               | 137,700           |
| 1803                  | 76,800                       | 61.0                               | 48,800                      | 39.0                               | 125,600           |
| 1804                  | 97,800                       | 67.0                               | 48,700                      | 33.0                               | 146,500           |
| 1805                  | 103,800                      | 60.4                               | 67,700                      | 39.6                               | 171,500           |
| 1806                  | 102,600                      | 54.3                               | 86,200                      | 45.7                               | 188,800           |
| 1807                  | 103,800                      | 51.4                               | 98,400                      | 48.6                               | 202,200           |
| 1808                  | 104,200                      | 46.9                               | 118,200                     | 53.1                               | 222,400           |
| 1809                  | 93,300                       | 40.7                               | 136,000                     | 59.3                               | 229,300           |
| 1810                  | 86,500                       | 38.4                               | 138,600                     | 61.6                               | 225,100           |
| 1811                  | 73,900                       | 32.3                               | 154,800                     | 67.7                               | 228,700           |
| 1812                  | 70,700                       | 29.2                               | 171,500                     | 70.8                               | 242,200           |
| 1813                  | 65,900                       | 26.2                               | 185,000                     | 73.8                               | 250,900           |
| 1814                  | 63,700                       | 25.0                               | 190,300                     | 75.0                               | 254,000           |
| Totals                | 1,284,000                    |                                    | 1,663,400                   |                                    | 2,947,400         |
| Annual )<br>Average ) | 85,600                       | 43.7                               | 110,893                     | 56.3                               | 196,493           |

Table (13).—Probability of Dying in a Year, and of Surviving a Year, at each Age. Normal  $(H^M)$  Mortality, Climate Risks, and War Risks.

|            | P  | ROBABILITY OF I       | YING IN A YE      | A B                      | Probability o                             |
|------------|--|-----------------------|-------------------|--------------------------|---|
| Age        | From Normal (II <sup>M</sup> ) Mortality | From Climate<br>Risks | From War<br>Risks | From Com-<br>bined Risks | Surviving a<br>Year (Com-<br>bined Risks) |
| 20         | .00633                                   | .00375                | .00375            | .01383                   | .98617                                    |
| 21         | .00673                                   | .00375                | 00375             | .01423                   | .98577                                    |
| 22         | .00684                                   | .09375                | .00375            | .01434                   | 98566                                     |
| 23         | .00676                                   | .00375                | 00375             | .01426                   | .98574                                    |
| 24         | .00664                                   | .00375                | .00375            | .01414                   | .98586                                    |
| 25         | .00663                                   | .00375                | $\cdot 00375$     | .01413                   | 98587                                     |
| 26         | .00669                                   | .00375                | .00375            | .01419                   | .98581                                    |
| 27         | .00690                                   | .00375                | 00375             | .01440                   | .98560                                    |
| 28         | .00717                                   | .00375                | .00375            | .01467                   | .98533                                    |
| 29         | .00743                                   | .00375                | .00375            | .01493                   | .98507                                    |
| 30         | .00772                                   | .00375                | .00375            | .01522                   | .98478                                    |
| 31         | .00792                                   | .00375                | .00375            | .01542                   | .98458                                    |
| 32         | .00811                                   | 00375                 | 00375             | .01561                   | 98439                                     |
| 33         | .00829                                   | .00375                | .09375            | .01579                   | .98421                                    |
| 34         | .00850                                   | 00375                 | 00375             | .01600                   | .98400                                    |
| 3 <b>5</b> | .00877                                   | .00375                | .00375            | 01627                    | .98373                                    |
| 36         | .00911                                   | .00375                | .00375            | .01661                   | .98339                                    |
| 37         | .00946                                   | .00375                | 00375             | .01696                   | .98304                                    |
| 38         | .00978                                   | .00375                | 00375             | .01728                   | .98272                                    |
| 39         | .01008                                   | ·00375                | 00375             | .01758                   | .98242                                    |
| 40         | .01031                                   | ·00375                | 00375             | .01781                   | .98219                                    |
| 41         | .01049                                   | ·00375                | .00375            | .01799                   | .98201                                    |
| 42         | 01073                                    | ·00375                | $\cdot 00375$     | .01823                   | .98177                                    |
| 43         | .01113                                   | .00375                | 00375             | .01863                   | 98137                                     |
| 4.1        | .01156                                   | ·00375                | 00375             | ·01906                   | 98094                                     |
| 45         | .01219                                   | .00375                | ·00375            | .01969                   | .98031                                    |
| 46         | .01294                                   | .00375                | .00375            | 02044                    | .97956                                    |
| 47         | .01370                                   | 00375                 | .00375            | ·02120                   | ·97880                                    |
| 48         | 01444                                    | 00375                 | .00375            | 02194                    | .97806                                    |
| 49         | .01522                                   | .00375                | 00375             | .02272                   | .97728                                    |
| 50         | .01595                                   | .00375                | .00375            | .02345                   | 97655                                     |
| 51         | .01667                                   | .00300                | .00300            | 02267                    | ·97733                                    |
| 52         | ·01755                                   | .00225                | $\cdot 00225$     | .02205                   | .97795                                    |
| 53         | .01860                                   | .00150                | .00150            | .02160                   | .97840                                    |
| 54         | 01973                                    | .00075                | .00075            | 02123                    | .97877                                    |
| 55         | .02103                                   | .00000                | .00000            | .02103                   | .97897                                    |

Table (14).—Numbers Living, and Numbers Dying, at each Age. Normal (H<sup>M</sup>) Mortality, Climate Risks, and War Risks.

|                       | Number                   | Number  | DYING AT                 | EACH AGE 13          | N A YEAR            |
|-----------------------|--------------------------|---|--------------------------|----------------------|---------------------|
| $\Lambda \mathrm{ge}$ | Living<br>at each<br>Age | From<br>Normal (H <sup>M</sup> )<br>Mortality | From<br>Climate<br>Risks | From<br>War<br>Risks | From Combined Risks |
| 20                    | 123,660                  | 783   | 464                      | 463                  | 1,710               |
| $\frac{1}{21}$        | 121,950                  | 820   | 457                      | 458                  | 1,735               |
| 22                    | 120,215                  | 822   | 451                      | 450                  | 1,723               |
| 23                    | 118,492                  | 802   | 111                      | 444                  | 1,690               |
| 24                    | 116,802                  | 774   | 438                      | 438                  | 1,650               |
| 25                    | $115,\!152$              | 763   | 432                      | 432                  | 1,627               |
| 26                    | 113,525                  | 760   | 426                      | 426                  | 1,612               |
| 27                    | 111,913                  | 772   | 420                      | 420                  | 1,612               |
| 28                    | 110,301                  | 790   | 414                      | 414                  | 1,618               |
| 29                    | 108,683                  | 807   | 407                      | 408                  | 1,622               |
| 30                    | 107,061                  | 827   | 402                      | 401                  | 1,630               |
| 31                    | 105,431                  | 835   | 395                      | 396                  | 1,626               |
| 32                    | 103,805                  | 840   | 390                      | 389                  | 1,619               |
| 33                    | 102,186                  | 847   | 383                      | 383                  | 1,613               |
| 34                    | 100,573                  | 854   | 377                      | 377                  | 1,608               |
| 35                    | 98,965                   | 868   | 371                      | 371                  | 1,610               |
| 36                    | 97,355                   | 886   | 365                      | 365                  | 1,616               |
| 37                    | 95,739                   | 906   | 359                      | 359                  | 1,624               |
| 38                    | 94,115                   | 920   | 353                      | 353                  | 1,626               |
| 39                    | 92,489                   | 932   | 346                      | 347                  | 1,625               |
| 40                    | 90,864                   | 937   | 341                      | 340                  | 1,618               |
| 41                    | 89,246                   | 935   | 334                      | 335                  | 1,604               |
| 42                    | 87,642                   | 941   | 329                      | 328                  | 1,598               |
| 43                    | 86,044                   | 959   | 322                      | 323                  | 1,604               |
| 11                    | 84,440                   | 977   | 317                      | 316                  | 1,610               |
| 45                    | 82,830                   | 1,010   | 310                      | 311                  | 1,631               |
| 46                    | 81,199                   | 1,051   | 305                      | 304                  | 1,660               |
| 47                    | 79,539                   | 1,090   | 298                      | 299                  | 1,687               |
| 48                    | 77,852                   | 1,123   | 292                      | 292                  | 1,707               |
| 49                    | 76,145                   | 1,159   | 286                      | 285                  | 1,730               |
| 50                    | 74,415                   | 1,188   | 279                      | 279                  | 1,746               |
| 51                    | 72,669                   | 1,211   | 218                      | 218                  | 1,647               |
| 52                    | 71,022                   | 1,246   | 160                      | 160                  | 1,566               |
| 53                    | 69,456                   | 1,292   | 104                      | 104                  | 1,500               |
| 54                    | 67,956                   | 1,341   | 51                       | 51                   | 1,443               |
| 55                    | 66,513                   | 1,399   |                          |                      | 1,399               |
|                       | *                        | *   |                          | 1                    | 1                   |

<sup>\*</sup> After Age 55 the values in these columns agree with those of  $\mathbf{H}^{\mathbf{M}}$  Table.

Table (15).—Combined Assurance, including Normal  $(H^M)$ , Climate, and War Risks—Net Premiums—Interest at  $3\frac{1}{2}$  percent.

| Age                        | Whole Life<br>Assurance                                  | Endowment Assurance at 60 or Death                    | Limited<br>Premiums<br>20 Payments                         |
|----------------------------|--|---|--|
| 20<br>21<br>22<br>23<br>24 | £ s. d.<br>1 17 9<br>1 18 4<br>1 19 0<br>1 19 7<br>2 0 3 | £ s. d.<br>2 5 6<br>2 6 7<br>2 7 9<br>2 9 0<br>2 10 4 | £ s. d.<br>2 14 10<br>2 15 6<br>2 16 1<br>2 16 9<br>2 17 5 |
| 25                         | 2 1 0  | 2 11 10   | 2 18 2   |
| 26                         | 2 1 10   | 2 13 4  | 2 19 0   |
| 27                         | 2 2 7  | 2 15 0  | 2 19 10  |
| 28                         | 2 3 6  | 2 16 9  | 3 0 8  |
| 29                         | 2 4 5  | 2 18 8  | 3 1 6  |
| 30                         | 2 5 4  | 3 0 S   | 3 2 5  |
| 31                         | 2 6 3  | 3 2 10  | 3 3 4  |
| 32                         | 2 7 3  | 3 5 2   | 3 4 4  |
| 33                         | 2 8 3  | 3 7 7   | 3 5 4  |
| 34                         | 2 9 5  | 3 10 4  | 3 6 4  |
| 35                         | 2 10 6   | 3 13 3  | 3 7 5  |
| 36                         | 2 11 9   | 3 16 5  | 3 8 6  |
| 37                         | 2 13 0   | 3 19 10   | 3 9 7  |
| 38                         | 2 14 3   | 4 3 7   | 3 10 9   |
| 39                         | 2 15 8   | 4 7 8   | 3 11 11  |
| 40                         | $\begin{array}{cccccccccccccccccccccccccccccccccccc$     | 4 12 2  | 3 13 2   |
| 41                         |  | 4 17 3  | 3 14 5   |
| 42                         |  | 5 2 10  | 3 15 9   |
| 43                         |  | 5 9 1   | 3 17 2   |
| 44                         |  | 5 16 3  | 3 18 7   |
| 45                         | 3 5 6  | 6 4 3   | 4 0 1  |

#### APPENDIX,

Being Extract from Report of May 1888.

The probability of death during the campaign.

Some interesting statistics as to campaigns during the last 30 years are given by Herr Klang. (See Table A). From these figures it results that the mortality in war has varied from  $1\frac{1}{2}$  to 10 per-cent, with an average value over 13 campaigns of  $4\frac{4}{5}$  percent.

Mr. W. B. Hodge investigated, in 1858, the whole question of the mortality in military operations, with especial reference to the casualties and deaths in the British army during the Napoleonic wars (1793–1815), and the Crimean war (1854–55). His general conclusions appear to be that during the years 1793–1815 (including 20½ years of war), the total average annual mortality was 56:21 per 1,000, of which 6:60 per 1,000 arose from casualties in action, and 49:61 per 1,000 from sickness and other causes. Mr. Hodge also dealt with the mortality of the British force in the Crimea, but the returns available at the date of his investigation (1858) were very partial and imperfect, and not published in such a form as to be of service for our present purpose.

Professor Karup investigated, in 1869, the mortality in campaigns from 1812 to 1864, and found the rates to vary from  $1\frac{1}{2}$  to 56 per-cent per annum. Upon the basis of these figures, he arrived at the conclusion that the rate of mortality in war may be considered to be 10 per-cent per annum; but we are of opinion that this estimate is decidedly too high, as the statistics upon which it is based include several exceptional campaigns, in which the mortality was very excessive. The mean mortality over the 19 campaigns investigated by Professor Karup, is stated by Herr Klang to be 5.37 per-cent, but he considers that this result is overstated by an error in the figures of the Austrian campaigns, and that the true average may be considered to be somewhat below 5 per-cent.

From the official statistics of the German losses in the Franco-German War of 1870-71, which are published in considerable detail, and to which we have given careful attention, it appears that the total force engaged was 887,876, of whom 44,752, or 5.05 per-cent were reported as dead or missing. The principal details are given in Table B, which shows the total force engaged according to rank, and the deaths in each class from the several causes specified; and in Table C, which gives the death-rate per

1,000 separately stated for each rank, and cause of death. It would have materially assisted our investigation had these official statistics included details of the force engaged and the deaths at each age, but although the returns are very minute and voluminous (extending over upwards of 300 pages), we do not find that they include any reference whatever to the ages of the officers and men engaged.

We have derived considerable assistance in our study of these German statistics from the able treatise by Mr. A. G. Mackenzie, F.F.A., in which he summarizes and compares the tabular results, and makes valuable deductions therefrom.

We are now in a position, from the different statistics of past campaigns, extending over nearly 100 years, to assess the probability of death in war. An examination and comparison of the results for these several campaigns lead to the conclusion that the average rate of death of the army in the field may be estimated at about 5 per-cent per annum of the force engaged. This rate includes the normal mortality, which may probably be estimated at about  $\frac{3}{4}$  per-cent; but we consider that it will be safer to conclude that the extra rate of mortality in war is equal to 5 per-cent, and that therefore the annual probability of death in consequence of war = 05.

Table A.

Rates of War Mortality in Different European Campaigns, 1854-1878.

(Extracted from Herr Klang's Kriegs-Versicherungs Vorlagen.)

|                  | Сашра   | ign and | Date    |     | Rate of<br>Mortality<br>per-cent |
|------------------|---------|---------|---------|-----|----------------------------------|
| France, in       | i the C | rimea,  | 1554-55 |     | 10.3                             |
| France, in       | ı Camı  | aign o  | f 1859  |     | 5.6                              |
| Italy            | ,,      | .,      |         |     | 5.5                              |
| $\Lambda$ ustria | ,,      | ,,      |         |     | 4.7                              |
| North Au         | ierica, | 1861-1  | 865 .   |     | 5.4*                             |
| Prussia, i       | n Cam   | paign o | f 1864  |     | 3.3                              |
| Austria          | ,,      | ٠,,     |         |     | 4:0                              |
| Italy            | ٠,      | **      | 1866    |     | 4:0                              |
| Prussia          | *1      | **      | 21      |     | 3.4                              |
| Bavaria          | ••      | ٠,      | **      |     | 5.1                              |
| Austria          | .,      | ,,      | ••      |     | 5.6                              |
| Germany          | **      | ••      | 1570-   | 71. | 4.2                              |
| Austria          |         | ٠,      | 1878    |     | 1.5                              |

<sup>\*</sup> See, however, Table (10).-ED. J.I.A.

TABLE B.—German Mortality in the Franco-German War, 1870-71—Numbers engaged and Deaths from Specified Causes. (Collated from the Official Statistics.)

| Total<br>Deaths and<br>Missing     |          |                | ely                          |             |              | 1,866                         | 12,886                            | 44,752       |
|------------------------------------|----------|----------------|------------------------------|-------------|--------------|-------------------------------|-----------------------------------|--------------|
| Deat                               |          |                | sparat<br>ssified            |             |              |                               | <u></u>                           | 7            |
| Finally<br>Missing                 |          |                | Not separately<br>classified |             |              | က                             | 4,006                             | 4,009        |
| Deaths<br>from<br>all Causes       | 6        | 137            | 363                          | 1,302       | 52           | 1,863                         | 38,880                            | 40,743       |
| Cause from from all Causes         | :        | -              | 11                           | 56          | G.           | 47                            | 921                               | 968          |
| Sudden<br>Death                    | 1        | П              | c1                           | +           | c1           | 10                            | 83                                | 93           |
| Acute Chronic<br>Discases Discases | :        | 4              | 7                            | က           | 4            | 15                            | 752                               | 292          |
| Arute<br>Diseases                  | က        | 9              | 17                           | 48          | 19           | 129                           | 10,190                            | 10,319       |
| Accident<br>and<br>Suleide         | :        | :              | 61                           | 1           | m            | 123                           | 307                               | 319          |
| Died<br>of<br>Wounds               | 6.1      | 58             | 148                          | 427         | າວ           | 640                           | 10,067                            | 10,707       |
| Fell<br>in<br>Battle               | က        | 67             | 182                          | 751         | 7            | 1,010                         | 16,560                            | 17,570       |
| Total<br>Force<br>engaged          | 195      | 1,350          | 4,279                        | 15,052      | 4,185        | 25,061                        | 862,815                           | 887,876      |
| Rank                               | Generals | Staff Officers | Captains, &c                 | Lieutenants | Surgeons, &c | Total Commissioned Officers . | Non-Commissioned Officers and Men | Grand Totals |

TABLE C.-German Mortality in the Franca-German War, 1870-71-Deaths per 1,000 from Sperified Causes.

(Collated from the Official Statistics.)

| Total<br>Deaths and<br>Missing       |          |                | Not, separately<br>classified |                  |              | 7-5                         | 49-7                               | 50.5         |
|--------------------------------------|----------|----------------|-------------------------------|------------------|--------------|-----------------------------|------------------------------------|--------------|
| Finally<br>Missing                   |          |                | Not separate<br>classified    |                  |              | - <del>-</del> -            | 9-4                                | <del>.</del> |
| Deaths<br>from<br>all Causes         | 7.91     | 7-101          | 6:18                          | 863              | 25.53        | 1:12                        | 1:2:1                              | 9.97         |
| Canse<br>Unknown                     | :        | 6.2            | 95                            | <u>-</u>         | 31<br>31     | <u> </u>                    | 9                                  | Ξ.           |
| Sudden                               | ë        | 0.7            | G:0                           | 8.0              | 5.5          | 0.0                         | 1.0                                | <u>-</u> 0   |
| Chronic   Sudden<br>Diseases   Death | :        | 3.0            | 21                            | ? <del>.</del> 0 | 1.7          | 9.0                         | 6.0                                | ç.<br>Ç.     |
| Acute                                | 7:21     | <u>;</u>       | 0.                            | 9.9              | ÷            | 10<br>51                    | $\frac{1}{x}$                      | 11.6         |
| Accident<br>and<br>Suicide           | :        | :              |                               | 0.5              | 2.0          | 10<br>0                     | 1:0                                | Ë            |
| Died<br>of<br>Wounds                 | 10-3     | 6.<br>27       | 31.6                          | 25<br>25<br>25   | <u>?1</u>    | 61<br>16                    | 11.7                               | ÷1           |
| Well<br>in<br>Battle                 | 2.5      | 2.68           | 5.<br>13.                     | 2.634            | <u>-</u>     | 50:3                        | 22.61                              | 8-61         |
| Rank                                 | Generals | Staff Officers | Captains, &c                  | Lieutenants      | Surgeons, &c | Total Commissioned Officers | Non-Commissioned Officers and Men. | Grand Totals |

#### REVIEWS.

#### De Witt and Actuarial Science in Holland.\*

Under the title of Mémoires pour servir à l'histoire des Assurances sur la Vie et des Rentes Viagères aux Pays-Bas. Réunis et publiés par la Direction de la Société Générale Nécrlandaise d'Assurances sur la Vie et de Rentes Viagères, Amsterdam, 1898, a volume of over 300 quarto pages, with photogravure reproductions of 11 portraits and other illustrations, has been presented to each of the members of the recently-held Second International Congress of Actuaries. The volume will serve as an agreeable reminder, in many assurance libraries, not only of the large share that Holland justly prides herself upon having taken in the past two centuries in the development of the science and practice of life assurance and annuities, but also of the public spirit of the Society which has so liberally distributed a work due to the research and careful literary editorship of its managers. It appears that for upwards of 15 years they have issued to their agents and correspondents a weekly octavo fly-sheet, sometimes running to 16 or more pages, containing matter on assurance subjects, treated, of course, from a Dutch point of view. The \$52nd number of this periodical is before us as we write, and evinces the same unflagging zeal that has so long kept up the undertaking. Mr. van Wickevoort Crommelin has contributed the results of his researches to the Dutch edition, at the instance and under the editorship of the Society's managers. The translations from Dutch have been made into French by Mr. K. R. Gallas, in the quarto volume presented to the members of the International Congress. It is, in point of fact, a selection from the historical and biographical papers and notices that had previously appeared in Dutch in the weekly periodical above referred to. It is divided into two parts, the first consisting of articles on the pioneer Dutch mathematicians and actuaries-De Witt, Hudde, and Huygens-and the second on Dutch actuaries of the eighteenth century and of times which have only recently passed away.

The papers on De Witt give some interesting details respecting his early educational training and association with eminent Dutch mathematicians, which prepared the way for his discovery that the calculus of probabilities could be applied to the estimation of the value of life annuities, as explained in his celebrated essay on that subject printed in 1672. We hardly need remind the readers of the Journal of the Institute of Actuaries that this essay was translated into English and printed in its pages (then known under the title of the Assurance Magazine) so long ago as in the years 1852 and 1853

<sup>\*</sup> Mémoires pour servir à l'histoire des Assurances sur la Vie et des Rentes Viagères aux Pays-Bas. Réunis et publiés par la Direction de la Société Générale Néerlandaise d'Assurances sur la Vie et de Rentes Viagères. Siège Social: Amsterdam, Dannak 74, 1898.

(vide vol. ii., pp. 121-150, and pp. 222-258, and vol. iii, pp. 93-120). This fact only came to the knowledge of the editors of the volume presented to the International Congress of Actuaries after it was printed and awaiting distribution among the members. This has, however, been acknowledged by them in the handsomest possible manner, in a printed letter from Dr. S. R. J. Van Schevichaven, E. W. Scott, and J. F. L. Blankenberg, the Managers of the Society, dated Amsterdam, June 1898, and sent to the Members of the Congress with the presentation copies of the volume.

From an English actuarial point of view, one of the most interesting papers is that numbered 675, of 23 March 1895, drawing attention to the edition of the complete works of Christiaan Huvgens which has been issued by the Dutch Society of Sciences (Hollandsche Maatschappij van Wetenschappen), and reproducing from that edition copies of this eminent mathematician's letters from Paris, written, in August to November 1669, to his brother Lodewijk Huygens at the Hague, in which a complete and correct method of constructing a table showing the average expectation of life at every age is for the first time expounded. The ball seems to have been set rolling by Lodewijk Huygens, who wrote to his brother as follows:-"The Hague, 22 August 1669. . . . . On the question of age, I have lately constructed a table of the time remaining to live for persons of every sort of age. It is a consequence I have drawn from that table of the English book of the Bills of Mortality of which I here send you a copy, in order that you may take the trouble to make a few of the like estimates, and that we may see how our calculations agree. I admit that I have had sufficient trouble in mastering it, but to you it will not be the same, and the consequences which result are very interesting, and may even be useful for fixing life annuities. The question is, up to what age should an infant naturally survive from the time of its conception, the same for one of 6 years old, or of 16 years, or of 26, and so on. If you find a difficulty, or too much trouble about it, I offer to communicate to you my method, which is a certain one, by the first opportunity. Adieu. According to my calculation, you will live to the age of  $56\frac{1}{2}$ years or thereabout, and I to the age of 55."

As Christiaan Huygens was then aged 40, this was an estimate of 155 years as the average expectation of life at that age—by no means an incorrect roughly-probable calculation. We refer to it, however, specially as showing that the rough sketch of a Life Table, contained in pages 61 and 62 of John Graunt's First Edition, London, 1662, of his "Natural and Political Observations", mentioned in a following Index, and made upon the "Bills of Mortality", had already commended the suggestion, to thinking minds like those of the brothers Lodewijk and Christiaan Huygens, of the applicability of such a Life or Survivorship Table to the calculation of the value of life annuities. The whole of the correspondence reprinted by the Dutch Society is of great interest, and in the last of the letters (Christiaan Huygens, Paris, 28 November 1669, to Lodewijk Huygens at the Hague) occurs this passage:—"The expectation or the value of the future age of a person, and the age to which there is an equal probability that he will reach it or will not reach it. are

two different things. The former is for the estimation of life annuities, the latter for that of bets." This distinction between the probable or average expectation, and the mean expectation, of life is thus early foreseen by the author of the "De Ratiociniis in ludo alea."

In the biographical sketch of Johan Hudde, Burgomaster and Councillor of Amsterdam, in the time of De Witt, and whose certificate of approval is appended to the latter's treatise on the value of Life Annuities, the editors of the volume presented to the International Congress of Actuaries have rendered a real service to their science in collecting all details that are as yet available about what Christiaan Huygens calls a "Table of Mortality drawn up by J. Hudde, derived from the registers of persons on whose lives annuity contracts were granted by the Government of the Netherlands in 1586-90." The figures given above the horizontal line in the table show the ages of the annuitants at the dates of the contracts. Under each age at entry are given, separately for each life, the number of years survived in numerical order from a minimum to a maximum term of years. The remark follows that, "If we consider that John de Witt communicated the results of his observations, but not these observations themselves, we have every reason to suppose that we have here to do with one of the most ancient, if not the most ancient, tables of scientifically-arranged observations, not only of the Netherlands, but of the whole world." A letter from Hudde to Huygens, dated 18 August 1671, explains that De Witt did not make use in his calculations of these observations on the lives of the City of Amsterdam annuitants (1586-90), but on a larger number of lives upon the registers of the Hague, and other places, and later in date than those of Amsterdam.

Next in the volume follow some interesting biographies of Dutch political arithmeticians and financial actuaries, if they are to be distinguished from assurance officials, of the eighteenth century, including Nicolaas Struyck (1687-1768?), Willem Kersseboom (1691-1771), Isaac de Graaf (1690-1732?), Johan van der Burch (1673-1738?) Nicolaas Duyn (1680-1745?), Abraham Gallas (1738-1788?), J. H. Van Swinden (1746-1823). Some of these dates are only approximative. The works of Struyck and of Kersseboom were frequently referred to by English writers of the eighteenth century, and by some of our actuaries of the first half of the present century. Kersseboom especially was most favourably thought of. It may serve to illustrate this if the writer of the present notice repeats the substance of a note to a paper on Government Life Annuities read by him to the Statistical Society in

1856.

When Mr. Pitt, in 1789, introduced, as Chaucellor of the Exchequer, his measure for raising the Tontine Loan of that year, of one million sterling, he stated to the House of Commons that the calculations had been made on the "most approved Tables of Lives." From the evidence of the original MS, calculations and outline of the scheme (formerly in the possession of Mr. Pitt's secretary, Rose, and now in that of the writer), it may be seen that the approved tables referred to by Mr. Pitt were those of Kersseboom of the Mortality of the State Annuitants in Holland. They did not differ very materially from those of De Parcieux of the mortality

amongst French Anuuity nominees. Mr. Baron Maseres, in his well-known work (London, 1782) had pointed out the near general correspondence of the two tables, the applicability of either of them, if Government should determine to grant Life Annuities either in the case of new loans, or in discharge of a part of old loans in perpetual 3 per-cents. This learned and spirited patron of science also pointed out the grounds of his preference for De Parcieux's tables of mortality, and published his own elaborate and laboriously calculated annuity tables deduced from them.

Next follow biographies of deceased Dutch actuaries of the present century: Rehuel Lobatto (1797-1866), the author of many works on Life Assurance, and who was a very eminent mathematician; Franciscus Johannes Stamkart (1805–1882) of whom the same may be said as of Lobatto; Marie Mathieu von Baumhauer (1816-1878), who may be said to have been the founder of Government Statisties in Holland, and the author of many researches in vital statistics. He represented his Government at all the International Statistical Congresses from 1853 to 1875, and of our personal knowledge we can fully confirm what his biographer says, that "Von Baumhauer was a sociable man, who loved to entertain relations with others. He was known to all the celebrated statisticians abroad. He was their friend, and kept up a close correspondence with them. He will always be cited with honour amongst Dutchmen who have pioneered the good way in the so-enlarged domain of statistics." He may be well compared, in many respects, to the late respected Dr. William Farr. F.R.S. The biographies close with that of David J. A. Samot (1837-1888), an actuary well known in London, and who was a Fellow of the Institute of Actuaries.

The last 112 pages of the book are occupied with reprints of old documents illustrative of Life Annuity business and tontines in Holland. These include (1) an Annuity Deed signed by Johan De Witt in 1665: (2) Certificates, from the archives of Tournay, of Annuities granted in 1228 and 1229; (3) the Rules and Regulations of many old Provident Associations in the Netherlands, and other

antiquarian remains concerning their practice and business.

F. H.

[It is with great pleasure that we are able to place before our readers the foregoing Review, contributed by Mr. Frederick Hendriks. That he should be the reviewer is eminently fitting, because the subject dealt with is specially his own, and in days long past his writings upon it attracted great attention.

The investigations and discoveries of Mr. Hendriks have been handsomely acknowledged by the authors of the volume, at whose immediate request we append the following letter, addressed to the Members of the Second International Congress of Actuaries, to each of whom a copy of the book was generously presented.—Ed. J.I.A.]

#### ALGEMEENE MAATSCHAPPIJ VAN LEVENSVERZEKERING EN LIJFRENTE,

AMSTERDAM, DAMRAK 74,

June 1898.

To the Members of the Second International Congress of Actuaries, held in London from 16—20 May 1898.

Gentlemen,—On the Second International Congress of Actuaries, held in London from 16—20 May last, the President, Mr. T. E. Young, B.A., announced, on our behalf, that we had the intention of offering to each of our fellow-members a copy of a French translation of a book, published by us in the Dutch language, and bearing upon the history of life assurance and annuities in the Netherlands.

This announcement procured us the honour and pleasure of making the acquaintance of Mr. Frederick Hendriks, F.S.S., F.I.A., —who informed us with reference to the five letters from De Witt to Hudde, printed at pages 24-33 of our book in French, that he had already interested himself, when a very young student, in a search for the long-lost treatise of De Witt on Life Annuities (Waardije van Lijf-renten naer proportie van Los-renten's Graven-Hage, 1671), and, having succeeded in discovering it, he had a small number of copies of his translation of that work privately printed in 1851. Very appreciative reviews of Mr. Hendriks's researches appeared in the Athenœum, London, 17 January 1852, in the Belgique Judiciaire, No. 20, 1852, and in the Weekblad van het Regt, The Hague, 24 June 1852. In France also they received attention through the interest taken in them by a correspondent and friend of Mr. Hendriks, the late Minister President of the Council of State, M. de Parieu. Encouraged by his success, Mr. Hendriks persevered in his recommendation that search should be made in the Dutch state archives for correspondence between De Witt and Hudde, which he intuitively felt certain would supply proof that the earlier discoveries of Pascal and Fermat, which promoted the foundation of the mathematical doctrine of probabilities, would be found to have paved the way for De Witt and Hudde's researches and inventions in the calculus of Life Annuities. Through the kindness of Mr. J. Heemskerk Azn., afterwards Dutch Prime Minister, Mr. Hendriks had the good fortune of perusing the correspondence between De Witt and Hudde, the seven letters and two memoirs (written between 27 September 1670 and 27 October 1671) having been copied from the Dutch originals in De Witt's autograph, preserved in the archives at the Hague and transmitted by Mr. Heemskerk, after collation by the archivist, Mr. J. A. de Zwaan, to Mr. Hendriks, through the intermediacy of the late

Mr. Fred. Muller, the learned publisher and author, of Amsterdam. The main contents of these documents Mr. Hendriks translated and published in English, in December 1852. He did us the favour of giving us his own copy for examination, which led us to the conclusion that Mr. Hendriks had performed an eminent piece of work. This, to our great regret, was wholly unknown to us before, so that we were unable to mention it, or to make use of it, in our work, as it would otherwise have been right to do. We expressed our surprise to the learned author that his remarkable articles had hitherto been left entirely out of mention by several authors, who have written on De Witt and Hudde, and that Mr. Heemskerk, with whom we also discussed these subjects, never mentioned his sending the documents to Mr. Hendriks. Mr. Hendriks suggested that his researches, made now nearly 50 years ago, probably had not become so publicly known on account of the fact that they were only printed originally for "private circulation", and, though afterwards published also in the Assurance Magazine (the predecessor of the Journal of the Institute of Actuaries), the numbers in which they appeared, namely: No. 6 and other quarterly numbers (1852-3), were soon out of print, and were never reprinted. He informed us also that another writer, an American named Barnwell (whom we have referred to in the note to page 7 of our work), printed a small book on De Witt in New York, in 1856, in which, without asking Mr. Hendriks's permission, he simply verbally transcribed Mr. Hendriks's translation of De Witt's treatise, merely stating in his preface that he had done Perhaps few copies of Mr. Barnwell's book reached Europe, and even Mr. Hendriks only saw it some ten years after its publication. And, as to Mr. Heemskerk's not having mentioned to us that he had sent Mr. Hendriks, in 1852, the transcripts of the De Witt letters certified by the Master of the Archives, Mr. Hendriks thought it probable that this gentleman may have lost recollection of the matter, after such a considerable lapse of time.

Mr. Hendriks's papers are entitled: "Contributions of the History of Insurance and of the theory of Life contingencies, by Frederick Hendriks, Esq., Actuary of the Globe Insurance Company. London, C. & E. Layton, 1851 and 1852." The first paper bears the sub-title: "With a restoration of the Grand Pensionary De Witt's

treatise of Life Annuities."

Now we feel bound to declare that the honour of bringing to light for the first time the work of De Witt and Hudde, and its influence upon financial operations in the last half of the 17th century, entirely belongs to Mr. Frederick Hendriks, only claiming for ourselves to have contributed by our present work to the revival of public interest in the historical contributions of Mr. Hendriks, which are by far too little known, according to the opinion of others; for example, it does not seem to have been known to Professor Eneström, of Stockholm. The paper of Mr. Hendriks only relating to the history of life insurance in our country in the last half of the 17th century, we further think that our work also will prove of some value, because it further comprises the period before and after the said end of the 17th century.

We now kindly request the honour of your accepting the present volume as a recollection of the Second International Congress of Actuaries, held with so great a success in London, and as a mark of personal esteem and friendship from

Yours truly,

Dr. S. R. J. VAN SCHEVICHAVEN, E. W. SCOTT, J. F. L. BLANKENBERG, Managers.

1. Lectures on The Law of Real Property. By G. Wood Hill.\*

2. Lectures on The London Daily Stock and Share List. By George Clare.†

The delivery and publication of these lectures mark, in a certain sense, a new epoch in the relations between the Institute of Actuaries and its Students. It is true that, for many years past, provision has been made for the instruction of Students in actuarial matters; but such instruction has hitherto been strictly confined to those more mathematical aspects of an actuary's work, with which, in the mind of the general public, he is chiefly, if not entirely concerned. Students have, however, been painfully aware, for a long time past, that much more than mere mathematical knowledge was required of them, before they could attain to the dignity of Fellows of the Institute; and their pathway in the search after such further knowledge has not always been an easy or a pleasant one. Some of the subjects now included in the Syllabus of Part IV of the Examination are of such a nature that reliable information concerning them is difficult to obtain, except by practical experience or from someone possessing that experience, and possessing, in addition, the power and the willingness to impart such information. The lectures recently delivered are apparently the outcome of the recognition by the Council of this felt want of information that could hardly be obtained from text-books, and they should, we think, be regarded as successes or failures according as they answer this end or fail to do so.

Looked at from this point of view, we find it difficult to speak very highly of the course of lectures on the Law of Real Property. Had such a thing been possible, we think the lectures would have proved much more useful to Students if the lecturer could have read beforehand the President's admirable introduction to them in their

† The Loudon Daily Stock and Share List. A Course of Lectures delivered at the Institute of Actuaries, by George Clarc. Published, for the Institute of Actuaries, by C. & E. Layton, London, 1898.

<sup>\*</sup> The Law of Real Property in England. A Course of Lectures delivered at the Institute of Actuaries, by G. Wood Hill, Barrister-at-Law. Published, for the Institute of Actuaries, by C. & E. Layton, London, 1898.

printed form, or if he could have given effect to the remarks with which Mr. T. E. Young prefaced the first lecture. They might then, perhaps, have taken the form of dissertations on those special aspects of the law of real property which more immediately concern the actuary in practice, instead of a somewhat bare outline of the law on

the subject generally. With regard to the matter of the lectures, considered apart from what we conceive should have been their object, there is much useful information to be gained from them. and fourth lectures are among the best from a Student's point of view. The remarks on legal and equitable estates, on pp. 28-32. and those on copyholds, on pp. 63-68, are specially worthy of note. The fifth lecture, dealing with leaseholds, and the sixth, on mortgages, are also very good. Considered generally, however, we think it is a fair criticism to make upon these lectures, that they do not go far enough to enable a Student to dispense with reading Williams or Strahan on the subject; and that if he reads one or other of these, which are as easy to understand as the lectures, the latter will be quite unnecessary. We quite agree with the lecturer's recommendation of Williams's Law of Real Property to actuarial students. It is true that Strahan's Law of Property furnishes all the information that a Student needs for his examination, and deals with both real and personal property in one volume of not more than half the size of Williams's Real Property alone. Nevertheless, we think that, to those who can spare the time, Williams's work is certainly to be preferred to Strahan's, not only on account of the fuller information which it contains and, as pointed out by the lecturer, of the illustrations given of the different points, but also on account of the more interesting way in which that information is imparted. This latter merit is due, to a great extent, to the adoption by Mr. Williams of the historical method, which is advocated by Mr. Wood Hill himself, in his remarks on the Feudal System. We cannot, however, quite agree with Mr. Hill when he states, as he does more than once, that certain parts of the law cannot be understood without a knowledge of that system. If he means that we could not understand why the law is what it is, without such knowledge, everyone would endorse his statement, but it is evidently possible to understand what the law is on any point, without any such knowledge. Two of the suggestions on page 3, seem a little contradictory, for if we adopt Mr. Wood Hill's advice, and add Williams on Real Property to our library as a book that will always be useful to us, we have the lecturer's assurance for the fact that when our edition has become an old one, which it will do about every three or four years, the best thing we can do with it is to give it away and buy a new edition. The latter advice is, however, no doubt sound.

There are one or two points in the first course of lectures which seem liable to be misunderstood as they stand at present. At the bottom of page 14, in speaking of the possibility of escheat in the case of real property, the lecturer remarks, in reference to goods, that the right of the owner's legal representative to them at his death is not defeated in any case by his

dying without heirs. We are certainly under the impression, however, that if a man dies intestate, and leaving no next-of-kin, his personal estate, after payment of all his debts, will pass to the Crown, the grant of administration being made in such cases to a nominee of the Crown, usually the Treasury Solicitor, and there is not much practical difference in the result between this and the escheat of real property. It is stated on page 17 that "any estate less than a tenancy for life is not a freehold estate," but, according to Williams, "where land is given to a widow during her widowhood, or to a man until he shall become bankrupt, or for any other definite period of time of uncertain duration, a freehold estate is conferred as in the case of a gift for life," and such estates are surely less than an estate for life. It is stated on page 20 that the method of barring an entail by a common recovery "existed in this country up to the year 1833, when an Act of Parliament was passed absolutely abolishing the fines that I mentioned to you in my last lecture." Now the only fines which we can find referred to in the first lecture are those forming incidents of tenancy, such as are found on pages 10 and 13. The fines, however, that were abolished by the "Act for the Abolition of Fines and Recoveries, 1833", were of a totally different nature, and had nothing in them of the nature of a penalty. According to Williams, "A fine was so called, because, having the effect of a judgment in a writ of right, the highest form of real action, it put an end, not only to the matter in dispute, but also to all claims to the land." On the lower part of page 20, there should not be a full stop before the expression, "After possibility of issue extinct." On page 54, 15 lines from the bottom, "D" should be read in place of "B." On page 57, the statement that "executors do not take real estate under a will", though true when the lecture was delivered, needs some revision now, in view of the appointment of a real representative under "The Land Transfer Act, 1897." On page 60, 15 lines from the top, the words "and died" should apparently be inserted after the words "if A survived the testator." In the last line of the first paragraph on page 79, "to the landlord" would be less likely to be misunderstood than the present form. On page 93, 17 lines from the top, for "mortgagor" should be read "mortgagee." In the middle of page 94, "principal" should be substituted for "principle." On page 98, it might, with advantage, have been pointed out that the receiver appointed by the mortgagee under his statutory power, is the agent, not of the mortgagee but of the mortgagor.

Little can be said of the lectures by Mr. Clare, except in praise of them. If we may single out any portions as being of special interest, reference may be made to the discussion of the variation of the price of rupee paper under the influence of accrued interest (pp. 7, 8), of arbitrage operations (pp. 8–14), to the remarks on the value of the  $2\frac{1}{2}$  per-cent Stock as a financial barometer (pp. 28, 29), the discussion on the redemption of the National Debt (pp. 43–45), on sinking funds (pp. 45–51), and that on Indian finance (pp. 59–67).

Perhaps on the whole, however, the most interesting and important of the lectures are the fifth and sixth, which are devoted to the discussion of local finance. Mr. Clare remarks, more than once, that the British Government securities constitute an absolute first charge on the wealth and industry of the country. It is to be presumed that he means a first charge after the necessary expenses for the maintenance of the government and the defence of the country have been provided for, as it can hardly be doubted that if, at any time. the funds available were only sufficient for these latter purposes, the payments in respect to the Debt would be sacrificed rather than the others. The same remarks apply to a similar reference to the Indian debt on page 65. The point, however, is of purely academical The discussion, on page 30, as to the merits or demerits of the borrowing methods adopted by the British Government towards the end of the last century. is interesting, but, in further defence of the method actually adopted, that of borrowing at a low rate of interest and at a considerable discount, it must be borne in mind that it was, to a great extent, forced upon them by the fact that the lenders practically insisted upon that plan being followed, by showing a great reluctance to take any other form, no doubt calculating upon the improvement of the Government credit and consequent gain in principal. On page 39, Mr. Clare declines to express an opinion as to whether Pitt was right or wrong in his contention as to the justice of taxing the fundholder in his capacity of income receiver. The arguments used by Pitt to justify his action seem, however, to be perfectly sound, nor are we altogether convinced that any breach of faith is committed in deducting the tax when the income is pavable abroad. Such tax is devoted mainly, if not entirely, to ensuring the carrying on of the Government and the defence of the country; and since anything that affects these will also affect the value of the security, it seems only right that all who benefit by any improvement in the security, should contribute to the cost of such improvement. If Pitt's argument is sound, the deduction is rather the payment of a tax collected in this particular way than deduction from dividend, and it is certainly difficult to see why those residing abroad, while reaping all the benefits of such a tax, as far as it applied to protect their security, should contribute nothing towards it. In reference to the discussion on Local Loan Stock, on page 56, it should be noted that this Stock does not form part of the Funded Debt of the United Kingdom. One remark may perhaps be made by way of adverse criticism of Mr. Clare's lectures. They seem, in view of their title, somewhat incomplete, seeing that only Government. Indian, Bank of England, and local securities are discussed. We would prefer, however, that this remark should be construed, not so much as adverse criticism, as the expression of the hope that Mr. Clare may see his way, at some future time, to continue the course of lectures which has commenced so auspiciously in the volume now published.

#### ORIGINAL TABLES.

To the Editor of the Journal of the Institute of Actuaries.

Dear Sir,—We beg to send you a set of tables, based on H<sup>M</sup> 3 per-cent, for the valuation of whole-term policies where premiums are limited to 10, 15, 20, 25, or 30 annual payments.

The values tabulated are for all ages from 15 to 60, and for all

years of duration until the premiums are paid up.

The calculations have been made according to the formula  $A_{x+n-t}P_x(1+a_{x+n|t-n-1})$ , the reversions and annuities being taken from Mr. R. P. Hardy's Valuation Tables, which are assumed to be rigorously correct. The net premiums are also those given by Mr. Hardy (excepting for  $_{30}P_{59}$ , which we have altered from 05721 to 05722, as we find the latter more accurate), but the premiums for ages 15 to 19 we have supplied ourselves.

The arrangement is similar in form to that adopted by Mr. Hardy, excepting that, with a view to economy of space, we have omitted the

line for year "0."

To ensure accuracy, the tables have been calculated by us separately: in the one case the products of premium and annuity were obtained by a continuous process, and in the other by direct multiplication. The products were then subtracted from the reversions, and the manuscripts read over.

The copy for the printer was written off from one manuscript, and read over with the other. The proof sheets, besides being read over, have been added up, and the results compared with the

totals of the original manuscript.

On account of the care taken over the calculations, and revision of proofs, we feel justified in thinking that the tables are entirely free from error (assuming always, as stated above, that Mr. Hardy is absolutely correct to the last figure), and we believe that the values here tabulated may be used throughout with confidence.

Trusting that the new tables will prove serviceable to the

members of the profession,

We remain,

Yours obediently,

A. S. HUME, W. STOTT.

HM

### VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura- | 15     | 16             | 17     | 18     | 19     | 20     | 21     | 22     | Dura- |
|-------|--------|----------------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 3'437  | 3.256          | 3.614  | 3.200  | 3.779  | 3*850  | 3.918  | 3.985  | tion  |
| 1     | 3.266  | 3.314          | 3.350  | 3.342  | 3.334  | 3.356  | 3.388  | 3.441  | 1     |
| 2     | 6.601  | 6.685          | 6.722  | 6.720  | 6.737  | 6.795  | 6.888  | 7.021  | 2     |
| 3     | 9.990  | 10.085         | 10.132 | 10.163 | 10.226 | 10.352 | 10.527 | 10.742 | 3     |
| 4     | 13.412 | 13.525         | 13.610 | 13.698 | 13·S37 | 14.046 | 14.306 | 14.603 | 4     |
| 5     | 16.880 | 17.034         | 17.184 | 17.357 | 17.588 | 17.886 | 18.231 | 18.601 | 5     |
| 6     | 20.421 | 20.644         | 20.883 | 21.153 | 21.486 | 21.873 | 22.294 | 22.729 | 6     |
| 7     | 24.064 | 24.380         | 24.720 | 25.100 | 25.529 | 25.997 | 26.489 | 26.993 | 7     |
| 8     | 27.834 | 28.258         | 28.714 | 29.198 | 29.717 | 30.263 | 30.823 | 31.398 | 8     |
| 9     | 31.746 | $32 \cdot 286$ | 32.851 | 33.439 | 34.045 | 34.668 | 35.303 | 35.949 | 9     |
| 10    | 35.812 | 36.465         | 37.139 | 37.824 | 38.518 | 39.221 | 39.934 | 40.662 | 10    |

| Dura- | 23     | 24     | 25     | 26     | 27     | 28     | 29     | 30     | Dura   |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| tion  | 4.023  | 4*124  | 4,501  | 4.581  | 4.362  | 4'450  | 4'536  | 4.623  | tion   |
| 1     | 3.517  | 3.612  | 3.685  | 3.764  | 3.829  | 3.893  | 3.954  | 4.023  | 1      |
| 2     | 7.182  | 7.354  | 7.505  | 7.652  | 7.775  | 7.906  | 8.037  | 8.176  | 2      |
| 3     | 10.980 | 11.228 | 11:446 | 11.655 | 11.852 | 12.046 | 12.253 | 12.476 | 3      |
| 4     | 14.916 | 15.224 | 15.513 | 15.791 | 16.053 | 16.329 | 16.618 | 16.924 | 4      |
| 5     | 18.978 | 19.354 | 19.707 | 20.053 | 20.400 | 20.759 | 21.137 | 21.527 | 4<br>5 |
| 6     | 23.173 | 23.612 | 24.037 | 24:467 | 24.899 | 25.349 | 25.808 | 26.284 | 6      |
| 7     | 27.504 | 28.012 | 28.521 | 29.035 | 29.560 | 30.098 | 30.650 | 31.211 | 7      |
| 8     | 31.978 | 32.567 | 33.160 | 33.766 | 34.356 | 35.014 | 35.653 | 36.306 | - 8    |
| 9     | 36.609 | 37.253 | 37.969 | 38.669 | 39.380 | 40.103 | 40.836 | 41.584 | 9      |
| 10    | 41.407 | 42.170 | 42.950 | 43.745 | 44.553 | 45.372 | 46.207 | 47.060 | 10     |

| Dura-       | 31                         | 32                         | 33                         | 34               | 35                         | 36                       | 37                         | 38                         | Dura-                                 |
|-------------|----------------------------|----------------------------|----------------------------|------------------|----------------------------|--------------------------|----------------------------|----------------------------|---------------------------------------|
| tion        | 4'712                      | 4.803                      | 4.896                      | 4.992            | 5,001                      | 5.103                    | 5,594                      | 5'399                      | tion                                  |
| 1           | 4.092                      | 4.169                      | 4.250                      | 4.332            | 4.407                      | 4.476                    | 4.555                      | 4.629                      | 1                                     |
| 3           | 8·324<br>12·706            | 8·482<br>12·941            | 8·644<br>13·184            | 8·801<br>13·418  | 8·952<br>13·645            | 9·101<br>13·876          | 9·255<br>14·121            | 9·413<br>14·375            | $\begin{bmatrix} 2\\ 3 \end{bmatrix}$ |
| 5           | 17·237<br>21·924           | 17.554<br>22.320           | 17·875<br>22·723           | 18·191<br>23·124 | 18·502<br>23·529           | 18·821<br>23·953         | 19·162<br>24·399           | 19·525<br>24·862           | 5                                     |
| 6<br>7<br>8 | 26·765<br>31·778<br>36·967 | 27·250<br>32·350           | 27·739<br>32·940           | 28·235<br>33·543 | 28·747<br>34·159           | 29·278<br>34·804         | 29.830<br>35.460           | 30·396<br>36·131           | 6 7                                   |
| 9           | 42·348<br>47·935           | 37·641<br>43·132<br>48·836 | 38·334<br>43·940<br>49·762 | 39·052<br>44·770 | 39·784<br>45·616<br>51·669 | 40.531 $46.477$ $52.642$ | 41·298<br>47·348<br>53·621 | 42·071<br>48·222<br>54·608 | 8   9                                 |
| 10          | 41 399                     | 40.290                     | 49.702                     | 50.707           | 91.009                     | 92.042                   | 55 621                     | 94.002                     | 10                                    |

 $H^{M}$ 

### VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura-           | 39                | 40               | 41               | 42               | 43                      | 44               | 45               | 46               | Dura-  |
|-----------------|-------------------|------------------|------------------|------------------|-------------------------|------------------|------------------|------------------|--------|
| tion            | 5.206             | 5.616            | 5'731            | 5.821            | 5'977                   | 6.108            | 6.243            | 6.381            | tion   |
| 1               | 4.708             | 4.804            | 4.902            | 5.008            | 5.102                   | 5.193            | 5.270            | 5.349            | 1      |
| $\frac{2}{3}$   | $9.591 \\ 14.655$ | 9·788<br>14·954  | 9·989<br>15·249  | 10·189<br>15·539 | 10·374<br>15·818        | 10.546<br>16.069 | 10·707<br>16·313 | 10·860<br>16·553 | 2 3    |
| 3<br>4          | 19.903            | 20.296           | 20.693           | 21.070           | 21.429                  | 21.771           | 22.105           | 22.443           | 4      |
| 5               | 25.341            | 25.835           | 26.314           | 26.782           | 27.233                  | 27.673           | 28.107           | 28.544           | 5      |
| 6               | 30·977<br>36·807  | 31·560<br>37·481 | 32·130<br>38·149 | 32·690<br>38·813 | $\frac{33.239}{39.477}$ | 33·787<br>40·136 | 34·330<br>40·802 | 34·888<br>41·479 | 6<br>7 |
| 8               | 42.840            | 43.617           | 44.395           | 45.174           | 45.950                  | 46.741           | 47.536           | 48.336           | 8      |
| 9<br><b>1</b> 0 | 49·102<br>55·605  | 49·989<br>56·613 | 50·882<br>57·635 | 51·784<br>58·676 | 52·699<br>59·729        | 53·621<br>60·792 | 54·549<br>61·863 | 55·482<br>62·939 | 9      |
| 10              | 99 009            | 00 019           | 0, 000           | 00 070           | 00 /20                  | 00 702           | 01 000           | 02 000           | " _    |

| Dura- | 47     | 48     | 49     | 50     | 51     | 52     | 53     | 54     | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 6.252  | 6.662  | 6.813  | 6.964  | 7.133  | 7.288  | 7.461  | 7.642  | tion  |
| 1     | 5·419  | 5·498  | 5·584  | 5·670  | 5·767  | 5·856  | 5·939  | 6·015  | 1     |
| 2     | 11·014 | 11·178 | 11·354 | 11·537 | 11·727 | 11·897 | 12·068 | 12·219 | 2     |
| 3     | 16·796 | 17·058 | 17·334 | 17·611 | 17·889 | 18·142 | 18·397 | 18·634 | 3     |
| 4     | 22·781 | 23·152 | 23·523 | 23·897 | 24·266 | 24·611 | 24·947 | 25·273 | 4     |
| 5     | 29·001 | 29·477 | 29·947 | 30·414 | 30·876 | 31·311 | 31·745 | 32·160 | 5     |
| 6     | 35·461 | 36·032 | 36·611 | 37·179 | 37·740 | 38·276 | 38·816 | 39·327 | 6     |
| 7     | 42·165 | 42·848 | 43·538 | 44·214 | 44·884 | 45·536 | 46·182 | 46·795 | 7     |
| 8     | 49·145 | 49·949 | 50·750 | 51·551 | 52·345 | 53·121 | 53·879 | 54·621 | 8     |
| 9     | 56·417 | 57·355 | 58·291 | 59·226 | 60·152 | 61·065 | 61·963 | 62·842 | 9     |
| 10    | 64·020 | 65·103 | 66·190 | 67·274 | 68·353 | 69·424 | 70·484 | 71·532 | 10    |

| Dura-                                     | 55  | 56  | 57  | 58  | 59  | 60  | Dura-                                     |
|---|---|---|---|---|---|---|---|
| tion                                      | 7.831   | 8.058   | 8.232   | 8.451   | 8.678   | 8.912   | tion                                      |
| 1<br>2<br>3<br>4<br>5<br>6<br>7<br>8<br>9 | 6.086<br>12:375<br>18:861<br>25:586<br>32:559<br>39:817<br>47:395<br>55:339<br>63:701<br>72:569 | 6·162<br>12·520<br>19·090<br>25·890<br>32·941<br>40·290<br>47·965<br>56·022<br>64·541<br>73·600 | 6:231<br>12:662<br>19:297<br>26:165<br>33:297<br>40:723<br>48:499<br>56:684<br>65:365<br>74:626 | 6:298<br>12:790<br>19:489<br>26:425<br>33:621<br>41:125<br>49:008<br>57:323<br>66:175<br>75:650 | 6·354<br>12·901<br>19·656<br>26·643<br>33·896<br>41·493<br>49·475<br>57·938<br>66·972<br>76·678 | 6:403<br>12:992<br>19:784<br>26:814<br>34:145<br>41:827<br>49:931<br>58:542<br>67:763<br>77:700 | 1<br>2<br>3<br>4<br>5<br>6<br>7<br>8<br>9 |

ΗM

### VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura- | 15     | 16     | 17     | 18     | 19              | 20     | 21     | 22     | Dura. |
|-------|--------|--------|--------|--------|-----------------|--------|--------|--------|-------|
| tion  | 2,490  | 2.555  | 2.620  | 2.683  | 2'741           | 2'794  | 2.845  | 2.894  | tion  |
| 1     | 2.281  | 2.311  | 2.320  | 2.295  | 2.266           | 2.262  | 2.269  | 2.310  | 1     |
| 2     | 4.602  | 4.643  | 4.630  | 4.580  | 4.553           | 4.565  | 4.615  | 4.718  | 2     |
| 3     | 6.944  | 6.967  | 6.932  | 6.889  | 6.886           | 6.943  | 7.054  | 7.222  | 3     |
| 4     | 9.282  | 9.288  | 9.266  | 9.250  | 9.296           | 9.417  | 9.595  | 9.823  | 4     |
| 5     | 11.619 | 11.639 | 11.647 | 11.687 | 11.800          | 11.993 | 12.231 | 12.514 | 5     |
| 6     | 13.987 | 14.041 | 14.109 | 14.221 | 14.410          | 14.663 | 14.959 | 15.286 | 6     |
| 7     | 16.409 | 16.523 | 16.667 | 16.859 | 1 <b>7</b> ·116 | 17.430 | 17.772 | 18.136 | 7     |
| 8     | 18.912 | 19.104 | 19.330 | 19.594 | 19.917          | 20.280 | 20.666 | 21.076 | 8     |
| 9     | 21.513 | 21.788 | 22.092 | 22.428 | 22.803          | 23.218 | 23.650 | 24.101 | 9     |
| 10    | 24.219 | 24.574 | 24.951 | 25.348 | $25.781 \pm$    | 26.243 | 26.722 | 27.228 | 10    |
| 11    | 27.028 | 27.458 | 27.899 | 28:360 | 28.846          | 29.362 | 29.899 | 30.462 | 11    |
| 12    | 29.933 | 30.432 | 30.941 | 31.464 | 32.010          | 32.587 | 33.185 | 33.809 | 12    |
| 13    | 32.934 | 33.500 | 34.078 | 34.667 | 35.281 -        | 35.922 | 36.585 | 37.272 | 13    |
| 14    | 36.028 | 36.666 | 37:314 | 37.979 | 38.666          | 39.376 | 40.105 | 40.851 | 14    |
| 15    | 39.221 | 39.934 | 40.662 | 41.407 | 42.170          | 42.950 | 43.745 | 44.553 | 15    |

| Dura- | 23     | 24     | 25             | 26     | 27     | 28           | 29     | 30     | Dura- |
|-------|--------|--------|----------------|--------|--------|--------------|--------|--------|-------|
| tion  | 2'944  | 2.998  | 3.022          | 3,112  | 3.128  | 3,541        | 3.306  | 3'372  | tion  |
| 1     | 2.378  | 2.438  | 2.500          | 2.556  | 2.596  | 2.643        | 2.683  | 2.717  | 1     |
| 2     | 4.847  | 4.971  | 5.089          | 5.188  | 5.267  | 5.359        | 5.436  | 5.526  | 2     |
| 3     | 7.411  | 7.591  | 7.751          | 7.891  | 8.013  | 8.146        | 8.280  | 8.422  | 3     |
| 4     | 10.066 | 10.284 | 10.488         | 10.672 | 10.839 | 11.026       | 11.213 | 11.414 | -4    |
| 5     | 12.797 | 13.059 | 13.302         | 13.534 | 13.757 | 13.998       | 14.244 | 14.500 | 5     |
| 6     | 15.611 | 15.912 | 16.203         | 16.486 | 16.768 | 17:068       | 17.375 | 17:690 | - 6   |
| 7     | 18.504 | 18.852 | 19.195         | 19.537 | 19.880 | 20.240       | 20.603 | 20.973 | 7     |
| 8     | 21.484 | 21.886 | 22.289         | 22.692 | 23.094 | 23.515       | 23.933 | 24.358 | 8     |
| 9     | 24.564 | 25.023 | 25.487         | 25.951 | 26.415 | 26.893       | 27:371 | 27.857 | 9     |
| 10    | 27:747 | 28.268 | 28.793         | 29.319 | 29.842 | 30.379       | 30.920 | 31.475 | 10    |
| 11    | 31.042 | 31.621 | $32 \cdot 206$ | 32.794 | 33:381 | 33.982       | 34.593 | 35.226 | 11    |
| 12    | 34.448 | 35.090 | 35.736         | 36.385 | 37.038 | 37.713       | 38.404 | 39.115 | 12    |
| 13    | 37.969 | 38.671 | 39.381         | 40.098 | 40.828 | $41.579 \pm$ | 42.356 | 43.153 | 13    |
| 14    | 41.609 | 42.374 | 43.152         | 43.945 | 44.757 | 45.595       | 46.456 | 47.335 | 14    |
| 15    | 45.372 | 46.207 | 47.060         | 47.935 | 48.836 | 49.762       | 50.707 | 51.669 | 15    |

| Dura- | 31     | 32     | 33     | 34     | 35     | 36     | 37     | 38     | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 3.438  | 3'507  | 3'577  | 3.650  | 3.726  | 3.803  | 3.883  | 3.964  | tion  |
| 1     | 2.775  | 2.823  | 2.884  | 2.939  | 2.983  | 3.039  | 3.078  | 3.134  | 1     |
| 2     | 5.635  | 5.739  | 5.853  | 5.960  | 6.056  | 6.159  | 6.255  | 6.370  | 2     |
| 3     | 8.588  | 8.746  | 8.916  | 9.071  | 9.215  | 9.375  | 9.531  | 9.710  | 3     |
| 4     | 11.633 | 11.849 | 12.067 | 12.274 | 12.476 | 12.692 | 12.918 | 13.176 | 4     |
| 5     | 14.776 | 15.042 | 15.311 | 15.576 | 15.840 | 16.127 | 16.428 | 16.758 | 5     |
| 6     | 18.015 | 18.333 | 18.661 | 18.984 | 19.319 | 19.682 | 20.061 | 20.462 | 6     |
| 7     | 21.354 | 21.728 | 22.118 | 22.516 | 22.929 | 23.369 | 23.817 | 24.282 | 7     |
| 8     | 24.799 | 25.239 | 25.700 | 26.177 | 26.668 | 27.178 | 27.691 | 28.220 | 8     |
| 9     | 28.361 | 28.874 | 29.416 | 29.968 | 30.531 | 31.114 | 31.689 | 32.271 | 9     |
| 10    | 32.055 | 32.648 | 33.265 | 33.891 | 34.529 | 35.171 | 35.810 | 36.453 | 10    |
| 11    | 35.885 | 36.558 | 37.250 | 37.952 | 38.655 | 39.360 | 40.062 | 40.772 | 11    |
| 12    | 39.857 | 40.607 | 41.374 | 42.145 | 42.912 | 43.686 | 44.461 | 45.248 | 12    |
| 13    | 43.969 | 44.799 | 45.638 | 46.474 | 47.316 | 48.166 | 49.022 | 49.885 | 13    |
| 14    | 48.231 | 49.135 | 50.044 | 50.958 | 51.879 | 52.810 | 53.752 | 54.712 | 14    |
| 15    | 52.642 | 53.621 | 54.608 | 55.605 | 56.613 | 57.635 | 58.676 | 59.729 | 15    |

ΗM

# Values of Policies for 100.

| Dura- | 39     | 40     | 41     | 42     | 43     | 44     | 45     | 46     | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 4.048  | 4.134  | 4.552  | 4.321  | 4,421  | 4.26   | 4.636  | 4.749  | tion  |
| 1     | 3.192  | 3.267  | 3.337  | 3.411  | 3.482  | 3.549  | 3.602  | 3.647  | 1     |
| 2     | 6.492  | 6.640  | 6.790  | 6.927  | 7.066  | 7.187  | 7.284  | 7.379  | 2     |
| 3     | 9.910  | 10.133 | 10.350 | 10.550 | 10.744 | 10.914 | 11.057 | 11.211 | 3     |
| 4     | 13.447 | 13.737 | 14.016 | 14.276 | 14.513 | 14.734 | 14.938 | 15.149 | 4     |
| 5     | 17.101 | 17.452 | 17.786 | 18.094 | 18:387 | 18.664 | 18.927 | 19.206 | 5     |
| 6     | 20.868 | 21.273 | 21.659 | 22.019 | 22.368 | 22.709 | 23.041 | 23.396 | 6     |
| 7     | 24.743 | 25.204 | 25.641 | 26.062 | 26.475 | 26.885 | 27.295 | 27.721 | 7     |
| 8     | 28.734 | 29.250 | 29.748 | 30.233 | 30.720 | 31.208 | 31.690 | 32.184 | 8     |
| 9     | 32.846 | 33.422 | 33.988 | 34.544 | 35.112 | 35.673 | 36.230 | 36.793 | 9     |
| 10    | 37.093 | 37.737 | 38.373 | 39.015 | 39.662 | 40.298 | 40.927 | 41.559 | 10    |
| 11    | 41.486 | 42.203 | 42.925 | 43.651 | 44.368 | 45.085 | 45.790 | 46.496 | 11    |
| 12    | 46.037 | 46.840 | 47.645 | 48.451 | 49.250 | 50.049 | 50.835 | 51.621 | 12    |
| 13    | 50.766 | 51.655 | 52.545 | 53.437 | 54.322 | 55.203 | 56.081 | 56.958 | 13    |
| 14    | 55.681 | 56.658 | 57.638 | 58.618 | 59.599 | 60.577 | 61.554 | 62.525 | 14    |
| 15    | 60.792 | 61.863 | 62.939 | 64:020 | 65.103 | 66.190 | 67.274 | 68.353 | 15    |

| Dura- | 47     | 48     | 49     | 50     | 51     | 52     | 53     | 54     | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 4.866  | 4.986  | 5,115  | 5'243  | 5.381  | 5.527  | 5.683  | 5.846  | tion  |
| 1     | 3.690  | 3.751  | 3.796  | 3.863  | 3.935  | 4.011  | 4.064  | 4.134  | 1     |
| 2     | 7.480  | 7.591  | 7.706  | 7.840  | 7.980  | 8.114  | 8.227  | 8.349  | 2     |
| 3     | 11.364 | 11.544 | 11.727 | 11.934 | 12.132 | 12.324 | 12.491 | 12.663 | 3     |
| 4     | 15.369 | 15.617 | 15.868 | 16.132 | 16.388 | 16.628 | 16.851 | 17.072 | 4     |
| 5     | 19.500 | 19.816 | 20.126 | 20.443 | 20.752 | 21.048 | 21.321 | 21.597 | 5     |
| 6     | 23.760 | 24.135 | 24.505 | 24.875 | 25.239 | 25.590 | 25.915 | 26.235 | 6     |
| 7     | 28.146 | 28.586 | 29.011 | 29.442 | 29.857 | 30.264 | 30.636 | 30.997 | 7     |
| 8     | 32.672 | 33.173 | 33.660 | 34.143 | 34.620 | 35.074 | 35.494 | 35.897 | 8     |
| 9     | 37.349 | 37.908 | 38.459 | 39.005 | 39.535 | 40.038 | 40.509 | 40.950 | 9     |
| 10    | 42.181 | 42.811 | 43.426 | 44.032 | 44.617 | 45.177 | 45.695 | 46.178 | 10    |
| ĩi l  | 47.191 | 47.896 | 48.579 | 49.248 | 49.896 | 50.509 | 51.085 | 51.634 | 11    |
| 12    | 52.400 | 53.179 | 53.932 | 54.675 | 55:390 | 56.073 | 56.725 | 57.348 | 12    |
| 13    | 57.824 | 58.680 | 59.522 | 60.344 | 61.136 | 61.907 | 62.655 | 63.384 | 13    |
| 14    | 63.487 | 64.438 | 65.372 | 66.289 | 67.188 | 68.073 | 68.943 | 69.804 | 14    |
| 15    | 69.424 | 70.484 | 71.532 | 72.569 | 73.600 | 74.626 | 75.650 | 76.678 | 15    |

| 6·820<br>4·394<br>8·842<br>13·357<br>17·936<br>22·598 | 7°53<br>4·432<br>8·909<br>13·446<br>18·042<br>22·718 | 1 2 3 4  |
|---|--|--|
| 8·842<br>13·357<br>17·936<br>22·598                   | 8·909<br>13·446<br>18·042                            | 2<br>3<br>4  |
| 13·357<br>17·936<br>22·598                            | 13·446<br>18·042                                     | 3<br>4   |
| 17.936 $22.598$                                       | 18.042   | 4  |
| 22.598  |  |  |
| 000   | 22.718   |  |
| a H a a a   |  | 5  |
| 27.366  | 27.523   | 6  |
| $32 \cdot 271$  | 32.463   | 7  |
| 37.348  | 37.592   | 8  |
| 42.628  | 42.944   | 9  |
| 48.170  | 48.550   | 10   |
| 54.007  | 54.437   | 11   |
| 60.169  | 60.656   | 12   |
| 66.741  | 67.293   | 13   |
| 73.803  | 74.457   | 14   |
| 81.510  | 82.345   | 15   |
|   | 54·007<br>60·169<br>66·741<br>73·803                 | 54·007 54·437<br>60·169 60·656<br>66·741 67·293<br>73·803 74·457 |

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## VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura- | 15     | 16     | 17     | 18     | 19     | 20     | 21     | 22      | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|---------|-------|
| tion  | 2.056  | 2.020  | 2,133  | 2.186  | 2.534  | 2.518  | 2,350  | 2.361   | tion  |
| 1     | 1.798  | 1.828  | 1.821  | 1.775  | 1.735  | 1.723  | 1.728  | 1.764   | 1     |
| 2     | 3.624  | 3.652  | 3.611  | 3.24   | 3.477  | 3.472  | 3.510  | 3.595   | 2 3   |
| 3     | 5.454  | 5.450  | 5.372  | 5.282  | 5.246  | 5.275  | 5.364  | 5.209   | 3     |
| 4     | 7.259  | 7.221  | 7.143  | 7.066  | 7.068  | 7.152  | 7.299  | 7.491   | 4     |
| 5     | 9.042  | 9.002  | 8.942  | 8.907  | 8.965  | 9.109  | 9.307  | 9.544   | 5     |
| 6     | 10.835 | 10.816 | 10.797 | 10.824 | 10.943 | 11.138 | 11.382 | 11.652  | 6     |
| 7     | 12.659 | 12:684 | 12.729 | 12.820 | 12.997 | 13.241 | 13.517 | 13.815  | 7     |
| 8     | 14.541 | 14:628 | 14.740 | 14.891 | 15.121 | 15.399 | 15.708 | 16.036  | 8     |
| 9     | 16:499 | 16.654 | 16.829 | 17.037 | 17:302 | 17.617 | 17.958 | 18.319  | 9     |
| 10    | 18.540 | 18.756 | 15:989 | 19.241 | 19.547 | 19.894 | 20.272 | 20.674  | 10    |
| 11    | 20.656 | 20.931 | 21.212 | 21.509 | 21.852 | 22.238 | 22.656 | 23.102  | 11    |
| 12    | 22.848 | 23.171 | 23.495 | 23.838 | 24.225 | 24.654 | 25.118 | 25.610  | 12    |
| 13    | 25.103 | 25.472 | 25.847 | 26.234 | 26:670 | 27.151 | 27:663 | 28.198  | 13    |
| 14    | 27.424 | 27.843 | 28.261 | 28:709 | 29.198 | 29.730 | 30.290 | 30.870  | 14    |
| 15    | 29.810 | 30.279 | 30.761 | 31.261 | 31.511 | 32.394 | 33.001 | 33.624  | 15    |
| 16    | 32.268 | 32.797 | 33.340 | 33.907 | 34.508 | 35.141 | 35.795 | 36.464  | 16    |
| 17    | 34.807 | 35.399 | 36.008 | 36.637 | 37.295 | 37.979 | 38.679 | 139.396 | 17    |
| 18    | 37:430 | 38.089 | 38.765 | 39.456 | 40.170 | 40.905 | 41.657 | 42.430  | 18    |
| 19    | 40.144 | 40.871 | 41.612 | 42.367 | 43.138 | 43.929 | 44.740 | 45.574  | 19    |
| 20    | 42.950 | 43.745 | 44.553 | 45.372 | 46.207 | 47.060 | 47.935 | 48.836  | 20    |

| Dura- | 23     | 24     | 25     | 26     | 27     | 28     | 29     | 30     | Dura          |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|---------------|
| tion  | 3,404  | 2,440  | 2,496  | 2'547  | 2.200  | 2.653  | 2.208  | 2.764  | tion          |
| 1     | 1.806  | 1.864  | 1.928  | 1.965  | 2.005  | 2.031  | 2.056  | 2.084  | 1             |
| 2     | 3.697  | 3.803  | 3.912  | 3.955  | 1.055  | 4.111  | 4.167  | 4.235  | 2<br>3        |
| 3     | 5.658  | 5.811  | 5.953  | 6.054  | 6.154  | 6.244  | 6.341  | 6.451  | 3             |
| 4     | 7.688  | 7.873  | 8.042  | 8.176  | 8.309  | 8.439  | 8.578  | 8.734  | 4             |
| 5     | 9.772  | 9.985  | 10.188 | 10.353 | 10.527 | 10.699 | 10.887 | 11.087 | $\frac{4}{5}$ |
| -6    | 11.908 | 12.155 | 12.386 | 12.594 | 12.813 | 13.032 | 13.265 | 13.507 | - 6           |
| 7     | 14.104 | 14.379 | 14.654 | 14.905 | 15.172 | 15.436 | 15.712 | 15.995 | 7             |
| 8     | 16.356 | 16.674 | 16.991 | 17:291 | 17:602 | 17.913 | 18.229 | 18.550 | 8             |
| 9     | 18.680 | 19.040 | 19.404 | 19:749 | 20.107 | 20.458 | 20.816 | 21.185 | - 9           |
| 10    | 21.076 | 21.483 | 21.891 | 22.284 | 22.682 | 23.076 | 23.481 | 23.900 | 10            |
| 11    | 23.551 | 24.005 | 24.458 | 24.893 | 25.331 | 25.776 | 26.230 | 26.707 | 11            |
| 12    | 26.105 | 26.602 | 27.098 | 27.579 | 28.066 | 28.561 | 29.076 | 29.618 | 12            |
| 13    | 28.739 | 29.279 | 29.817 | 30.347 | 30.889 | 31.444 | 32.025 | 32.631 | 13            |
| 1.4   | 31.454 | 32.037 | 32.624 | 33.207 | 33.809 | 34.433 | 35.077 | 35.740 | 14            |
| 15    | 34.251 | 34.883 | 35.523 | 36.170 | 36.839 | 37.526 | 38.231 | 38.955 | 15            |
| 16    | 37.139 | 37.825 | 38.528 | 39.241 | 39:977 | 40.726 | 41.492 | 12.266 | 16            |
| 17    | 40.127 | 40.875 | 41.640 | 42.424 | 43.222 | 44.034 | 44.854 | 45.677 | 17            |
| 18    | 43.221 | 11 036 | 44.870 | 45.715 | 46.578 | 47:447 | 48.319 | 49.199 | 18            |
| 19    | 46.432 | 47:313 | 48.211 | 49.122 | 50.043 | 50.968 | 51.900 | 52.841 | 19            |
| 20    | 49.762 | 50.707 | 51.669 | 52.642 | 53.621 | 54.608 | 55.605 | 56.613 | 20            |

Original Tables.

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## VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura- | 31     | 32     | 33     | 34     | 35     | 36     | 37     | 38     | Dura- |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion  | 2.850  | 2.879  | 2'940  | 3.003  | 3.068  | 3.136  | 3.502  | 3'277  | tion  |
| 1     | 2.135  | 2.172  | 2.215  | 2.259  | 2.306  | 2.339  | 2.380  | 2.420  | 1     |
| 2     | 4.327  | 4.412  | 4.498  | 4.583  | 4.669  | 4.736  | 4.824  | 4.908  | 2     |
| 3     | 6.587  | 6.715  | 6.842  | 6.967  | 7.086  | 7.201  | 7.334  | 7.477  | 3     |
| 4     | 8.915  | 9.085  | 9.249  | 9.411  | 9.575  | 9.738  | 9.927  | 10.135 | 4     |
| 5     | 11.309 | 11.516 | 11.721 | 11.925 | 12.137 | 12.357 | 12.610 | 12.879 | 5     |
| 6     | 13.770 | 14.014 | 14.261 | 14.516 | 14.785 | 15.064 | 15.382 | 15.702 | 6     |
| 7     | 16.295 | 16.585 | 16.881 | 17.190 | 17.521 | 17.866 | 18.234 | 18.607 | 7     |
| 8     | 18.895 | 19.234 | 19.585 | 19.959 | 20.352 | 20.752 | 21.170 | 21.586 | 8     |
| 9     | 21.576 | 21.972 | 22.388 | 22.822 | 23.270 | 23.721 | 24.185 | 24.633 | 9     |
| 10    | 24.349 | 24.808 | 25.284 | 25.776 | 26.275 | 26.770 | 27.269 | 27.760 | 10    |
| 11    | 27.218 | 27.741 | 28.275 | 28.819 | 29.362 | 29.897 | 30.436 | 30.969 | 11    |
| 12    | 30.191 | 30.770 | 31.357 | 31.945 | 32.532 | 33.108 | 33.692 | 34.274 | 12    |
| 13    | 33.260 | 33.891 | 34.529 | 35.159 | 35.789 | 36.413 | 37.046 | 37.685 | 13    |
| 14    | 36.427 | 37.107 | 37.786 | 38.464 | 39.142 | 39.820 | 40.507 | 41.210 | 1.4   |
| 15    | 39.687 | 40.415 | 41.143 | 41.872 | 42.605 | 43.338 | 44.093 | 44.855 | 15    |
| 16    | 43.046 | 43.823 | 44.604 | 45.391 | 46.182 | 46.985 | 47.803 | 48.618 | 16    |
| 17    | 46.509 | 47.342 | 48.184 | 49.031 | 49.892 | 50.760 | 51.639 | 52.514 | 17    |
| 18    | 50.089 | 50.985 | 51.887 | 52.808 | 53.737 | 54.671 | 55.613 | 56.552 | 18    |
| 19    | 53.793 | 54.756 | 55.736 | 56.726 | 57.724 | 58.727 | 59.734 | 60.743 | 19    |
| 20    | 57.635 | 58.676 | 59.729 | 60.792 | 61.863 | 62.939 | 64.020 | 65.103 | 20    |

| Dura- | 39     | 40     | 41     | 42     | 43     | 44     | 45       | 46       | Dura-    |
|-------|--------|--------|--------|--------|--------|--------|----------|----------|----------|
| tion  | 3,321  | 3,450  | 3.210  | 3.597  | 3.688  | 3.482  | 3.887    | 3.993    | tion     |
| 1     | 2.472  | 2.521  | 2.595  | 2.656  | 2.722  | 2.771  | 2.811    | 2.850    | 1        |
| 2     | 5.019  | 5.133  | 5.273  | 5.388  | 5.510  | 5.602  | 5.679    | 5.758    | 2 3      |
| 3     | 7.652  | 7.832  | 8.022  | 8.192  | 8.356  | 8.485  | 8.601    | 8.723    | 4        |
| 4     | 10.371 | 10.605 | 10.848 | 11.061 | 11.257 | 11.425 | 11.585   | 11.756   | 5        |
| 5     | 13.166 | 13.453 | 13.737 | 13.982 | 14.217 | 14.432 | 14.637   | 14.862   | 6        |
| 6     | 16.041 | 16.370 | 16.685 | 16.969 | 17.250 | 17.506 | 17.770   | 18.055   | 7        |
| 7     | 18.988 | 19.348 | 19.701 | 20.031 | 20.353 | 20.667 | 20.988   | 21.328   | (        |
| 8     | 22.001 | 22.396 | 22.794 | 23.164 | 23.543 | 23.915 | 24.295   | 24.683   | 8<br>9   |
| 9     | 25.086 | 25.526 | 25.967 | 26.391 | 26.826 | 27.254 | 27.683   | 28.118   |          |
| 10    | 28.254 | 28.739 | 29.232 | 29.717 | 30.207 | 30.682 | 31.156   | 31.634   | 10       |
| 11    | 31.511 | 32.048 | 32.600 | 33.140 | 33.678 | 34.202 | 34.719   | 35.242   | 11       |
| 12    | 34.868 | 35.465 | 36.072 | 36.660 | 37.249 | 37.818 | 38.385   | 38.949   | 12       |
| 13    | 38.339 | 38.990 | 39.648 | 40.285 | 40.922 | 41.541 | 42.150   | 42.763   | 13       |
| 14    | 41.918 | 42.625 | 43.334 | 44.022 | 44.706 | 45.376 | 46.036   | 46.690   | 14       |
| 15    | 45.619 | 46.378 | 47.137 | 47.877 | 48.614 | 49.335 | 50.043   | 50.740   | 15       |
| 16    | 49.441 | 50.255 | 51.068 | 51.862 | 52.659 | 53.432 | 54.189   | 54.933   | 16       |
| 17    | 53:395 | 54.268 | 55.138 | 55.996 | 56.848 | 57.676 | 58.490   | 59.284   | 17       |
| 18    | 57.492 | 58.430 | 59.367 | 60.289 | 61.198 | 62.092 | 62.967   | 63.818   | 18       |
| 19    | 61.752 | 62.761 | 63.764 | 64.756 | 65.736 | 66.699 | 67.645   | 68.576   | 19       |
| 20    | 66.190 | 67.274 | 68.353 | 69.424 | 70.484 | 71.532 | 72.569   | 73.600   | 20       |
|       | 1      |        |        |        |        |        | <u> </u> | <u> </u> | <u> </u> |

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## VALUES OF POLICIES FOR 100.

3 PER-CENT.

| Dura- | 47     | 48            | 49     | 50     | 51     | 52     | 53     | 54      | Dura- |
|-------|--------|---------------|--------|--------|--------|--------|--------|---------|-------|
| tion  | 4'103  | 4,518         | 4'339  | 4.462  | 4.603  | 4.141  | 4.005  | 5.062   | tion  |
| 1     | 2.894  | <b>2</b> ·943 | 2.996  | 3.053  | 3.130  | 3.197  | 3.252  | 3.314   | 1     |
| 2     | 5.848  | 5.946         | 6.057  | 6.180  | 6.318  | 6.439  | 6.554  | 6.670   | 2     |
| 3     | 8.862  | 9.018         | 9.194  | 9.377  | 9.567  | 9.741  | 9.904  | 10.067  | 3     |
| 4     | 11.948 | 12.168        | 12.404 | 12.633 | 12.875 | 13.095 | 13:304 | 13.516  | 4     |
| 5     | 15.119 | 15.394        | 15.675 | 15.956 | 16.238 | 16.503 | 16.755 | 17.015  | 5     |
| 6     | 18:367 | 18.688        | 19.015 | 19:337 | 19:666 | 19.969 | 20.263 | 20.556  | 6     |
| 7     | 21.686 | 22.054        | 22.421 | 22.785 | 23.151 | 23.495 | 23.823 | 24:138  | 7     |
| 8     | 25.088 | 25.496        | 25:902 | 26.303 | 26.705 | 27:081 | 27:431 | 27.768  | 8     |
| 9     | 28.560 | 29.011        | 29.454 | 29.896 | 30.329 | 30.728 | 31.095 | 31.443  | 9     |
| 10    | 32.119 | 32.608        | 33.092 | 33.566 | 31.022 | 34.439 | 34.822 | 35.176  | 10    |
| 11    | 35.771 | 36.297        | 36.814 | 37:312 | 37.785 | 38.219 | 38.615 | 38.985  | 11    |
| 12    | 39.520 | 40.051        | 40.627 | 41.148 | 41.639 | 42.086 | 42.500 | 42.894  | 12    |
| 13    | 43.370 | 43.965        | 44.540 | 15.0S0 | 45.591 | 46.057 | 46 497 | -46.930 | 13    |
| 14    | 47.333 | 47.963        | 48.563 | 49.126 | 49.665 | 50.164 | 50.615 | 51.116  | 14    |
| 15    | 51.424 | 52.085        | 52.714 | 53.316 | 53.890 | 54:437 | 54.968 | 55.498  | 15    |
| 16    | 55.656 | 56.356        | 57.031 | 57.680 | 58.307 | 58.912 | 59.506 | 60.097  | 16    |
| 17    | 60.056 | 60.809        | 61.538 | 62.252 | 62.948 | 63.633 | 64.298 | 64.934  | 17    |
| 18    | 64.654 | 65.476        | 66.282 | 67:078 | 67:865 | 68.633 | 69:368 | 70.068  | 18    |
| 19    | 69.497 | 70.408        | 71.311 | 72.211 | 73.098 | 73.959 | 74.783 | 75.556  | 19    |
| 20    | 74.626 | 75.650        | 76.678 | 77:700 | 78.706 | 79.685 | 80.623 | 81.510  | 20    |

| Dura-         | 55     | 56     | 57     | 58     | 59     | 60     | Dura |
|---------------|--------|--------|--------|--------|--------|--------|------|
| tion          | 5°244  | 5.433  | 5.635  | 5.852  | 6.084  | 6.338  | tion |
| 1             | 3.367  | 3.426  | 3.490  | 3.562  | 3.610  | 3.667  | 1    |
| $\tilde{2}$   | 6.776  | 6.888  | 7.017  | 7:139  | 7.233  | 7.337  | 2    |
| 3             | 10.225 | 10.393 | 10.569 | 10.728 | 10.867 | 11.002 | 3    |
| 4             | 13.724 | 13.933 | 14.139 | 14.333 | 14.502 | 14.661 | -4   |
| $\frac{4}{5}$ | 17.257 | 17.495 | 17:734 | 17.951 | 18.137 | 18.328 | 5    |
| 6             | 20.827 | 21.092 | 21.341 | 21.579 | 21.791 | 22.021 | 6    |
| 7             | 24.430 | 24.711 | 24.976 | 25.232 | 25.476 | 25.754 | 7    |
| 8             | 28.076 | 28.361 | 28.644 | 28.926 | 29.211 | 29.535 | 8    |
| 9             | 31.762 | 32.066 | 32.374 | 32.691 | 33.017 | 33.408 | 9    |
| 10            | 35.504 | 35.835 | 36.173 | 36.529 | 36.918 | 37.346 | 10   |
| 11            | 39.341 | 39.697 | 40.076 | 10.489 | 40.910 | 41.343 | 11   |
| 12            | 43.277 | 43.677 | 44.108 | 44.548 | 44.984 | 45.396 | 12   |
| 13            | 47.353 | 47.802 | 48.268 | 48.726 | 49.140 | 49.503 | 13   |
| 14            | 51.596 | 52.089 | 52.571 | 53.017 | 53:396 | 53.680 | 14   |
| 15            | 56.027 | 56.539 | 57.027 | 57.443 | 57.765 | 57.988 | 15   |
| 16            | 60.661 | 61.191 | 61.667 | 62.052 | 62.331 | 62.573 | 16   |
| 17            | 65.537 | 66.084 | 66.549 | 66.925 | 67:254 | 67.524 | 17   |
| 18            | 70.712 | 71.285 | 71.779 | 72.227 | 72.630 | 72.979 | 18   |
| 19            | 76.266 | 76.912 | 77.524 | 78.094 | 78.624 | 79.123 | 19   |
| 20            | 82.345 | 83.159 | 83.946 | 84.711 | 85.461 | 86.187 | 20   |

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Values of Policies for 100.

Premiums limited to 25 Annual Payments.

| 3 | PER-CENT |
|---|----------|
|---|----------|

| Dura- | 15     | 16     | 17     | 18     | 19     | 20     | 21     | 22     | Dura |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| tion  | 1,422  | 1.803  | 1.850  | 1.896  | 1,030  | 1.978  | 5,010  | 2.025  | tion |
| 1     | 1.526  | 1.530  | 1.528  | 1.484  | 1.432  | 1.415  | 1.404  | 1.111  | 1    |
| 2     | 3.064  | 3.058  | 3.013  | 2.925  | 2.857  | 2.842  | 2.858  | 2.944  | 2    |
| 3     | 4.596  | 4.248  | 4.461  | 4.361  | 4.296  | 4.309  | 4.374  | 4.513  | 3    |
| 4     | 6.089  | 6.003  | 5.904  | 5.810  | 5.778  | 5.839  | 5.957  | 6.138  | 4    |
| 5     | 7.549  | 7.454  | 7.364  | 7:304  | 7.322  | 7.438  | 7.600  | 7.820  | 5    |
| 6     | 9.007  | 8.922  | 8.867  | 8.859  | 8.934  | 9.095  | 9.298  | 9.543  | 6    |
| 7     | 10.483 | 10.435 | 10.432 | 10.483 | 10.605 | 10.808 | 11.039 | 11.308 | 7    |
| 8     | 12.004 | 12.009 | 12.067 | 12.168 | 12.335 | 12.567 | 12.822 | 13.113 | 8    |
| 9     | 13.588 | 13.653 | 13.762 | 13.911 | 14.110 | 14 367 | 14.648 | 14.965 | 9    |
| 10    | 15.240 | 15.359 | 15.516 | 15.700 | 15.927 | 16.213 | 16.520 | 16.869 | 10   |
| 11    | 16.953 | 17.122 | 17:317 | 17.531 | 17:791 | 18.103 | 18:446 | 18.831 | 11   |
| 12    | 18.727 | 18.934 | 19.161 | 19.412 | 19.701 | 20.051 | 20.431 | 20.854 | 12   |
| 13    | 20.548 | 20.791 | 21.054 | 21.338 | 21.669 | 22.058 | 22.478 | 22.937 | 13   |
| 14    | 22.416 | 22.697 | 22.997 | 23.325 | 23.698 | 24.129 | 24.586 | 25.082 | 14   |
| 15    | 24.332 | 24.654 | 25.000 | 25.373 | 25.791 | 26.262 | 26.758 | 27.285 | 15   |
| 16    | 26.301 | 26.669 | 27.064 | 27.485 | 27.950 | 28.461 | 28.991 | 29.551 | 16   |
| 17    | 28.330 | 28.748 | 29.194 | 29.665 | 30.172 | 30.721 | 31.288 | 31.884 | 17   |
| 18    | 30.424 | 30.896 | 31.391 | 31.910 | 32.459 | 33.045 | 33.653 | 34.292 | 18   |
| 19    | 32.586 | 33.111 | 33.657 | 34.222 | 34.814 | 35.443 | 36.095 | 36.782 | 19   |
| 20    | 34.817 | 35.395 | 35.989 | 36.601 | 37.241 | 37.918 | 38.623 | 39.364 | 20   |
| 21    | 37.116 | 37.747 | 38.392 | 39.055 | 39.748 | 40.480 | 41.242 | 42.036 | 21   |
| 22    | 39.488 | 40.170 | 40.870 | 41.592 | 42.345 | 43.133 | 43.954 | 44.797 | 22   |
| 23    | 41.930 | 42.671 | 43.432 | 44.217 | 45.036 | 45.885 | 46.756 | 47.649 | 23   |
| 24    | 44.452 | 45.257 | 46.085 | 46.940 | 47.823 | 48.729 | 49.653 | 50.590 | 24   |
| 25    | 47.060 | 47.935 | 48.836 | 49.762 | 50.707 | 51.669 | 52.642 | 53.621 | 25   |

| Dura-   | 23             | 24     | 25     | 26     | 27     | 28     | 29     | 30     | Dura- |
|---------|----------------|--------|--------|--------|--------|--------|--------|--------|-------|
| tion    | 2.000          | 2,130  | 2,1,3  | 5,510  | 2.266  | 2,312  | 2.364  | 2,412  | tion  |
| 1       | 1.488          | 1.545  | 1.591  | 1.622  | 1.656  | 1.675  | 1.711  | 1.734  | 1     |
| 2       | 3.041          | 3.143  | 3.232  | 3.290  | 3.347  | 3.389  | 3.450  | 3.510  | 2     |
| 3       | 4.653          | 4.795  | 4.910  | 4.991  | 5.075  | 5.144  | 5.240  | 5.334  | 3     |
| 4       | 6.320          | 6.487  | 6.625  | 6.733  | 6.841  | 6.946  | 7.079  | 7.213  | 4     |
| 5       | 8.027          | 8.219  | 8.380  | 8.515  | 8.657  | 8.800  | 8.972  | 9.145  | 5     |
| 6       | 9.774          | 9.989  | 10.177 | 10.346 | 10.526 | 10.709 | 10.918 | 11.126 | 6     |
| 7       | 11.562         | 11.801 | 12.024 | 12.229 | 12.451 | 12.672 | 12.916 | 13.158 | 7     |
| 8       | 13.393         | 13.666 | 13.925 | 14.171 | 14.430 | 14.687 | 14.965 | 15.235 | 8     |
| 9       | 15.277         | 15.585 | 15.883 | 16.169 | 16.465 | 16.754 | 17:063 | 17:372 | 9     |
| 10      | 17.215         | 17.564 | 17.900 | 18.222 | 18.551 | 18.875 | 19.220 | 19.570 | 10    |
| 11      | 19.216         | 19.601 | 19.972 | 20.330 | 20.691 | 21.052 | 21.441 | 21.841 | 11    |
| 12      | 21.275         | 21.695 | 22.102 | 22.492 | 22.892 | 23.297 | 23.732 | 24.186 | 12    |
| 13      | 23.395         | 23.847 | 24.285 | 24.716 | 25.159 | 25.611 | 26.104 | 26.612 | 13    |
| 14      | 25.572         | 26.057 | 26.535 | 27:007 | 27.500 | 28.010 | 28.555 | 29.110 | 14    |
| 15      | 27.808         | 28.332 | 28.852 | 29.375 | 29.924 | 30.487 | 31.081 | 31.680 | 15    |
| 16      | 30.112         | 30.676 | 31.246 | 31.825 | 32.429 | 33.044 | 33.681 | 34.317 | 16    |
| 17      | 32.486         | 33.102 | 33.727 | 34.362 | 35.015 | 35.675 | 36.349 | 37.020 | 17    |
| 18      | 34.944         | 35.613 | 36.294 | 36.978 | 37.676 | 38.379 | 39.087 | 39.794 | 18    |
| 19      | 37.489         | 38.211 | 38.940 | 39.675 | 40.415 | 41.152 | 41.899 | 42.646 | 19    |
| 20      | 40.123         | 40.894 | 41.673 | 42.448 | 43.227 | 44.005 | 44.794 | 45.586 | 20    |
| 21      | 42.844         | 43.664 | 44.485 | 45.300 | 46.120 | 46.942 | 47.779 | 48.620 | 21    |
| 22      | 45.654         | 46.516 | 47.376 | 48.235 | 49.102 | 49.976 | 50.862 | 51.762 | 22    |
| 23      | 48.550         | 49.450 | 50.355 | 51.265 | 52.183 | 53.109 | 54.057 | 55.013 | 23    |
| $^{24}$ | 51.531         | 52.478 | 53.432 | 54.394 | 55.369 | 56.361 | 57.365 | 58.377 | 24    |
| 25      | <b>54</b> ·608 | 55.605 | 56.613 | 57.635 | 58.676 | 59.729 | 60.792 | 61.863 | 25    |

Original Tables.

 $H^{M}$ 

Values of Policies for 100.

Premiums limited to 25 Annual Payments.

3 PER-CENT.

| Dura-   | 31     | 32     | 33     | 34             | 35             | 36     | 37     | 38             | Dura- |
|---------|--------|--------|--------|----------------|----------------|--------|--------|----------------|-------|
| tion    | 2,462  | 2,21   | 2.211  | 2.636          | 2.697          | 2.260  | 2'826  | 2,894          | tion  |
| 1       | 1.770  | 1.805  | 1.848  | 1.878          | 1.915          | 1.955  | 1.982  | 2.021          | 1     |
| 2       | 3.550  | 3.659  | 3.738  | 3·S07          | 3.877          | 3.947  | 4.012  | 4.096          | 2     |
| 3       | 5.446  | 5.561  | 5.678  | 5.777          | 5.880          | 5.988  | 6.097  | 6.235          | 3     |
| 4       | 7.364  | 7.513  | 7.661  | 7.793          | 7.934          | 8.083  | 8.247  | 8.412          | 4 5   |
| 5       | 9.330  | 9.511  | 9.691  | 9.862          | 10.041         | 10.244 | 10.465 | 10.716         |       |
| 6       | 11.345 | 11.555 | 11.773 | 11.985         | 12.216         | 12:476 | 12.753 | 13.053         | 6     |
| 7       | 13.405 | 13.656 | 13.912 | 14:176         | 14.463         | 14.779 | 15.102 | 15.448         | 7     |
| 8       | 15.522 | 15.812 | 16.119 | 16.440         | 16.783         | 17:143 | 17.511 | 17.891         | 8     |
| 9       | 17:700 | 18.038 | 15:402 | 18.776         | 19.166         | 19.570 | 19.973 | 20.378         | 9     |
| 10      | 19.947 | 20.341 | 20.758 | 21.176         | 21.611         | 22.050 | 22.478 | 22.913         | 10    |
| 11      | 22.269 | 22:719 | 23.179 | 23.643         | 24.113         | 24.578 | 25.037 | 25.507         | 11    |
| 12      | 24.670 | 25.164 | 25.670 | 26.169         | 26.664         | 27:160 | 27.656 | 28.159         | 12    |
| 13      | 27.142 | 27.679 | 28.220 | 28.748         | 29.272         | 29.805 | 30.334 | 30.883         | 13    |
| 14      | 29.683 | 30.258 | 30.825 | 31.385         | 31.947         | 32.515 | 33.088 | 33.683         | 14    |
| 15      | 32.289 | 32.893 | 33.495 | 34.090         | 34.659         | 35.301 | 35.924 | 36.563         | 15    |
| 16      | 34.958 | 35.595 | 36.231 | 36.867         | 37.510         | 35.172 | 38.839 | 39.515         | 16    |
| 17      | 37.694 | 38:369 | 39.046 | 39.726         | 40.420         | 41.127 | 41.832 | 42.548         | 17    |
| 18      | 40.507 | 41.222 | 41.946 | 42.678         | 43.418         | 44.166 | 44.910 | 45.662         | 18    |
| 19      | 43.402 | 44.163 | 44.941 | 45.719         | 46.503         | 47.293 | 45.077 | 48.864         | 19    |
| 20      | 46.388 | 47:205 | 48.032 | 48.856         | 49.683         | 50.513 | 51.337 | $52 \cdot 164$ | 20    |
| 21      | 49:479 | 50.348 | 51.218 | 52.091         | 52.963         | 53.836 | 54.700 | 55.572         | 21    |
| 22      | 52.673 | 53.592 | 54.511 | 55.432         | 56.350         | 57:267 | 58.181 | 59.093         | 22    |
| 23      | 55.976 | 56.947 | 57.916 | 58.885         | 59.855         | 60.825 | 61.786 | 62.739         | 23    |
| $^{24}$ | 59:396 | 60.418 | 61.443 | $62 \cdot 467$ | $63 \cdot 493$ | 64.514 | 65.527 | 66.530         | 24    |
| $^{25}$ | 62.939 | 64.020 | 65.103 | 66.190         | 67.274         | 68.353 | 69.424 | 70.484         | 25    |

| Dura- | 39     | 40     | 41     | 42     | 43     | 44     | 45     | 46     | Dura-   |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| tion  | 2.965  | 3.040  | 3,110  | 3.504  | 3*294  | 3.389  | 3*490  | 3.296  | tion    |
| 1     | 2:069  | 2.119  | 2.185  | 2.243  | 2.302  | 2.362  | 2.407  | 2.446  | 1       |
| 2     | 4.198  | 4.309  | 4.437  | 4.550  | 4.660  | 4.762  | 4.841  | 4.922  | 2       |
| 3     | 6.395  | 6.567  | 6.745  | 6:909  | 7:060  | 7:195  | 7:312  | 7.437  | 3       |
| 4     | S·661  | 8.883  | 9.110  | 9.311  | 9.491  | 9.666  | 9.823  | 9.994  | 4       |
| 5     | 10.985 | 11.255 | 11:515 | 11.748 | 11.964 | 12.178 | 12:381 | 12:602 | 5       |
| 6     | 13.364 | 13.669 | 13.963 | 14.225 | 14.479 | 14.735 | 14.991 | 15.272 | 6       |
| 7     | 15.793 | 16.126 | 16.450 | 16.750 | 17.045 | 17:350 | 17:662 | 17:994 | 7       |
| 8     | 18.263 | 18.627 | 18.985 | 19.325 | 19.668 | 20.028 | 20.390 | 20.765 | 5       |
| 9     | 20.780 | 21.177 | 21.574 | 21.958 | 22.356 | 22.765 | 23.166 | 23.584 | 9       |
| 10    | 23.352 | 23.784 | 24.224 | 24.662 | 25.106 | 25.553 | 25.996 | 26.447 | 10      |
| 11    | 25.979 | 26.457 | 26.946 | 27.429 | 27:911 | 28.397 | 28.873 | 29:362 | 11      |
| 12    | 28.676 | 29.200 | 29.734 | 30.255 | 30.774 | 31.293 | 31.804 | 32.322 | 12      |
| 13    | 31.445 | 32.016 | 32.587 | 33.146 | 33.698 | 34.248 | 34.789 | 35.336 | 13      |
| 14    | 34.289 | 34.897 | 35.504 | 36.093 | 36.680 | 37.259 | 37.834 | 35.405 | 14      |
| 15    | 37.205 | 37.847 | 38.486 | 39.109 | 39.726 | 40.339 | 40.938 | 41.527 | 15      |
| 16    | 40.195 | 40.869 | 41.541 | 42.198 | 42.845 | 43.483 | 44.101 | 44:701 | 16      |
| 17    | 43.260 | 43.971 | 44.674 | 45.364 | 46.038 | 46.697 | 47:332 | 47:944 | 17      |
| 18    | 46.411 | 47.152 | 47.891 | 48.607 | 49.307 | 49.988 | 50.636 | 51.261 | 18      |
| 19    | 49.649 | 50.428 | 51.196 | 51.939 | 52.664 | 53.363 | 54.026 | 54.672 | 19      |
| 20    | 52.987 | 53.798 | 54.595 | 55.368 | 56.116 | 56.834 | 57.527 | 58:198 | 20      |
| 21    | 56.431 | 57:275 | 58.105 | 58.905 | 59.680 | 60.433 | 61.162 | 61.875 | 21      |
| 22    | 59.989 | 60.872 | 61.735 | 62.570 | 63.385 | 64:179 | 64.959 | 65.725 | 22      |
| 23    | 63.681 | 64.605 | 65.506 | 66.388 | 67:256 | 68.109 | 68.953 | 69.792 | 23      |
| 24    | 67:519 | 68.492 | 69.450 | 70.396 | 71.332 | 72.261 | 73.188 | 74.104 | $^{24}$ |
| 25    | 71.532 | 72.569 | 73.600 | 74.626 | 75.650 | 76.678 | 77.700 | 78.706 | 25      |

 $H_M$ 

# Values of Policies for 100. Premiums limited to 25 Annual Payments.

3 PER-CENT.

| Dura- | 47     | 48     | 49     | 50     | 51     | 52     | 53     | 54     | Dura   |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| tion  | 3.208  | 3.825  | 3'949  | 4.081  | 4.333  | 4'374  | 4.238  | 4.714  | tion   |
| 1     | 2.477  | 2.529  | 2.583  | 2.644  | 2.726  | 2.801  | 2.869  | 2.933  | 1      |
| 2     | 4.998  | 5.098  | 5.212  | 5.350  | 5.497  | 5.636  | 5.769  | 5.900  | 2      |
| 3     | 7.560  | 7.719  | 7.904  | 8.100  | 8.308  | 8.505  | 8.696  | 8.885  | 3      |
| 4     | 10.173 | 10.397 | 10.639 | 10.890 | 11.153 | 11.405 | 11.647 | 11.896 |        |
| 5     | 12.845 | 13.125 | 13.416 | 13.719 | 14.027 | 14.327 | 14.622 | 14.925 | 4<br>5 |
| 6     | 15.570 | 15.894 | 16.232 | 16.579 | 16.932 | 17.278 | 17.624 | 17.970 | 6      |
| 7     | 18.335 | 18.706 | 19.085 | 19.472 | 19.864 | 20.259 | 20.642 | 21.020 | 7      |
| 8     | 21.149 | 21.556 | 21.971 | 22.395 | 22.830 | 23.258 | 23.667 | 24.075 | 8      |
| 9     | 24.005 | 24.447 | 24.898 | 25.360 | 25.822 | 26.271 | 26.703 | 27.129 | 9      |
| 10    | 26.903 | 27:377 | 27.865 | 28.349 | 28.831 | 29.301 | 29.746 | 30.181 | 10     |
| 11    | 29.847 | 30.354 | 30.864 | 31.367 | 31.865 | 32.338 | 32.796 | 33.245 | 11     |
| 12    | 32.840 | 33.369 | 33.894 | 34.409 | 34.914 | 35.394 | 35.861 | 36.331 | 12     |
| 13    | 35.878 | 36.422 | 36.959 | 37:481 | 37.984 | 38.474 | 38.957 | 39.455 | 13     |
| 14    | 38.963 | 39.516 | 10.057 | 40.582 | 41.094 | 41.600 | 42.107 | 42.633 | 14     |
| 15    | 42.096 | 42.657 | 43.198 | 43.724 | 44.253 | 14.778 | 45.313 | 45.886 | 15     |
| 16    | 45.281 | 45.847 | 46.395 | 46.939 | 47.483 | 48.037 | 48.615 | 49.209 | 16     |
| 17    | 48.531 | 49.106 | 49.669 | 50.230 | 50.804 | 51.396 | 51.992 | 52.590 | 17     |
| 18    | 51.860 | 52.452 | 53.041 | 53.629 | 54.238 | 54.855 | 55.458 | 56.030 | 18     |
| 19    | 55.294 | 55.914 | 56.529 | 57.159 | 57.797 | 58.419 | 59.005 | 59.537 | 19     |
| 20    | 58.856 | 59.512 | 60.171 | 60.833 | 61.480 | 62.097 | 62.648 | 63.121 | 20     |
| 21    | 62.576 | 63.279 | 63.981 | 64.663 | 65.313 | 65.909 | 66.421 | 66.845 | 21     |
| 22    | 66.488 | 67.242 | 67.973 | 68.674 | 69.325 | 69.897 | 70.387 | 70.841 | 22     |
| 23    | 70.618 | 71.419 | 72.190 | 72.910 | 73.564 | 74.144 | 74.682 | 75.183 | 23     |
| 24    | 74.998 | 75.860 | 76.674 | 77.429 | 78.123 | 78.785 | 79.408 | 79.997 | 24     |
| 25    | 79.685 | 80.623 | 81.510 | 82.345 | 83.159 | 83.946 | 84.711 | 85.461 | 25     |

| Dura-         | 55     | 56     | 57             | 58     | 59     | 60     | Dura-   |
|---------------|--------|--------|----------------|--------|--------|--------|---------|
| tion          | 4.003  | 5,104  | 5.322          | 5.222  | 5.806  | 6.026  | tion    |
| 1             | 3.012  | 3.088  | 3.150          | 3.241  | 3.321  | 3.394  | 1       |
| 2             | 6.044  | 6.188  | 6.328          | 6.486  | 6.629  | 6.768  | 2       |
| $\frac{2}{3}$ | 9.093  | 9.306  | 9.509          | 9.720  | 9.924  | 10.113 | 3       |
| 4             | 12.160 | 12.426 | 12.685         | 12.946 | 13.196 | 13.433 | 4       |
| 5             | 15.239 | 15.552 | 15.847         | 16.151 | 16.439 | 16.731 | 4<br>5  |
| 6             | 18.323 | 18.670 | 18.999         | 19.332 | 19.665 | 20.016 | 6       |
| 7             | 21.404 | 21.777 | $22 \cdot 133$ | 22.502 | 22.885 | 23.302 | 7       |
| 8             | 24.479 | 24.871 | 25.256         | 25.671 | 26.111 | 26.598 | 8       |
| 9             | 27.546 | 27.965 | 28.384         | 28.847 | 29.353 | 29.923 | 9       |
| 10            | 30.618 | 31.068 | 31.528         | 32.043 | 32.628 | 33.254 | 10      |
| 11            | 33.708 | 34.192 | 34.703         | 35.282 | 35.908 | 36.563 | 11      |
| 12            | 36.827 | 37.350 | 37.918         | 38.543 | 39.185 | 39.820 | 12      |
| 13            | 39.983 | 40.562 | 41.164         | 41.799 | 42.422 | 43.006 | 13      |
| 14            | 43.212 | 43.815 | 44.427         | 45.033 | 45.595 | 46.087 | 14      |
| 15            | 46.489 | 47.097 | 47.678         | 48.226 | 48.694 | 49.085 | 15      |
| 16            | 49.809 | 50.388 | 50.916         | 51.369 | 51.736 | 52.105 | 16      |
| 17            | 53.165 | 53.689 | 54.123         | 54.476 | 54.820 | 55.152 | 17      |
| 18            | 56.554 | 56.995 | 57.337         | 57.667 | 57.976 | 58.280 | 18      |
| 19            | 59.985 | 60.342 | 60.668         | 60.971 | 61.255 | 61.552 | 19      |
| 20            | 63.507 | 63.856 | 64.159         | 64.441 | 64.722 | 65.000 | 20      |
| 21            | 67.237 | 67:577 | 67.883         | 68:174 | 68.444 | 68.676 | 21      |
| 22            | 71.245 | 71.604 | 71.938         | 72.250 | 72.504 | 72.681 | 22      |
| 23            | 75.637 | 76.059 | 76.447         | 76.781 | 77.026 | 77.214 | 23      |
| 24            | 80.559 | 81.083 | 81.552         | 81.951 | 82.284 | 82.541 | $^{24}$ |
| 25            | 86.187 | 86.874 | 87.506         | 88.090 | 88.617 | 89.110 | 25      |

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## VALUES OF POLICIES FOR 100.

3 PER-CENT.

407

| _             | 15     | 16             | 17     | 18     | 19     | 20     | 21     | 22     | D            |
|---------------|--------|----------------|--------|--------|--------|--------|--------|--------|--------------|
| Dura-<br>tion |        |                |        |        |        |        |        |        | Dura<br>tion |
| tion          | 1'582  | 1.652          | 1.669  | 1,111  | 1,121  | 1.484  | 1.822  | 1.826  |              |
| 1             | 1.344  | 1:361          | 1.336  | 1.295  | 1.231  | 1.214  | 1.206  | 1.235  | 1            |
| 2             | 2.698  | 2.698          | 2.629  | 2.536  | 2.453  | 2.435  | 2.451  | 2.525  | 2            |
| 3             | 4.036  | 3.993          | 3.574  | 3.765  | 3.684  | 3:659  | 3.750  | 3.574  | 2<br>3       |
| 4             | 5.332  | 5.241          | 5.108  | 5.003  | 4.947  | 4.997  | 5.109  | 5.275  | 4            |
| 5             | 6.585  | 6.481          | 6.352  | 6.274  | 6.265  | 6.366  | 6.519  | 6.721  | 5            |
| 6             | 7.529  | 7:728          | 7.630  | 7:598  | 7.642  | 7:785  | 7:976  | S:200  | 6            |
| 7             | 9.082  | 9.011          | S:961  | 5.954  | 9.072  | 9.255  | 9.468  | 9.711  | 7            |
| S             | 10.370 | 10.350         | 10:355 | 10.422 | 10.550 | 10.756 | 10.989 | 11.253 | $\mathbf{s}$ |
| 9             | 11.714 | 11:747         | 11:799 | 11:909 | 12.063 | 12.290 | 12.545 | 12.829 | - 9          |
| 10            | 13.118 | 13.198         | 13.292 | 13.430 | 13.610 | 13.858 | 14.136 | 14:445 | 10           |
| 11            | 14:575 | 14:698         | 14.522 | 14.987 | 15.190 | 15.463 | 15:770 | 16.113 | 11           |
| 12            | 16:079 | 16.234         | 16:357 | 16.578 | 16.507 | 17:111 | 17.450 | 17:829 | 12           |
| 13            | 17:623 | 17.506         | 17:986 | 18:206 | 15:470 | 15:897 | 19.180 | 19:590 | 13           |
| 14            | 19.202 | 19:414         | 19.626 | 19.550 | 20.180 | 20.553 | 20.960 | 21:401 | 14           |
| 15            | 20·S19 | 21.061         | 21.310 | 21.602 | 21.942 | 22:349 | 22:758 | 23.255 | 15           |
| 16            | 22.473 | 22.754         | 23.045 | 23.375 | 23.756 | 24.195 | 24.664 | 25.156 | 16           |
| 17            | 24:178 | $24 \cdot 499$ | 24.832 | 25:205 | 25.619 | 26:090 | 26.555 | 27.111 | 17           |
| 18            | 25.932 | 26:297         | 26.672 | 27:085 | 27.532 | 25:031 | 25.561 | 29:122 | 18           |
| 19            | 27:741 | 28.148         | 2S·565 | 29:014 | 29:494 | 30:029 | 30.595 | 31:198 | 19           |
| 20            | 29.603 | 30.053         | 30.508 | 30.553 | 31.513 | 32.087 | 32.696 | 33.346 | 20           |
| 21            | 31.521 | 32:010         | 32.504 | 33.031 | 33.591 | 34.211 | 34.869 | 35.565 | 21           |
| 22            | 33.490 | 34:020         | 34.557 | 35:129 | 35.741 | 36.411 | 37:117 | 37.854 | 22           |
| 23            | 35.513 | 36.058         | 36.675 | 37.299 | 37:966 | 38.656 | 39.434 | 40.208 | 23           |
| 24            | 37.598 | 35.222         | 35.564 | 39.547 | 40.266 | 41.030 | 41.S21 | 42.627 | 24           |
| 25            | 39.748 | 40.429         | 41.132 | 41.871 | 42.640 | 43.449 | 44.272 | 45·10S | 25           |
| 26            | 41.972 | 42.715         | 43.478 | 44.270 | 45.089 | 45.934 | 46.788 | 47.655 | 26           |
| 27            | 44.275 | 45.050         | 45.900 | 46.745 | 47.606 | 45.485 | 49.375 | 50.278 | 27           |
| 28            | 46.661 | 47.522         | 48.399 | 49.292 | 50.193 | 51.111 | 52.041 | 52.985 | 28           |
| 29            | 49.125 | 50.044         | 50.973 | 51.910 | 52.857 | 53.818 | 54.791 | 55.779 | 29           |
| 30            | 51.669 | 52.642         | 53.621 | 54.608 | 55.605 | 56.613 | 57.635 | 58.676 | 30           |

 $H^{M}$ 

### Values of Policies for 100.

3 PER-CENT.

| Dura. | 23     | 24     | 25     | 26     | 27     | 28     | 29     | 30     | Dura       |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|------------|
| tion  | 1,891  | 1,050  | 1.970  | 2.015  | 2.022  | 2.103  | 2.120  | 3,199  | tion       |
| 1     | 1.289  | 1.335  | 1.367  | 1.418  | 1.440  | 1.463  | 1.491  | 1.506  | 1          |
| 2     | 2.629  | 2.717  | 2.788  | 2.863  | 2.905  | 2.951  | 3.001  | 3.048  | 2          |
| 3     | 4.019  | 4.145  | 4.239  | 4.336  | 4.400  | 4.467  | 4.548  | 4.633  | 3          |
| 4     | 5.455  | 5.604  | 5.719  | 5.838  | 5.924  | 6.023  | 6.138  | 6.260  | 4          |
| 5     | 6.923  | 7.093  | 7.230  | 7.370  | 7.489  | 7.622  | 7.774  | 7.929  | 5          |
| 6     | 8.423  | 8.614  | 8.773  | 8.943  | 9.094  | 9.264  | 9.451  | 9.639  | 6          |
| 7     | 9.952  | 10.164 | 10.356 | 10.557 | 10.747 | 10.951 | 11.170 | 11.388 | 7          |
| 8     | 11.516 | 11.758 | 11.981 | 12.219 | 12.443 | 12.679 | 12.926 | 13.172 | 8          |
| 9     | 13.121 | 13:396 | 13.654 | 13.927 | 14.182 | 14.446 | 14.722 | 15.001 | 9          |
| 10    | 14.772 | 15.081 | 15.374 | 15.678 | 15.961 | 16.254 | 16.563 | 16.877 | 10         |
| 11    | 16.471 | 16.812 | 17.136 | 17.469 | 17:779 | 18.107 | 18.452 | 18.811 | 11         |
| 12    | 18.218 | 18.589 | 18.943 | 19:301 | 19.647 | 20.009 | 20.400 | 20.810 | 12         |
| 13    | 20:009 | 20.410 | 20.788 | 21.182 | 21.563 | 21.971 | 22.412 | 22.873 | 13         |
| 14    | 21.846 | 22.272 | 22.685 | 23.115 | 23.541 | 24.000 | 24.489 | 24.990 | 14         |
| 15    | 23.728 | 24.186 | 24.635 | 25.109 | 25.586 | 26.093 | 26.621 | 27.161 | 15         |
| 16    | 25.659 | 26.155 | 26.649 | 27.169 | 27.697 | 28.243 | 28.810 | 29.381 | 16         |
| 17    | 27.649 | 28.186 | 28.728 | 29.298 | 29.865 | 30.452 | 31.050 | 31.642 | 17         |
| 18    | 29.700 | 30.287 | 30.878 | 31.488 | 32.095 | 32.712 | 33.334 | 33.951 | 18         |
| 19    | 31.825 | 32.457 | 33.089 | 33.738 | 34.378 | 35.018 | 35.666 | 36.315 | 19         |
| 20    | 34.018 | 34.692 | 35.363 | 36.043 | 36.708 | 37.378 | 38.057 | 38.737 | 20         |
| 21    | 36.279 | 36.991 | 37.694 | 38.400 | 39.094 | 39.795 | 40.507 | 41.226 | 21         |
| 22    | 38.604 | 39.347 | 40.079 | 40.814 | 41.541 | 42.277 | 43.028 | 43.791 | 22         |
| 23    | 40.992 | 41.762 | 42.524 | 43.292 | 44.055 | 44.832 | 45.628 | 46.429 | 23         |
| 24    | 43.436 | 44.238 | 45.034 | 45.839 | 46.642 | 47.467 | 48.302 | 49.142 | 24         |
| 25    | 45.947 | 46.784 | 47.618 | 48.462 | 49.317 | 50.183 | 51.057 | 51.932 | 25         |
| 26    | 48.529 | 49.404 | 50.281 | 51.175 | 52.075 | 52.979 | 53.893 | 54.805 | 26         |
| 27    | 51.192 | 52.108 | 53.036 | 53.975 | 54.917 | 55.863 | 56.816 | 57.766 | 27         |
| 28    | 53.938 | 54.907 | 55.882 | 56.865 | 57.852 | 58.840 | 59.832 | 60.824 | 28         |
| 29    | 56.785 | 57.800 | 58.822 | 59.851 | 60.882 | 61.917 | 62.953 | 63.991 | <b>2</b> 9 |
| 30    | 59.729 | 60.792 | 61.863 | 62.939 | 64.020 | 65.103 | 66.190 | 67.274 | 30         |

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### VALUES OF POLICIES FOR 100.

3 PER-CENT.

| D             | 31             | 32             | 33     | 34     | 35     | 36     | 37     | 38     | Dune          |
|---------------|----------------|----------------|--------|--------|--------|--------|--------|--------|---------------|
| Dura-<br>tion | 5,540          | 2,301          | 2.356  | 2'413  | 2'473  | 2.535  | 2.600  | 2.669  | Dura-<br>tion |
| 1             | 1.538          | 1.577          | 1.607  | 1.644  | 1.677  | 1.719  | 1.754  | 1.779  | 1             |
| 2             | 3.118          | 3.191          | 3.258  | 3.332  | 3.397  | 3.468  | 3.544  | 3.611  | 2             |
| 3             | 4.738          | 4.848          | 4.949  | 5.052  | 5.146  | 5.257  | 5.374  | 5.495  | 3             |
| 4             | 6.399          | 6.545          | 6.676  | 6.807  | 6.938  | 7.088  | 7.258  | 7:437  | 4             |
| 5             | 8.103          | 8.275          | 8.435  | 8.603  | 8.775  | 8.975  | 9.199  | 9.436  | 5             |
| 6             | 9.842          | 10.042         | 10.236 | 10.445 | 10.664 | 10.920 | 11.201 | 11.486 | 6             |
| 7             | 11.617         | 11.851         | 12.083 | 12.341 | 12.614 | 12.923 | 13.251 | 13.580 | 7             |
| 8             | 13.435         | 13.707         | 13.987 | 14.296 | 14.623 | 14.979 | 15.347 | 15.708 | l s           |
| 9             | 15.300         | 15.620         | 15.951 | 16.313 | 16.683 | 17.079 | 17:480 | 17:864 | 9             |
| 10            | 17.223         | 17.593         | 17.975 | 18.380 | 18.793 | 19.218 | 19.642 | 20.055 | 10            |
| 11            | 19.208         | 19.628         | 20.053 | 20.495 | 20.938 | 21.388 | 21.838 | 22.283 | 11            |
| 12            | 21.256         | 21.717         | 22.181 | 22.653 | 23.119 | 23.595 | 24.075 | 24.553 | 12            |
| 13            | 23.357         | 23.857         | 24.351 | 24.846 | 25.338 | 25.842 | 26.354 | 26.872 | 13            |
| 14            | 25.513         | 26.042         | 26.558 | 27.078 | 27.598 | 28.135 | 28.684 | 29.248 | 14            |
| 15            | 27.714         | 28.264         | 28.805 | 29.356 | 29.907 | 30.480 | 31.074 | 31.675 | 15            |
| 16            | 29.955         | 30.530         | 31.103 | 31.682 | 32.272 | 32.887 | 33.518 | 34.150 | 16            |
| 17            | $32 \cdot 242$ | 32.848         | 33.449 | 34.067 | 34.698 | 35.350 | 36.011 | 36.673 | 17            |
| 18            | 34.581         | 35.216         | 35.856 | 36.515 | 37.185 | 37.868 | 38.559 | 39.246 | 18            |
| 19            | 36.977         | 37.649         | 38.330 | 39.025 | 39.727 | 40.440 | 41.154 | 41.867 | 19            |
| 20            | 39.436         | 40.151         | 40.869 | 41.597 | 42.326 | 43.065 | 43·S05 | 44.541 | 20            |
| 21            | 41.968         | 42.720         | 43.471 | 44.229 | 44.985 | 45.750 | 46.516 | 47.275 | 21            |
| 22            | 44.571         | 45.355         | 46.139 | 46.924 | 47.711 | 48.499 | 49.290 | 50.067 | 22            |
| 23            | 47.244         | 48.059         | 48.874 | 49.689 | 50:500 | 51.317 | 52.126 | 52.921 | 23            |
| $^{24}$       | 49.991         | 50.838         | 51.682 | 52.526 | 53.367 | 54.206 | 55.033 | 55.844 | 24            |
| 25            | 52.814         | 53.693         | 54.569 | 55.445 | 56.311 | 57:171 | 58.018 | 58.842 | 25            |
| 26            | 55.721         | 56.633         | 57.546 | 58.450 | 59.341 | 60.224 | 61.088 | 61.929 | 26            |
| 27            | 58.718         | 59.669         | 60.614 | 61.546 | 62.467 | 63:373 | 64.260 | 65.128 | 27            |
| 28            | 61.818         | $62 \cdot 805$ | 63.782 | 64.750 | 65.701 | 66.634 | 67.554 | 68:460 | 28            |
| 29            | 65.025         | 66.052         | 67.068 | 68.071 | 69.059 | 70.034 | 71.000 | 71.957 | 29            |
| 30            | 68.353         | 69.424         | 70.484 | 71.532 | 72.569 | 73.600 | 74.626 | 75.650 | 30            |

 $H_M$ 

### Values of Policies for 100.

3 PER-CENT.

| _             | 39     | 40     | 41     | 42     | 43     | 44     | 45     | 46     | Dura-     |
|---------------|--------|--------|--------|--------|--------|--------|--------|--------|-----------|
| Dura-<br>tion | 2.740  | 2.816  | 2.896  | 2.085  | 3.014  | 3'172  | 3'276  | 3*387  | tion      |
| 1             | 1.834  | 1.879  | 1.950  | 2.017  | 2.076  | 2.135  | 2.188  | 2.223  | 1         |
|               | 3.717  | 3.828  | 3.962  | 4.083  | 4.197  | 4.301  | 4.389  | 4.470  | $\hat{2}$ |
| $\frac{2}{3}$ | 5.662  | 5.832  | 6.019  | 6.191  | 6.348  | 6.485  | 6.618  | 6.744  | 3         |
| 4             | 7.662  | 7.884  | 8.119  | 8.332  | 8.519  | 8.696  | 8.873  | 9.053  | 4         |
| 5             | 9.711  | 9.981  | 10.250 | 10.495 | 10.717 | 10.937 | 11.162 | 11.399 | 5         |
| 6             | 11.805 | 12.108 | 12.408 | 12.684 | 12.950 | 13.214 | 13.491 | 13.791 | 6         |
| 7             | 13.931 | 14.262 | 14.594 | 14.907 | 15.212 | 15.527 | 15.869 | 16.223 | 7         |
| 8             | 16.086 | 16.446 | 16.813 | 17:164 | 17.519 | 17.894 | 18.284 | 18.688 | 8         |
| 9             | 18.273 | 18.665 | 19.069 | 19.465 | 19.876 | 20.298 | 20.733 | 21.182 | 9         |
| 10            | 20.495 | 20.923 | 21.368 | 21.818 | 22.275 | 22.738 | 23.216 | 23.701 | 10        |
| 11            | 22.755 | 23.226 | 23.721 | 24.216 | 24.709 | 25.211 | 25.724 | 26.245 | 11        |
| 12            | 25 065 | 25.582 | 26.118 | 26.648 | 27:179 | 27.714 | 28.259 | 28.811 | 12        |
| 13            | 27.429 | 27.987 | 28.557 | 29.121 | 29.681 | 30.245 | 30.820 | 31.406 | 13        |
| 14            | 29.844 | 30.433 | 31.035 | 31.628 | 32.216 | 32.809 | 33.414 | 34.020 | 14        |
| 15            | 32.301 | 32.923 | 33.550 | 34.170 | 34.784 | 35.406 | 36.028 | 36.647 | 15        |
| 16            | 34.805 | 35.452 | 36.108 | 36.750 | 37.390 | 38.028 | 38.663 | 39.293 | 16        |
| 17            | 37:353 | 38.026 | 38.703 | 39.370 | 40.026 | 40.674 | 41.316 | 41.949 | 17        |
| 18            | 39 949 | 40.643 | 41.342 | 42.025 | 42.691 | 43.347 | 43.991 | 44.619 | 18        |
| 19            | 42.591 | 43.310 | 44.025 | 14.718 | 45.388 | 46.041 | 46.687 | 47:319 | 19        |
| 20            | 45.289 | 46.024 | 46.749 | 47:447 | 48.121 | 4S.773 | 49.414 | 50.057 | 20        |
| 21            | 48.039 | 48.785 | 49.514 | 50.215 | 50.890 | 51.545 | 52.198 | 52.851 | 21        |
| 22            | 50.844 | 51.599 | 52.332 | 53.035 | 53.713 | 54.378 | 55.042 | 55.718 | 22        |
| 23            | 53.710 | 54.469 | 55.207 | 55.915 | 56.604 | 57.288 | 57.973 | 58.676 | 23        |
| $^{24}$       | 56.642 | 57.407 | 58.156 | 58.878 | 59.588 | 60.291 | 61.009 | 61.734 | $^{24}$   |
| 25            | 59.649 | 60.432 | 61.196 | 61.944 | 62.681 | 63:419 | 64.160 | 64.887 | 25        |
| 26            | 62.757 | 63.564 | 64.357 | 65.135 | 65.910 | 66.680 | 67.433 | 68.156 | 26        |
| 27            | 65.983 | 66.826 | 67:657 | 68.483 | 69.296 | 70.085 | 70.846 | 71.559 | 27        |
| 28            | 69.357 | 70.246 | 71.132 | 72.004 | 72.850 | 73.665 | 74.431 | 75.136 | 28        |
| 29            | 72.910 | 73.862 | 74.801 | 75.724 | 76.611 | 77.451 | 78.234 | 78.958 | 29        |
| 30            | 76.678 | 77.700 | 78.706 | 79.685 | 80.623 | 81.510 | 82.345 | 83.159 | 30        |

 $H_M$ 

## Values of Policies for 100.

3 PER-CENT.

## Premiums limited to 30 Annual Payments.

| Dura- | 47     | 48     | 49     | 50            | 51     | 52     | 53     | 54     | Dura |
|-------|--------|--------|--------|---------------|--------|--------|--------|--------|------|
| tion  | 3.204  | 3.627  | 3.758  | 3.898         | 4,048  | 4.510  | 4,384  | 4.211  | tion |
| 1     | 2.262  | 2:324  | 2:393  | <b>2</b> ·463 | 2:541  | 2.633  | 2.708  | 2.783  | 1    |
| 2     | 4.562  | 4.682  | 4.816  | 4.969         | 5.120  | 5.287  | 5.441  | 5.587  | 2    |
| 3     | 6.891  | 7.075  | 7.285  | 7.511         | 7.730  | 7:967  | 8.187  | 8.409  | 3    |
| 4     | 9.259  | 9.516  | 9.793  | 10.079        | 10.364 | 10.659 | 10.947 | 11.238 | 4    |
| 5     | 11.673 | 11.990 | 12.325 | 12.674        | 13.015 | 13:374 | 13.723 | 14.081 | 5    |
| 6     | 14.124 | 14.497 | 14.884 | 15.286        | 15.683 | 16.094 | 16.511 | 16.924 | 6    |
| 7     | 16.607 | 17:030 | 17:467 | 17.918        | 18:365 | 18.836 | 19.300 | 19.759 | 7    |
| 8     | 19.118 | 19.585 | 20.066 | 20.564        | 21.064 | 21.579 | 22.080 | 22.580 | 8    |
| 9     | 21.655 | 22.161 | 22.656 | 23.226        | 23.763 | 24.316 | 24.852 | 25.382 | I 9  |
| 10    | 24.211 | 24.756 | 25.322 | 25.897        | 26.466 | 27.042 | 27.604 | 28.158 | 10   |
| 11    | 26.790 | 27:374 | 27.969 | 28.570        | 29.161 | 29.756 | 30.336 | 30.918 | 11   |
| 12    | 29.394 | 30.003 | 30.619 | 31.239        | 31.844 | 32.455 | 33.056 | 33.672 | 12   |
| 13    | 32.013 | 32.638 | 33.272 | 33.901        | 34.516 | 35.142 | 35.775 | 36.431 | 13   |
| 14    | 34.641 | 35.279 | 35.922 | 36.555        | 37.185 | 37.832 | 38.497 | 39.201 | 14   |
| 15    | 37.282 | 37.924 | 38.567 | 39.210        | 39.860 | 40.533 | 41.236 | 41.998 | 1 15 |
| 16    | 39.926 | 40.572 | 41.220 | 41.878        | 42.550 | 43.258 | 44.008 | 41.798 | 16   |
| 17    | 42.582 | 43.230 | 43.889 | 44.569        | 45.266 | 46.017 | 46.797 | 47.591 | 17   |
| 18    | 45.255 | 45.911 | 46.588 | 47.288        | 48.027 | 48.798 | 49.579 | 50.348 | 18   |
| 19    | 47.965 | 48.635 | 49.329 | 50.066        | 50.819 | 51.589 | 52.338 | 53.046 | 19   |
| 20    | 50.715 | 51.404 | 52.131 | 52.881        | 53.631 | 51.366 | 55.056 | 55.675 | 20   |
| 21    | 53.529 | 54.249 | 54.987 | 55.727        | 56.444 | 57.123 | 57.722 | 58.247 | 21   |
| 22    | 56.425 | 57.153 | 57.887 | 58.597        | 59.256 | 59.845 | 60.350 | 60.848 | 22   |
| 23    | 59.399 | 60.125 | 60.827 | 61.484        | 62.063 | 62.562 | 63.011 | 63.500 | 23   |
| 24    | 62.454 | 63.157 | 63.813 | 64.394        | 64.890 | 65.368 | 65.814 | 66.244 | 24   |
| 25    | 65.595 | 66.256 | 66.850 | 67.365        | 67.846 | 68.293 | 68.714 | 69.133 | 25   |
| 26    | 68.836 | 69.450 | 69.959 | 70.498        | 70.961 | 71.399 | 71.818 | 72.218 | 26   |
| 27    | 72.207 | 72.788 | 73.339 | 73.846        | 74.313 | 74.763 | 75.188 | 75.561 | 27   |
| 28    | 75.775 | 76.384 | 76.960 | 77.496        | 78.003 | 78.453 | 78.908 | 79.255 | 28   |
| 29    | 79.655 | 80.319 | 80.953 | 81.563        | 82.138 | 82.664 | 83.122 | 83.219 | 29   |
| 30    | 83.946 | 84.711 | 85.461 | 86.187        | 56.874 | 87.506 | 88.090 | 88.617 | 30   |

 $H^{M}$ 

## VALUES OF POLICIES FOR 100.

3 PER-CENT.

# Premiums limited to 30 Annual Payments.

| Dura-  | 55     | 56     | 57     | 58     | 59     | 60     | Dura |
|--------|--------|--------|--------|--------|--------|--------|------|
| tion   | 4.220  | 4.984  | 5,513  | 5.459  | 5.455  | 6.005  | tion |
| 1      | 2.875  | 2.961  | 3.047  | 3.139  | 3.222  | 3.315  | 1    |
| 2<br>3 | 5.759  | 5.923  | 6.105  | 6.270  | 6.435  | 6.607  | 2    |
| 3      | 8.655  | 8.904  | 9.154  | 9.390  | 9.629  | 9.864  | 3    |
| 4      | 11.559 | 11.877 | 12.193 | 12.492 | 12.789 | 13.085 | 4    |
| 5      | 14.465 | 14.835 | 15.209 | 15.566 | 15.914 | 16.276 | 5    |
| 6      | 17:357 | 17.780 | 18.203 | 18.607 | 19.011 | 19.444 | 6    |
| 7      | 20.236 | 20.699 | 21.164 | 21.620 | 22.091 | 22.607 | 7    |
| 8      | 23.089 | 23.591 | 24.099 | 24.616 | 25.165 | 25.767 | 8    |
| 9      | 25.921 | 26.462 | 27.022 | 27.608 | 28.232 | 28.938 | 9    |
| 10     | 28.733 | 29.317 | 29.940 | 30.597 | 31.320 | 32.091 | 10   |
| 11     | 31.533 | 32.172 | 32.856 | 33.601 | 34.390 | 35.210 | 11   |
| 12     | 34.334 | 35.030 | 35.792 | 36.594 | 37.422 | 38.247 | 12   |
| 13     | 37.142 | 37.907 | 38.722 | 39.554 | 40.381 | 41.166 | 13   |
| 14     | 39.973 | 40.784 | 41.626 | 42.149 | 43.230 | 43.937 | 14   |
| 15     | 42.807 | 43.634 | 44.466 | 45.243 | 45.948 | 46.561 | 15   |
| 16     | 45.621 | 46.432 | 47.213 | 47.910 | 48.522 | 49.140 | 16   |
| 17     | 48.389 | 49.149 | 49.841 | 50.448 | 51.059 | 51.655 | 17   |
| 18     | 51.092 | 51.761 | 52.360 | 52.960 | 53.539 | 54.125 | 18   |
| 19     | 53.701 | 54.280 | 54.863 | 55.428 | 55.992 | 56.579 | 19   |
| 20     | 56.234 | 56.789 | 57:339 | 57.885 | 58.447 | 59.004 | 20   |
| 21     | 58.780 | 59.295 | 59.819 | 60.355 | 60.890 | 61.372 | 21   |
| 22     | 61.341 | 61.834 | 62:341 | 62.844 | 63.299 | 63.648 | 22   |
| 23     | 63.961 | 64.438 | 64.913 | 65.344 | 65.665 | 65.895 | 23   |
| 24     | 66.691 | 67.133 | 67.539 | 67.843 | 68.057 | 68.126 | 24   |
| 25     | 69.554 | 69.933 | 70.225 | 70.136 | 70.513 | 70.492 | 25   |
| 26     | 72.588 | 72.878 | 73.092 | 73.190 | 73.197 | 73.150 | 26   |
| 27     | 75.867 | 76.098 | 76.236 | 76.298 | 76.309 | 76.331 | 27   |
| 28     | 79.552 | 79.750 | 79.899 | 80.015 | 80.127 | 80.226 | 28   |
| 29     | 83.847 | 84.126 | 84.388 | 84.659 | 84.941 | 85.299 | 29   |
| 30     | 89.110 | 89.601 | 90.118 | 90.663 | 91.301 | 92.020 | 30   |

### THE INSTITUTE OF ACTUARIES.

### REVISED RULES AND SYLLABUS OF EXAMINATIONS.

## REGULATIONS FOR THE CLASS OF PROBATIONERS.

The question of the Examinations of the Institute has for some time past engaged the attention of the Council, with the result that new Rules, and a new Syllabus have been adopted, together with Regulations for a new Class, to be styled *Probationers*.

Experience has shown that the intention of the framers of the Charter has been frustrated by some persons, who have been elected Students, publicly using the title "Member of the Institute of Actuaries." Clause 36 of the Charter provides that "a Fellow may use after his name the initials F.I.A., and an Associate may use after his name the initials A.I.A."; and the Council of the Institute desire it to be understood that they strongly disapprove of any other title being used by Members, unless at the same time the class of membership be mentioned.

To prevent any misuse of the privileges of the Institute in future, by those who have no intention of passing the Examinations, it has been decided that henceforth a candidate for admission as a Student must pass Part I of the Syllabus of Examinations.

In order, however, that those who bond fide desire to pass the Examinations of the Institute may enjoy all the advantages which they have hitherto possessed, the new Class called Probationers has been established. A Probationer will not be a Member of the Institute, but will be entitled to join the Classes for Students, to avail himself of the Library, and to attend the Ordinary General Meetings. He will also, on passing Part I of the Syllabus, be at once eligible for election as a Student of the Institute. The entrance fee and the annual subscription for this Class are half those paid by Students.

These are the principal changes introduced by the new Rules. There are also minor alterations which it is unnecessary to recapitulate.

With regard to the Syllabus, Part I has been remodelled, by excluding Euclid, and by including the Elements of the Calculus of Finite Differences, so as to introduce the student early to the

special subjects with which he will meet in the course of his work.

Part II has been generally restricted in its scope to the subjects of the *Text-Book*, but excluding Graduation and some of the more advanced portions. The elementary application of the Calculus of Finite Differences, and of the Differential and Integral Calculus, to Life Contingencies, has been inserted, as a stepping-stone to the advanced portions of the theoretical work covered by Part III; while all subjects of a general nature, such as Law and Finance, have been relegated to Part IV.

Parts III and IV take the place of Part III, Sections A and B, of the old Syllabus, and have been arranged on the broad plan of including in Part III advanced Special or Actuarial subjects, and in Part IV Legal, Financial, and other general matters.

To render the change of Syllabus as little inconvenient to Students as possible, its application is deferred, and will be gradual. Part I will come into force in 1900, Part II in 1901, and Parts III and IV in 1902. Meantime the Examinations will be held under the old Syllabus.

The revised Rules of Examination, and the new Syllabus, are appended, as also the Regulations for the Class of Probationers.

Ed. J.I.A.

RULES prescribed by the Council of the Institute to regulate Examinations qualifying for admission to the Classes of Students, Associates, and Fellows respectively.

#### GENERAL REGULATIONS.

- 1. These Rules shall come into force as from the 1st October 1898.
- 2. The Examinations shall, until further notice, be conducted in writing, under such conditions as the Examination Committee may prescribe, subject to the approval of the Council.

3. The Examiners shall place successful Candidates in three Classes, according to merit, and the names in each Class shall be

arranged in alphabetical order.

4. No Candidate will be allowed to present himself for Examination until he has paid all Subscriptions, Entrance Fees, and Examination Fees that may be due.

## Examinations in the United Kingdom.

5. Examinations will be held in April of each year, or at such other time as the Examination Committee may prescribe.

At least one month's public notice will be given of the days and

hours when, and of the place or places where, the Examinations will

take place.

6. Candidates for any Examination must give fourteen days' notice in writing, addressed to the Honorary Secretaries, of their intention to present themselves for Examination, specifying the particular Examination for which they intend to present themselves.

#### Examinations in the Colonies.

7. Examinations will be held in April of each year, or at such

other time as the Examination Committee may prescribe.

Public notice will be given, not later than in November in each year, by advertisement in at least one newspaper of the Colony in which an Examination is to be held, of the date and place of such Examination.

8. Candidates for any Examination in a Colony must give notice in writing, addressed to the Honorary Secretaries, of their intention to present themselves for Examination, specifying the particular Examination for which they intend to present themselves. Such notice must be posted in time to reach the Honorary Secretaries in London not later than the 31st January preceding the date of the Examination.

#### STUDENTS.

- 1. Candidates for admission to the Class of Students must, at the time of giving notice of their intention to present themselves for Examination, pay an Entrance Fee and an Examination Fee, each of £1.1s.
- 2. Candidates who have paid the above Fees, and whose applications for admission have been approved by the Council, shall, unless exempted under the next clause, be examined in the subjects set forth in Part I of the annexed Syllabus.

3. Graduates in Mathematical Honours of any University in the United Kingdom will be exempt from Examination in Part I of the

Syllabus.

4. In the event of a Candidate passing the Examination, he will be admitted a Student after he has signed the Form of Obligation prescribed by the Council, and paid his Subscription of £1. 1s. for the current year.

5. In the event of a Candidate failing to pass the Examination, he shall, subject to the approval of the Council, and on payment of the Examination Fees, be permitted to be examined in a subsequent

year.

N.B.—At the Examination in Part I in April 1899 the Syllabus hitherto in force, and dated 9 January 1894, will apply.

#### Associates.

1. Students shall be required to pass Parts I and II of the Syllabus to qualify for admission to the Class of Associates, and, subject to the exceptions contained in these Rules, no Student shall present himself for Examination in the second part unless he has previously succeeded in passing the first part of the said Examination.

- 2. Candidates for Examination in either Part must, at the time of giving notice of their intention to present themselves, pay a fee of £1. 1s.
- 3. Students of the Institute admitted after 1st October 1898, shall be required to pass only Part II of the Syllabus to qualify for admission to the class of Associates.
- 4. Fellows of the Faculty of Actuaries by Examination will be considered by the Council, other circumstances in their view being favourable, to be eligible under Bye-Law No. 44, as Associates, without passing the Examinations for admission to the Class of Associates.
  - N.B.—At the Examination in Part II in April 1899 and April 1900 the Syllabus hitherto in force, and dated 9 January 1894, will apply.

#### Fellows.

I. Students or Associates shall be required to pass Parts I, II, III and IV of the Syllabus to qualify for admission to the Class of Fellows, subject to the exceptions contained in these Rules. No Candidate shall present himself for Parts III and IV until the expiration of at least 12 months after passing Part II.

2. Candidates may take Parts III and IV in the same year, or they may take Part III in one year and postpone Part IV to a subsequent year. In the event of a Candidate passing in Part IV but failing in Part III he may present himself again in Part III in

a subsequent year.

3. Candidates for Examination in Parts III and IV must, at the time of giving notice of their intention to present themselves, pay

a fee of £1.1s. in respect of each part for which they enter.

4. Associates who, prior to the commencement of the Bye-Laws (26th day of February 1886), had passed Part I or Parts I and II, shall be required to pass only Parts II, III, and IV. or only Parts III and IV, respectively, of the Examination to qualify for admission to the Class of Fellows.

5. Associates admitted after the 26th February 1886, shall be required to pass only Parts III and IV of the Examination to qualify for admission to the Class of Fellows; and Students admitted after 1st October 1898 shall be required to pass only Parts II, III, and IV of the Examination to qualify for admission to the Class of Fellows.

N.B.—At the Examination in Parts III and IV in April 1899, April 1900, and April 1901, the Syllabus hitherto in force, and dated 9 January 1894, will apply; and thereafter Part III, Section A, of such Syllabus shall be held to be equivalent to Part IV, and Part III. Section B, to Part III of this Syllabus.

By order of the Council, 11 October 1898.

A. F. BURRIDGE, Hon. Secs.

#### SYLLABUS OF EXAMINATIONS

Referred to in the annexed Rules.

#### Part I.

(1) Arithmetic and Algebra.

(2) The Theory and use of Logarithms.(3) The Elements of the Theory of Probabilities.

(4) The Elements of the Calculus of Finite Differences, including Interpolation and Summation.

#### PART II.

(1) Compound Interest and Annuities-Certain.

(2) The application of the Theory of Probabilities of Life Contingeneies.

(3) The Theory of Annuities and Assurances on Lives and Survivor-

- (4) The principles of the construction of Mortality Tables (excluding graduation); and the construction of monetary and other Tables involving the Contingencies of Life.
- (5) The elementary application of the Calculus of Finite Differences and of the Differential and Integral Calculus to Life Contingencies.

#### Part III.

(1) The methods of constructing and graduating Mortality, Sickness and other Tables.

(2) The history and distinctive features of existing Tables.

(3) The Valuation of the Liabilities and Assets of Life Assurance Companies.

(4) The Distribution of Surplus.

- (5) The Calculation of Office Rates for Assurance, Annuity, Sickness and other risks.
- (6) The practical valuation of Life Interests and Reversions, and of Policies for surrender or purchase.

#### PART IV.

(1) The elements of the Law of Real and Personal Property.

- (2) The Law relating to Life Assurance Companies and Life Assurance Contracts.
- (3) The Constitution and Valuation of Friendly Societies and Pension Funds, and the Laws relating to such Institutions.
- (4) Life Assurance Book-keeping; prejuration of Schedules, Statements and Reports.
- (5) The Principles of Banking and Finance, including a knowledge of the Constitution and Operations of the Bank of England, and of the National and Local Debts of the United Kingdom.
- (6) The Investments of Life Assurance Companies.

#### REGULATIONS FOR THE CLASS OF PROBATIONERS.

For the benefit of persons who desire to qualify themselves for admission to the Institute, it has been decided by the Council to establish a Class of Probationers who, while not being Members of the Institute, shall be allowed some of the privileges of the Members.

A person desiring to become a Probationer shall apply to the Council, and. if his application be approved, shall become a Probationer on payment of a fee of 10s. 6d., but the Council may at any time withdraw their approval, and thereupon the person shall eease to be a Probationer. Should the Probationer subsequently be admitted to the Institute, this fee of 10s. 6d. will be taken as paid on account of the entrance fee.

The annual Subscription for Probationers is 10s. 6d., payable in advance on 1st October in each year. If the subscription for any year be not paid before the 31st December, then the defaulter shall no longer be a Probationer.

Probationers will be entitled to join the Classes for Students, in accordance with the rules prescribed for such Classes, and to attend the Ordinary General Meetings of the Institute, but not to vote or take part in the discussions thereat.

Probationers may borrow books from the Library for the purposes of their studies, but this privilege is subject to the discretion of the Librarians and to the rules which they may from time to time prescribe.

BY ORDER OF THE COUNCIL.

#### LECTURES.

The Lectures on the Law of Real Property, and on the London Daily Stock and Share List, having proved so acceptable to the Members of the Institute, the Council have arranged for a Third Course, on the Companies Acts, as follows:

Syllabus of a Course of Six Lectures, by Mr. A. C. Clauson, Barrister-at-Law, on "The Companies Acts",

TO BE DELIVERED BEFORE THE MEMBERS OF THE INSTITUTE

On Monday, 9 and 23 January 1899, at 5.30 p.m.

6 ,, 20 FEBRUARY 1899, ,

6 ,, 20 MARCH 1899,

#### INTRODUCTORY.

Association in (a) partnerships, (b) corporations created by Royal Charter—Relation of "partnerships" and "companies"—"The

Bubble Act, 1719"—" Parliamentary" Companies, with quasi-public objects—Special Acts extending privileges to Companies with private objects—General Acts (1826, 1834) extending privileges to Companies with private objects—" Letters Patent" Companies: introduction of limited liability (1837)—Formation by registration (with or without limited liability) involving (a) incorporation, (b) easy transfer of rights, or shares. (c) easy transfer of liabilities, (d) limitation of objects (intra and ultra vires).

#### FORMATION OF COMPANY.

Registration of memorandum of association referring to (a) objects, (b) extent of liability, (c) capital—Registration of companies formed under previous legislation (Act of 1862, part vii): deed of settlement: supersession of deed by memorandum ("Memorandum of Association Act, 1890")—Certificate of incorporation—Deposit on formation of insurance Company—Objects: alteration under "Memorandum of Association Act, 1890"—Liability: extent and enforcement (Act of 1862, s. 38): past members: limitation by guarantee: mixed liability: contractual limitation in insurance Companies—Capital: Act of 1862, part ii: meanings of word: underwriting: transferability of shares (sec. 22): subdivision and consolidation: stock: share warrants—Members: definition of (sec. 23): register of (ss. 25. 33, 35)—"Colonial Registers Act, 1883"—Trusts of Shares (sec. 30)—Certificate (sec. 31) as evidence of shares held—Promotion of Companies: promoters: prospectus (Act of 1867, sec. 38)—"Monopoly" of Companies Acts (Act of 1862, sec. 4).

#### MANAGEMENT AND ADMINISTRATION OF COMPANY.

Articles or regulations: alteration (Act of 1862, ss. 50, 51): Table A-Registered Office-Name of Company: Act of 1862, ss. 20, 41, 42-Use of word "limited": Act of 1867, s. 23-Periodical statements and returns-Registers: inspection of registers -Examination of affairs by Board of Trade inspectors-Meetings: Act of 1862, sec. 49: of 1867, sec. 39: notice of meeting: proxies: resolutions: minutes: rights and remedies of minority—Shares: application and allotment: issue as fully paid (Act of 1867, sec. 25): issue at discount: joint holdings: devolution on death: calls: lien: forfeiture: surrender: purchase by company—Transfer of Shares: restrictions on transfer: certification: certificates—Preferred and Deferred Shares: alteration of rights attached to shares—Increase and reduction of capital—Directors: qualification: election: powers: "intra and ultra vires": how far trustees: "Directors' Liability Act, 1890"-Form of contracts: contracts under seal, in writing: signature by directors: "Companies Seals Act"—Borrowing powers: securities on uncalled capital, on chattels: debentures, bearer and registered: debenture stock: registration of mortgages and charges— Dividends: capital and income: wasting assets: dividends not payable out of capital: interim dividends: preference dividends, cumulative and non-cumulative-Audit and accounts-Insurance companies: reports, abstracts and statements ("Life Assurance Companies Acts, 1870, 1872"): "Payment into Court Act. 1896."

#### WINDING-UP.

Voluntary winding-up: Act of 1862, sec. 129: liquidation controlled by members—Winding-up under supervision: Act of 1862, sec. 147: Court can be set in motion by creditor—Winding-up under order of Court ("compulsory"): "Winding-up Act of 1890": Official Receiver and Board of Trade—Petition for winding-up order: "paying off" and "substituting" petitioner: grounds for petition: qualification of petitioner: "Life Assurance Companies Act, 1870", sec. 21; 1872, sec. 4: "wishes of creditors and contributories", Act of 1862, sec. 91—Effect of winding-up on (a) business, (b) liabilities, (c) actions, distresses, &c.—Fraudulent preference: dealings with property pending winding-up proceedings—Secured creditors in winding-up: "majority" clause in debenture trust deeds—Proof of debts: Act of 1862, sec. 158—Estimate of liabilities: "Hardy v. Fothergill": "Life Assurance Companies Act, 1872", sec. 5—Set off: debt and call—Misfeasance of directors and officers: public examinations—Surplus assets.

#### RECONSTRUCTION AND AMALGAMATION.

Reconstruction: (a) without winding-up: "Cotton v. Imperial Agency": "Life Assurance Companies Act, 1870", sec. 14: reduction of capital: Acts of 1867 and 1877; (b) winding-up, without formation of new company: Act of 1862, ss. 89, 136, 159, 160: "Joint Stock Companies Arrangement Act 1870"; (c) winding-up, and transfer to another Company: Act of 1862, ss. 161, 162: "Life Assurance Companies Act, 1870", sec. 14: "Joint Stock Companies Arrangement Act, 1870; (d) Life Assurance Companies: (i) amalgamation: "Life Assurance Companies Act, 1870", sec. 14; (ii) reduction of contracts: "Life Assurance Companies Act, 1870", sec. 22—Novation on amalgamation of companies: "Life Assurance Companies Act, 1872", sec. 7.

#### CONCLUSION.

Projected reforms as to (a) Companies generally, (b) Life Assurance Companies—Extension of principle of limited liability: "one man" companies: fraudulent and quasi-fraudulent use of limited liability.

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES.

Debentures of Trading Companies as Investments for Life Assurance Funds, considered in their Legal and Financial Aspects. By ARTHUR RHYS BARRAND, F.I.A., of the Prudential Assurance Company.

[Read before the Institute, 19 December 1898.]

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m THE}$  subject I have chosen for my paper is one which needs no apology for its introduction to this Institute, closely connected as it is with one of the most pressing problems which those responsible for the management of a life assurance office have to deal with at the present time. It is, no doubt, difficult, in the face of the eager competition which has to be encountered, to increase the business of an office; but it is scarcely less difficult to satisfactorily invest the surplus arising from the premiums when they are obtained. One is reminded of the picture which appeared a few years ago in Punch, in which a Dean, importuned for alms by a beggar, exclaims, "Ah, my good "woman, it is not the poor only who have their troubles; you, " for instance, have probably never experienced the difficulty of "finding investments, combining adequate security with a "remunerative rate of interest"! It is probable that she had not, but it is equally probable that the actuary of every assurance company in this country has already experienced the difficulty, and is likely to experience it increasingly in the future. The investments which, in the past, have found favour with life

assurance companies, have not increased in proportion to the increase of the funds to be invested. The comparatively rapid reduction of the National Debt which has taken place in recent years has not only had the effect of diminishing the amount of that debt available for investment purposes, but, in combination with conversion schemes, has lowered the rate of interest to such an extent as to render that debt altogether unsuitable for the investment of life assurance funds. The same cause has also affected other securities, for trustees, with the wider powers given to them by modern legislation, and deterred by the prohibitive price of Consols, have entered into competition for those other securities which in the past have furnished safe and profitable investments for life offices. It would be foreign to my purpose to enter now into a minute discussion of the various causes of the decline in the rate of interest which all assurance companies have now to face, and as to the probability of a continuance of such decline. I have indicated one cause, and this, together with the great increase in the wealth of the country in recent years, and the rapidly growing funds of life assurance companies themselves, would seem to be mainly responsible for this decline. changed character of life assurance business has accentuated the difficulty arising from the increase of assurance funds, for the great increase in the proportion of endowment assurances to whole life assurances, means that a much larger proportion of the premium income has to be invested than was the case a few years ago; and although this tendency will ultimately tend to correct itself as far as the investments are concerned, yet it will affect them in the manner referred to for many years to come, As to the fact of the decline there can be no dispute, and it seems probable that the extent of the fall in the rate of interest is in reality greater than at first sight appears.

In an office whose funds are growing but slowly from year to year, the low rate yielded by new investments is to a great extent masked by the rate still realized on the old investments, these latter frequently showing an artificial rate by reason of the fact that the securities comprised in them were purchased at low prices many years ago, and are still maintained in the balance sheet at those prices. The truth of this view of the extent to which the rate of interest has fallen can be seen to some extent from the fact that several companies having large funds to invest from year to year, have only succeeded in preventing a very appreciable fall in the rate realized on their assurance fund, by

investing in securities of a totally different nature from those that have found favour in the past; and where such precautions have not been adopted, the result is to be seen in an extremely low rate of interest.

The conditions under which life assurance business can be carried on most successfully have undergone considerable change in many directions during recent years, but perhaps in no direction has the change been more marked than in the region of investments. If we take what may be considered as typical investments for life assurance funds, of a few years ago, such as life interests and reversions, mortgages, and railway debenture stock, we find that the fall in the rate of interest realized on such securities during the past 30 years has been about 1½ per-cent. changed eireumstances in respect to the acquisition of business have been fully recognized, and every device and expedient that keen business minds could suggest for meeting new surroundings with new methods have been brought into service. It is therefore not a little singular that this willingness on the part of companies to adapt their methods to their changed environment, which has been such a prominent characteristic in connection with the extension of life assurance business, should have penetrated so slowly into the investment side of the offices. It is true that quite recently, there has been a tendency on the part of many companies who have been restricted by their constitution in the matter of investments, to take more extensive powers in this direction, and a few large companies have shown considerable enterprise in the matter of new investments. Speaking generally, however, I think it will be admitted that an undue spirit of conservatism has characterized the investments of life assurance companies. The theory of evolution by natural selection, which has exercised such a deep and widespread influence on every aspect of life and thought, seems to have made little headway here, although it is as true of life assurance companies as of animal or vegetable nature, that any organism which cannot or does not adapt itself to a changed environment, will fail in the struggle for life, as compared with an organism which is capable of such adaptation, and does actually adapt itself to its changed circumstances. It is true that such failure may be slow in manifesting itself in the case of an assurance company, even as it is slow in revealing its effects in the natural world, but it is none the less sure; and life assurance officials should be the last people in the world to be satisfied with the present, and to shut their

eyes to the fact that the institution under their care is drifting towards difficulties that might easily be averted by the adoption of measures now within their reach.

At the present time actuaries are brought face to face with the alternative of submitting to a lower and, apparently, still falling rate of interest, or of seeking other investments, which, while yielding a rate of interest comparable with that realized in the past, shall, at the same time, afford adequate security, though perhaps of not quite the same order in this respect as that for which they had previously been accustomed to look. If the former alternative is accepted, and a lower and falling rate of interest is regarded as inevitable, other unpleasant consequences are also entailed, for it can hardly be hoped that the loss of surplus in this respect can be compensated for by a gain in other directions. The rate of expenditure does not seem to be appreciably on the down grade, nor, on the whole, does the profit from mortality seem likely to prove an increasing quantity. This latter may be partly attributed to the growing proportion of endowment assurances, on which there is less room for mortality profit than on whole life assurances, and partly, perhaps, to greater laxity in accepting lives owing to the stress of competition, and to under-average lives being frequently accepted on much too favourable terms. Many of the extraneous sources of profit may also be regarded as diminishing, if not as vanishing, quantities, in these days of liberal surrender-values and non-forfeiture regulations, and it is difficult, therefore, to see any direction in which we can reasonably look for compensation for a falling rate of interest. Under these circumstances, such a fall must mean, in the near future, a diminished surplus in proportion to the amount at risk, bringing with it a reduced bonus, and, perhaps, in the case of proprietary companies, reduced dividends. The prospect is not a pleasant one, but, fortunately, it appears to be avoidable to some extent by a timely recognition of the fact that, although the old rate of interest can no longer be realized on the old class of securities, yet there are still securities to be obtained, affording ample scope as far as amount is concerned, for the investments of assurance companies for many years to come, and which, while offering adequate security, will, if carefully selected, yield a rate of interest varying from  $3\frac{3}{4}$  to  $4\frac{1}{2}$  per-cent, with the further advantage of there being a fair prospect of an appreciation in price in the future. There are possibly many securities which come within this description, and, no doubt, those British

companies which have sought and found new fields for investment in the gold and sterling bonds of certain American railways and in colonial mortgages, would claim that their securities did so. It does not, however, come within the scope of my present paper to discuss these, but rather, as the title I have taken indicates, to discuss one special class of securities which, in my opinion, certainly comes within the above description, and which has been practically neglected by all assurance companies up to the present time. The securities I refer to are the debentures and debenture stock issued by companies registered under the Companies Act, 1862, and commonly referred to as trading companies. It is true that, according to the last Board of Trade returns, the balance sheets of assurance companies show about £41,000,000 invested in debentures and debenture stock; but it will be found on investigation that, in most cases at any rate, these are issued by companies incorporated by special Act of Parliament for the purpose of carrying on business of a public nature, such as waterworks, gas works, railways, &c., and these securities differ considerably in their legal and financial aspects from the debentures of companies registered under the Act of 1862. I do not propose to discuss the former class of debentures, as they cannot be said to offer a new field for the investment of assurance funds; and their eligibility can be judged from the fact that the rate of interest vielded by them, at present prices, in very few cases exceeds 23 per-cent.

The debentures and debenture stock of trading companies stand, however, on a very different footing, for not only can a rate of interest, varying from  $3\frac{3}{4}$  to  $4\frac{1}{7}$  per-cent be obtained on good security, but the amount of such securities in existence is so large as to render it probable that a considerable portion of the surplus funds of assurance companies could be invested in them for some years to come, without thereby causing much rise in price and consequent fall in the rate yielded by them. It is estimated by Mr. F. B. Palmer, than whom there is no greater authority living on joint stock companies, that the amount of the debentures and debenture stock, issued by companies registered under the Companies Act, 1862, is at least £400,000,000. Nor is this amount, like Consols, a diminishing quantity, but, on the contrary, as the Lord Chancellor pointed out recently, in referring to the work of the Committee on the new Companies' Bill, "In the present "state of things almost all the great commercial interests of the "country are tending towards the establishment of joint stock

"companies", a process that renders the supply of debentures available for investment a continually increasing quantity. Under these circumstances, it has seemed to me that I should do wisely to discuss this particular class of security in some detail, rather than glance superficially at a wider range of securities.

It was remarked, at the outset, that no apology seemed needed for bringing this subject before the Institute. I am not sure however that the same can be said of the method I propose to adopt; indeed I feel called upon to give some explanation beforehand of the fact that, in the portion of my paper dealing with the legal aspects of the debentures, I have deemed it necessary to enter into details which are doubtless familiar to the senior members of the Institute. Even if those details were not already known to many, I should, perhaps, be told that it is not usual, when investing in such securities, to consider all their legal aspects, but that it is customary, in practice, to take it for granted that when a security has been upon the market for some years, everything is in order from a legal point of view, and that every necessary precaution has been taken for the protection of the investor. My reasons for entering, in some detail, into the legal aspects of debentures are two-fold. In the first place, one of the things that directed my attention specially to the subject of debentures, was the difficulty which many students have in connection with the subject. From some little acquaintance with the difficulties which beset that unfortunate class in their preparation for the financial subjects of the final examination of the Institute, I have found that, as a rule, the subject of debentures presents more obstacles, and ideas with regard to them are more hazy, than in respect to any other securities. To quote one instance, it is extremely common to find the idea that a debenture of necessity represents a secured debt, whereas, according to an authoritative definition, "a debenture means a document which either creates a " debt or acknowledges it, and any document which fulfils either of "these conditions is a debenture", and although the debentures subsequently discussed are all secured on some form of property more or less tangible, that is not an essential characteristic of a debenture. The legal part of my paper is therefore intended, at any rate to some extent, to assist students in their preparation; and in view of the great interest which the Institute has always taken in its students, an interest never more manifest than at the present time, I feel sure that the elementary nature of some of my remarks in that portion will be overlooked. With regard to the

second possible objection to which I have referred above, it is doubtless true that ordinary investors, when purchasing debentures, do not concern themselves as to whether all the necessary legal formalities have been observed with regard to the issue of such securities, and in effect, assume that everything is straightforward and that any necessary enquiries were made by the first holder of the security. Such a course may not involve any serious consequences where only a small investment is concerned, but the investments now contemplated are upon a large scale, and should not be made until the assurance company has satisfied itself that the security, in both its legal and financial aspects, is quite sound. Moreover, if such investments as are here discussed, commend themselves as being suitable for purchase in the open market, there is no reason why they should not be taken at the outset from the company issuing the debentures; indeed there is every reason why this latter course should be adopted wherever practicable. As the members of this Institute are aware, it is a common practice for companies issuing debentures, to get them underwritten by one or more financial firms, paying them a small commission, varying according to the probable popularity of the security offered, for so doing. If, however, the debentures are such that they would be considered a satisfactory investment for life assurance funds, there is no reason why this underwriting should not be done by the assurance company or, if necessary, by a combination of assurance companies, and an additional profit made thereby. In fact, if the debentures of trading companies should be deemed suitable investments for life assurance funds, there can be little doubt that the business of underwriting debenture issues, as far as sound companies are concerned, would fall almost entirely into the hands of assurance companies, to the advantage of both parties. In such a case, however, it would be very necessary that the assurance company making the investment should satisfy itself that the debentures were in every respect perfectly secured, and hence a knowledge of the legal aspects of such securities is not only very desirable but absolutely necessary in those who may be called upon to deal with such investments. When, to these remarks in extenuation of the method adopted, I add that the law relating to debentures is still in a somewhat unsettled condition, and that even within the past few months decisions of great importance, affecting debentures, have been given by the Courts, I trust that I shall be considered as having sufficiently justified the inclusion in my paper of the

portion now referred to. I need hardly remind you that the advisability of such knowledge has been recently emphasized, both by our President and by our ex-President, Mr. Young, in the weighty remarks made by them in the introductions to the two courses of lectures delivered before this Institute and lately published. The necessity for a practical acquaintance with finance in all its many aspects has there been pointed out; and although, perhaps, in the remote past of life assurance, when gilt-edged investments could, without difficulty, be obtained to yield from four to five per-cent, the necessity for such knowledge may not have been so apparent, yet there can be no doubt as to its being essential under the changed financial conditions of the present time; and the lectures to which I have referred, and those to be delivered in the near future, are themselves evidence that the Institute recognizes the necessity for its students to be fully informed in these matters.

With these introductory remarks, I will now proceed to consider the subject more in detail in its legal and financial aspects, though I may perhaps point out that, with regard to the former, there are many legal points affecting debentures which, although of considerable importance under certain circumstances, yet have little or no bearing on the question of investing in such securities; and as I am dealing with the subject entirely from that point of view, I have omitted all reference to such portions of the subject as seem to come within this description. I have also abstained from discussing the special legal points affecting the debentures and debenture stock of public companies incorporated under special Acts of Parliament, and also those in connection with similar securities issued by local authorities, as these all differ essentially from the debentures and debenture stock issued by companies incorporated under the Companies Act, 1862, to which, as already indicated, I propose to confine myself. I may also point out that as the debenture stock to be here considered differs from a debenture in form rather than in substance, and as the law relating to them is in most cases identical, I shall, as a rule, use the word "debenture" to signify either or both, only referring to debenture stock when it is necessary to distinguish between them.

## THE LEGAL ASPECTS OF DEBENTURES.

As already stated, debentures, although usually representing a secured debt, do not necessarily do so; and it has been stated

on authority that a debenture merely means an instrument which shows that the party owes and is bound to pay. If, however, we add to this definition that, either by the debentures or some other instrument, security is given for the payment of the debt, we shall have, perhaps, a fair description of the debentures commonly issued by trading companies. Not only is it true that a debenture does not necessarily imply a secured debt, but the fact that it is described in the Stock Exchange official list as a debenture does not in any way guarantee that it is entitled to be called a debenture at all, whether secured or unsecured, since it is the practice to describe securities in that list according to the description given by the company issuing them. These points are, however, of little importance to the subject of my paper, since I only propose to consider debentures properly so called and secured more or less effectually upon certain property of the company. It may here be pointed out that the covering deed which nearly always accompanies debenture stock and is frequently used also to secure debentures, has been declared not to be a debenture, although apparently coming within the description of such securities, but the reason for this decision is not clear.

The principal aspect in which debentures differ from debenture stock is that whereas in the former case, each lender usually has a separate bond, a number of such bonds being issued, all of the same amount, in the latter case the lender has a certificate entitling him to a certain share in the whole loan. In the case of debenture stock, the property on which the debt is secured is, almost invariably, conveved to trustees by a deed called the covering deed, by which the trustees are empowered, on the happening of certain contingencies, to realize the property and pay off the holders of the stock. One great advantage of debenture stock as compared with debentures lies in the fact that the holder can transfer fractions of his holding, whereas a debenture is always for a certain fixed sum, forming one of a series of debentures of similar amount, and can only be transferred in its entirety. There need not, however, necessarily be several debentures forming a series, as it has been held that, where there was only a single security and a single lender, and it did not purport to be a debenture, yet it was to be regarded as such. The convenience arising from the ability to transfer in any fraction is so great that the issue of debenture stock is now much more usual than that of debentures. Where debentures are issued in a series, it is usual to provide that they shall rank pari passu. In default of such a provision they will rank according to their date of issue and, if sealed in numerical order, they will, primá facie, rank in priority according to their numbers.

Leaving out of account debentures which are not secured in any way, and considering only those which, either in themselves or by means of a covering deed, constitute a charge on the assets of the company issuing them, and which are properly called mortgage debentures, we can divide them into two main classes, namely, debentures payable to bearer, and debentures payable to registered holder, each of which classes is itself subdivided into perpetual or irredeemable debentures, and determinable deben-The descriptions explain themselves, except that it should be noted that the words "perpetual" and "irredeemable" are used in a somewhat technical sense, and simply imply that the principal is not repayable at any fixed date but only becomes payable on the happening of certain indeterminate events, such as the company being wound up or default being made in the payment of interest for a given period. Such debentures, however, frequently give power to the company issuing them, to redeem on giving six months' notice. Even without such a provision, there is some doubt as to whether the company could not redeem on giving due notice, on the ground that any condition to the contrary was an undue clogging of the equity of redemption. The point, however, is not of any great importance as, in the absence of express statutory provisions to the contrary, the company can certainly redeem its perpetual debentures at any time by going into voluntary liquidation, and it seems very doubtful whether it is possible for a company, by any stipulations in the debentures or covering deed, to contract itself out of this right. If therefore an investor in perpetual debentures wishes to protect himself against having to find a new investment at short notice, and at, perhaps, a lower rate of interest, the most effectual method appears to be to stipulate that the debentures shall be redeemable only at a considerable premium. Otherwise, on the company's credit improving, or the money market becoming favorable, the company may find it profitable to go into voluntary liquidation simply for the purpose of redeeming its debentures. This applies also to determinable debentures.

Of the two main classes of debentures, those payable to bearer were formerly the more favoured, no doubt on account of the ease of transfer, they being as a rule, under their conditions, negotiable instruments for most practical purposes. As a matter of fact, debentures payable to bearer, issued under the seal of an English company registered under the Companies Act, 1862, have, for many years, been commonly treated as being negotiable instruments, but until last August this practice had not been recognized by a legal decision. It was then decided, however, in the case of The Bechuanaland Exploration Company v. The London Trading Bank, that it was permissible to show that a custom to regard such instruments as negotiable existed amongst merchants, and that sufficient evidence was forthcoming to prove the existence of such a custom. It was therefore held that debentures payable to bearer, and professing to be transferable by delivery, were negotiable instruments by the "law merchant", and that the defendants, who had taken such bonds for value and without notice, from an official of the plaintiff company who had stolen them, had therefore a good title.

In spite, however, of the superior case of transfer, debentures payable to bearer have declined in favour since 1885. year the stamp duty on such securities was raised to 10s. per £100, while the stamp duty on debentures payable to registered holder remained, and still remains, at the ordinary mortgage rate of 2s. 6d. per £100. The result of this has been that the former class of security has declined in popular favour, while the latter class has increased in popularity, and is now the favorite form of debenture. While referring to the subject of stamping, attention may be called to the very important alteration in the practice relating to the stamping of debentures, which has been brought about by the decision in the case of Rowell & Son, Limited, v. The Commissioners of Inland Revenue, in 1897. Until that decision, debentures redeemable at a premium had been considered as sufficiently stamped if the stamp covered the sum advanced, exclusive of the premium; but it was decided in that case that this hitherto universal practice was wrong, and that the stamping must include the premium. The effect of this decision has been widespread, it being extremely common to issue such debentures, and the Inland Revenue authorities have therefore made special provisions for the due stamping of such securities without penalty, the payment of the extra duty being made, in many cases, in a lump sum by the company issuing the debentures. Holders of debentures would therefore be well advised to examine their securities, and, if they come within the scope of the decision referred to, have them put in order without delay. It should be pointed out that the decision only applies to those debentures that are redeemable at a premium in any event, and not to those which are only so redeemable if the redemption takes place before some particular date.

One or two other questions relating to the stamping of debentures may perhaps be referred to here. When debentures are further secured by a covering deed, the debentures themselves are stamped with the ordinary ad valorem duty for such securities. The covering deed is not treated in practice as liable to the rate for collateral security of 6d. per £100, but is considered as sufficiently stamped with 10s. Where, however, debenture stock is secured by a covering deed only, that deed must be stamped with the ordinary ad valorem duty. Debenture stock certificates to registered holder, which are secured by a covering deed, do not require stamping. On the renewal of determinable debentures, the endorsement effecting such renewal, if under seal, must be stamped with 10s, where the ad valorem duty would exceed that amount, and with ad valorem duty if less than 10s. If the endorsement is under hand, a 6d. stamp is sufficient. Coupons are exempt from stamp duty. It may also be pointed out that although, as already seen, an instrument may be described in itself as a debenture and yet not be one, nevertheless if it is so called and bears on its face the outward and visible signs of a debenture, it will require stamping as if it were one in reality. The debenture holder is responsible for the due stamping of his debenture and the trustees for the stamping of the covering deed.

Debentures to registered holder, not being transferable by delivery like those to bearer, require in theory, that a deed of transfer shall be executed every time that they are transferred; and this, at first sight, would seem to be a decided objection to a security which may frequently change hands. In practice, however, this difficulty is frequently got over by passing on the debenture with a transfer in blank; and in this form it may pass from hand to hand in much the same way as if payable to bearer. It must be borne in mind, however, that the Courts will consider that anyone taking a debenture under such circumstances is warned by the blank transfer to make enquiries as to the right of the party from whom it is received, to transfer it. To quote the words of Lord Selbourne when delivering judgment in the case of France v. Clark, "The defence of purchaser for

"valuable consideration without notice, by anyone who takes "from another, without enquiry, an instrument signed in blank by a third party and then himself fills up the blanks, appears to "us to be altogether untenable." It has been held, however, that a transfer in blank confers a good title on the assignee as against the assignor's trustee in bankruptcy, although no notice may have been given until after the commencement of the bankruptcy.

In considering the question of the security offered by debentures of any description, the first point which arises is as to whether the company issuing them has power to do so, as if there is no such power, the most elaborately drawn-up debentures, professing to give the most extensive charge possible, will be valueless. Not only is this so, but if a company borrows without having power to do so, or borrows in excess of its powers, it is not only able to repudiate the debt, but has no option in the matter: and should it have repaid any money so borrowed, the liquidator of the company, on its being wound up, can recover any sum so paid, and this even if the transaction should have been subsequently ratified by all the shareholders. It is perhaps more common for a company to borrow in excess of its powers than to borrow without having any power to do so at all; indeed, the latter process, in the case of most companies now under consideration. is difficult, if not impossible, since, in the absence of express power to borrow in the memorandum or articles of association, a borrowing power is implied in trading companies, according to the decision in the General Auction Company v. Smith. There is no limit to the borrowing power so implied, which will extend to the full extent necessary for carrying on the company.

Although power to borrow, where not specifically referred to, is implied in the case of a trading company, it is not an incident of every company's objects: and where it is not, no borrowing power is implied. Where there is no borrowing power, express or implied, and it is desired to issue debentures, the necessary steps should be taken for an alteration in the memorandum of association to that end, although it would appear that unless prohibited by its memorandum of association, it can obtain such powers by means of a special resolution and subsequent alteration in its articles of association. This latter method is, however, considered by some to be inadvisable on account of the comparative case with which such borrowing powers can be varied. Where the company has express limited borrowing powers, the

Court will not imply any borrowing power beyond the prescribed limit or conditions. In the case of limited borrowing powers, where, as is not infrequent, the power is limited to some fraction of the uncalled capital for the time being, this is not confined to uncalled capital on issued shares, but includes share capital at present unissued. In connection with the subject of limited borrowing powers, it may be noted that a building society constituted before, and not incorporated under, the Building Societies Act, 1874, has no implied borrowing power, and therefore cannot borrow in the absence of express power to do so; but a society duly constituted under that Act has, by Statute, certain limited borrowing powers. Should these be exceeded, the society, as already seen, cannot by any means repay the sum so borrowed in excess of its powers. In such a case, however, an artifice known as subrogation can sometimes be resorted to by the lender as a means of at least mitigating his loss. principle of subrogation is, that where a company owes debts which would be recoverable at law and borrows money to pay them without any power to do so, the lender, although he cannot maintain an action for the recovery of the debt, yet, in equity, can claim to stand in the place of the creditors whose debts have been paid with his money. The doctrine is based on a legal fiction, by which the money lent is supposed to be advanced by the lender to the creditor of the company, in exchange for an assignment of the debt owing to the latter. As has been said in reference to this subject, "The Court, as the author of this "benevolent fiction, can fix its own time and place for the "enactment of the supposed bargain between the two parties, "who have met and contracted together only in the imagination " of the Court. And the test which the Court applies is not a "test of time, but the test whether the amount of the company's "liabilities has been really increased." It may also be added that where the directors have borrowed without power to do so, or in excess of their power, it is frequently possible for the lenders to hold them personally liable in damages. The Stock Exchange Committee usually require some limitation of the directors' borrowing powers before they will grant an official quotation.

Power to borrow, whether express or implied, confers power to issue debentures, but unless there is express power to borrow by means of debentures, a special resolution must be passed, authorizing the issue of such securities. There is a difference

of opinion among authorities as to whether a company registered under the Act of 1862 can issue perpetual debentures or debenture stock in the absence of express power to do so in the memorandum of association. The reasoning by which those who contend that power to borrow confers power to issue such securities support that contention, seems, however, to be sound, although there appears to be no direct decision on the point.

Not only are there frequently limitations of the extent to which a company can borrow, but there are also formalities in connection with the issue of the debentures, prescribed in the memorandum or articles of association, and the question arises as to how far the validity of debentures is affected by the nonobservance of the conditions so laid down. Such formalities are said to be either imperative or directory. If the former, the non-observance will render the issue void; but if the latter, it will hold good. It is impossible to give a definition of what constitutes an imperative provision and what a directory one, but, as illustrations of the latter, it may be stated that power to borrow on being authorized by a general resolution, a direction that the scal of the company should only be affixed in the presence of a director in pursuance of a resolution of the Board, and an instruction that the company was to be carried on by twelve directors, of whom five were to form a quorum, have all been held to be directory provisions only.

Assuming that the company has power to borrow by debentures, and that the necessary formalities have been observed with regard to their issue, the next point to be noticed is their registration. It is enacted by Section 43 of the Companies Act, 1862, that every company registered under that Act shall keep a register of all mortgages and charges affecting its property. The entry is to include a description of the property charged, the amount of the charge, and the name of the person entitled to such charge. Debentures, coming as they do, within the description of this Section, require to be registered under it; and by means of this register, which is open to the inspection of any creditor or member of the company, it should always be possible to ascertain the secured indebtedness of the company. Where there is a covering deed, it is usual to register the trustees of this deed as the persons entitled to the charge, instead of the actual debenture, or debenture stock. holders. It may here be pointed out that for many years it was held that the covering deed, if it included chattels, required to

be in the form prescribed by, and registered under, the Bills of Sale Act, 1882. This, however, has been altered by a recent decision, and the covering deed now only requires to be registered under the Companies Act, 1862. Another point to be noted in reference to registration is, that societies registered under the Industrial and Provident Societies Act, 1893, are not "companies", and there being no provision for the registration of charges created by them, as there is for companies registered under the Act of 1862, any debentures affecting chattels, issued by such societies, are subject to the provisions of the Bills of Sale Act as regards such chattels, and will be inoperative with respect to them, unless duly registered under the Act. It has been decided that Section 43 of the Act of 1862 is directory only, and not imperative, so that the non-registration of debentures or covering deed, will not invalidate or postpone the charge, or, apparently, affect it in any way. There are special precautions to be observed. relating to registration where the property charged comes within the provisions of The Stannaries Act, 1887, or includes land in either Middlesex or Yorkshire, but it will suffice to call attention to the matter here, without going into details of such registration.

The next point to be considered is the nature of the charge usually given by debentures. This will, to a great extent, depend upon the nature of the company issuing them, but it will, perhaps, be sufficient if brief reference is made to floating charges, specific charges, and charges on uncalled capital. I must not, of course, be understood as suggesting that these are mutually exclusive. On the contrary, all three may be, and frequently are, combined in the charge given on the property of the company to the debenture holders, but as each form of charge presents special features, I have deemed it best to examine them separately.

What is termed a floating charge is a charge on the assets for the time being, of the company, without in any way interfering with the sale or exchange of those assets in the ordinary course of business. According to the usual conditions of such debentures, the holders are entitled, on the happening of certain contingencies, such as the non-payment of principal or interest when due, or the winding-up of the company, to take possession of the assets comprised in the charge, in which case the charge becomes specific; but until the happening of such contingencies, or of similar ones, to which the Court may give like effect, the debenture holders will not be entitled to interfere with such assets.

A charge on "the undertaking" has been held to be sufficient to give a valid floating charge on all the property of the company; but if it is desired to give the fullest possible floating charge, a clause should be inserted in the debenture to the effect that the company thereby charges with the payment by it of principal and interest, its undertaking and all its property, both present and future, including all unpaid calls and its uncalled capital for the time being. In spite, however, of the complete nature of the charge contained in such words as these, there are certain liabilities of the company that will take precedence of it. Thus, a floating charge will not protect the chattels comprised in it from distress for poor rate or from certain other debts which are given priority over such a charge by the Preferential Payments in Bankruptcy Amendment Act, 1897. This Act is not, however. retrospective, so that, where debenture holders have commenced proceedings to enforce their security before the Act came into operation, it will not apply. Such a charge will not, before the commencement of winding-up proceedings, protect the property from the landlord's right to distrain for arrears of rent, and it should also be noted that such a charge is ineffectual in Scotland.

There are other difficulties in connection with floating charges arising from their conflict with specific charges. A specific charge arises, as its name indicates, when some definite portion of the property of the company is specifically charged with the repayment of a certain amount. It need hardly be said that where some particular part of the company's property is the subject of a specific charge, and a floating charge is subsequently given over the whole of the undertaking, the specific charge takes precedence of the floating one. It is not, however, so obvious that this may also be true when the order of the creation of the charges is reversed, but this is nevertheless the case, and constitutes one of the chief dangers of a floating charge. An example of this occurred very recently in a case where a company gave a floating charge over all its property in favor of the debenture holders. The charge included certain leasehold property, the title deeds for which remained in the possession of the company. Subsequently the directors deposited these title deeds with their bankers, by way of equitable mortgage, and on the debenture holders seeking to enforce their security, the question arose as to whether they or the bank were entitled to priority; and it was decided in favour of the bank. It has been suggested that in this case the debenture holders were, to some

extent, guilty of negligence and that they might have protected themselves by endorsing on the deeds particulars of their charge, so that no third party could claim to have taken them as security in ignorance of the prior lien on them; but it is by no means certain that, even had this been done, the decision would have been different. In the case of a floating charge, the company has usually, until default has been made, express power to deal with its property in any way it may think fit in the ordinary course of business, as if the charge had not been given; and it seems to be fairly well established that such acts as the giving of scenrity to secure a banking account, or the raising money by way of mortgage in order the better to carry on its business, come within the scope of the powers usually conferred on the company by the terms of the debentures or trust deed. In order to get over this difficulty, most debentures drawn up in recent years have contained a clause to the effect that, while the charge created by them is a floating one, the company shall not be at liberty to create any mortgage or charge on its property in priority to it. In spite of such a clause, however, a solicitor's lien, or rights acquired adversely to the company by a garnishee order, will still have priority over the debentures. It has been pointed out, moreover, that although such a clause as this may be effectual as between the company and the debenture holders, yet if the former should, in spite of it, sell or specifically charge some of the property to a third party who was in ignorance of this clause, though he might know of the existence of the debentures, such sale or charge would still prove effectual as against the debenture holders. The decision on which this view is based was however given in 1892, when the clause for the protection of debenture holders referred to, was by no means so common as it It is quite possible that a different view might be taken at the present time, and that it might be considered that subsequent incumbrancers, having notice of the issue of debentures, were put upon enquiry as to whether the company had power to create a prior charge to such debentures. In theory, the position of the debenture holders is not altered by such a sale or mortgage, as their charge will extend to the consideration received in respect of the transaction; but, in practice, their position will often be changed for the worse. If the company expressly binds itself not to part with the property charged in favour of the debenture holders, otherwise than in the ordinary course of its business, the holders may, on the company attempting to deal with such

property in any other way, obtain an injunction restraining them from so doing; and such an injunction may sometimes be obtained even where there is no such stipulation. It will also be possible, in many cases, to hold the directors of the company personally liable, if they misapply the proceeds of the sale of the company's property, when it is charged with the repayment of a debenture debt, unless they have first made due provision for the debenture holders' claim. It may be pointed out that debentures secured by an equitable charge on ships rank after a legal mortgage of the ship in the statutory form, executed at a later date, in spite of the fact that the legal mortgagee may have had notice of the debenture holder's charge. A company may, after having given a floating charge on all its assets, sell off a considerable portion of its stock-in-trade in order to raise money to carry on its business. Such a transaction will not be upset at the instance of the debenture holders, who, if they feel aggrieved, must resort to the remedies provided by their security or by the Courts, such as the appointment of a receiver. Although it is in the ordinary course of business for a company to pay its debts, and it is not therefore prevented from so doing by reason of a floating charge on all its property, yet this does not apply when a creditor has obtained judgment and levied execution; in fact, the levying of an execution is held to jeopardize the security to such an extent as to warrant the debenture holders in at once taking steps to enforce their security. When a company has given a floating charge over all its property, including real estate. it must, on selling any portion of the latter, if required by the purchaser to do so, furnish evidence that no default has been made under the debentures, although, as has been seen, in many cases the purchaser could obtain a good title without such evidence. The foregoing considerations lead to the conclusion that where the company has property capable of being specifically charged without undue interference with its business, such property should be specifically charged by means of a covering deed, vesting the legal estate of the property charged in the trustees, and securing to the debenture holders, by this means, an effectual first charge on the property comprised in the deed, as against equitable encumbrancers.

One special class of property, if it may be so described, which is frequently included in the charge given by debentures, and which presents, in some respects, special features of its own, remains to be considered—I refer to uncalled capital. If a

company has power to charge its property with the payment of loans contracted by it, this will include power to charge any capital called up, whether such calls are, or are not, payable at the time; but if it is desired to charge capital not vet called up, the company must possess special powers to that end. Where these special powers take the form of a distinct authority to charge the future calls or uncalled capital, as they frequently do, no question can arise; but some difficulty occurs when the power is less explicit, and it is sought to determine whether the authority conferred is sufficient for the purpose. To discuss this point at all fully would necessitate the quotation of the various cases at length, but briefly, it may be said, as specimens of what have been held to be insufficient or sufficient for this purpose, that power merely to charge the property, property and effects, funds or property, estate and undertaking, or property present or future, of the company, would not appear to confer power to charge the uncalled capital; but power to mortgage the property and assets, or the property and rights of the company, or to borrow money on any security of the company, would appear to confer such powers. Where the memorandum of association confers explicitly or implicitly a general borrowing power, but contains no reference to uncalled capital, power to charge the latter can be conferred by the articles of association, either in their original form or as subsequently modified by a special resolution.

An interesting point in connection with a charge on uncalled capital arises in connection with the application of the Companies Act, 1879, Section 5, which enacts that "a limited company may, " by special resolution, declare that any portion of its capital "which has not already been called up, shall not be capable of "being called up, except in the event of, and for the purpose " of, the company being wound up; and that thereupon, such " portion of capital shall not be capable of being called up except " in the event of, and for the purpose of, the company being "wound up." Where a resolution in accordance with this section has been passed, it has been decided that the company cannot give a valid charge on the uncalled capital so reserved; and that when such reserved capital is called up on the company being wound up, the amount paid in respect of such calls is not subject to the debenture holders' charge. In considering, therefore, the security offered by debentures to be partly secured on uncalled capital, it is necessary, first of all, to ascertain whether a resolution in accordance with the above section has been passed by the company. This can readily be done, since the resolution, being a special one, has to be registered with the Registrar of Joint Stock Companies, in accordance with the Act of 1862. A copy of such a resolution must also be annexed to the articles of association. I may add that Mr. F. B. Palmer is of opinion that the case referred to above has been wrongly decided, and that the passing of a resolution in accordance with Section 5 of the Act of 1879 was not intended to interfere with any power that the company might possess to charge its uncalled capital. In connection with a charge on uncalled capital, it is stated that, if the shareholders liable for such calls reside in Scotland, notice of the charge should be served on them, as, otherwise, the amount of such calls may be arrested by Scotch creditors of the company, and the claim of the latter will then take precedence of that of the debenture holders.

It not infrequently happens that more than one series of debentures is issued by a company, such series ranking according to the order of their creation, while the debentures of any given series will usually rank pari passu with each other. It does not, however, necessarily follow that when the amount of the debentures issued, say, in the first series and ranking pari passu, is known, the extent of the prior charge that can be created by such debentures is also known; this will be seen from the following considerations, which however affect the holders of second and subsequent series of debentures rather than the holders of the first series, and therefore are not of much importance from the life assurance official's point of view. Where a company has created a certain series of debentures ranking pari passu, but has not issued the whole of the series, and subsequently creates a second series of debentures, the company may afterwards issue the remainder of the first series, and the debentures so issued will, in the absence of special provisions to the contrary, rank before all the debentures of the second series. A company having power to issue debentures has also power to redeem them, subject of course to the conditions on which they were issued, and also power to re-issue them. It may thus happen that when a second series of debentures is created, a considerable portion of the first series has been paid off, and the holders of the former may think that only the amount of the outstanding debentures will have priority over their charge. This, however, is not necessarily the case, as unless the second debenture holders are specially protected, the company can re-issue the debentures that have been redeemed, and when so re-issued, they will have priority over all the second debentures.

The rights of debenture holders may also be interfered with by the action of a majority of such holders. It is not unusual for the debentures, or covering deed, to confer power on a stated majority of the holders to modify or compromise the rights of such holders; and where the powers conferred by the deed are sufficient for the purpose, the action of the majority will bind the minority. Power to modify the contract or compromise the claims of the debenture holders has been held to enable the majority to postpone the charge to a new one to be created by the company, to agree for the exchange of the debentures for preference or ordinary shares in a new company, and to an alteration in the date at which the debentures are payable. If, however, the company is being wound up, it is not necessary that special powers like those referred to should be contained in the debentures or eovering deed, as under the Joint Stock Companies Arrangement Act, 1870, power is given for a majority in number of the debenture holders, representing three-fourths in value of such creditors, present in person or by proxy at a meeting called for the purpose, to agree to any arrangement or compromise of their rights, and such agreement, on receiving the sanction of the Court, will bind the minority. The majority must be a majority of those present, not of those present and voting. Each series of debenture holders will represent a different class of creditors for the purposes of this Act. Holders of debentures which pass by delivery are not entitled to vote at such a meeting unless they produce their debentures at or before the meeting. Where paid-up shares are taken in exchange for debentures under an arrangement, care should be taken that the contract is duly registered under Section 25 of the Act of 1867, as otherwise, the holders of such shares may be held liable for their whole amount on the subsequent winding-up of the company; but an attempt under such circumstances to prove for the original debenture debt on the ground that the consideration for the exchange had failed, has recently proved successful.

Apart from the question of rights and priorities existing between the different debenture holders, the company issuing the debentures can sometimes set up a claim that interferes considerably with the value of the debentures in the hands of subsequent holders. The debentures of a company, being choses in action not assignable at law, are, primo facie, subject in the hands of an assignce to all the equities to which they were liable in the hands of the assignor. If therefore there is any liability, on the part of the original holder, to the company in respect of the issue of the debentures, such liability may attach to them in the hands of a subsequent holder, unless it appears from the nature or terms of the contract, that the securities were intended to be assignable free from any such equities. It is usual, however, to insert a provision in the debentures, implying that they are to be transferable free from all equities existing between the company and the original holder, and such a provision will effectually protect the subsequent holder. It is advisable to see that a clause of this nature is contained in all debentures payable to registered holder, but such a precaution is unnecessary in debentures payable to bearer, in view of the decision which declares such debentures to be negotiable instruments. If a company accepts notice of the transfer of debentures, and, while not registering it, does not object to the transfer, it will not be able to set up against the transferee equities which it might have set up against the transferor.

The question of the transfer of debentures has already been incidentally referred to and does not seem to call for further discussion. It may, however, be pointed out that where it is provided by Act of Parliament, or by the memorandum or articles of association, that the transfer shall be by deed, such a provision is an imperative one and not directory merely.

It now remains to consider briefly the remedies available for the holders of debentures secured by a floating charge on the property of the company, when it becomes necessary to enforce their security. The circumstances under which they are entitled to take such action, depend to a great extent on the terms of the debentures or covering deed, but it would appear that, without any special provisions, the security becomes enforceable on the non-payment of the principal at the date specified in the deed. on the interest being in arrears for a certain period, on the company passing a winding-up resolution or being ordered by the Court to be wound-up, or on the company parting with, substantially, the whole of its assets except in the ordinary course of business. A debenture which does not purport to be perpetual, but does not specify any date for the repayment of the principal, will become enforceable on giving the company six months' notice. Even if none of the circumstances enumerated

have occurred, the Court will, on application, appoint a receiver, provided it can be shown that the assets on which the debentures are secured are being jeopardized, and this will be held to be the case if there are judgments against the company. If no direct, available powers are conferred by the debentures or covering deed, the debenture holders may apply to the Court for the purpose of enforcing their security (1) By an action to enforce their charge, (2) By a petition to wind up the company, (3) By proof in the winding-up of such company. A company may be plaintiff in a debenture holder's action if it holds debentures. If the company issuing the debentures has its registered office in Scotland or Ireland, it must be sued in the country where such registered office is situated, although it may have a place of business in England and trade there.

The usual remedies given to debenture holders by their security are power to appoint a receiver, or receiver and manager, where it is desirable that the receiver should be able to carry on the business, and power of sale. The powers conferred on mortgagees by the Conveyancing Act, 1881, of appointing a receiver or selling the property mortgaged, do not extend to debenture holders. If, therefore, there are no provisions to these ends contained in the debentures or covering deed, application must be made to the Court. The latter will not appoint a receiver and manager unless the creditor has, under his security, power of sale. Where the security is a floating one and there is no covering deed, the Court will, at the request of all the holders, make a foreclosure order. This will not, however, be granted unless the security is deficient. Where a receiver is appointed under powers conferred by the security, he will be, in almost every ease, the agent, not of the debenture holders, but of the company issuing the debentures. It would appear, however, from a recent case, that if the company is wound-up, such winding-up will operate as a revocation of the agency, and the receiver may then become personally responsible for any liabilities incurred by him after the date of the winding-up order. Where the receiver is appointed by the Court, he will be personally responsible for any liabilities he may incur, but with a right to indemnity out of the assets of the business. The appointment of a receiver has the effect of converting the floating charge into a specific one on the assets comprised in the charge and at that time in the possession of the company; and on this taking place the debenture holders'

charge will have priority not only over the ordinary judgment creditors but also over the execution creditors of the company. It should be noted, however, that although, as seen, under certain circumstances a floating security ceases to float and becomes specific, yet a security that is issued on the condition that it shall float until default, does not necessarily cease to float on default. It will only do so on the conditions already referred to, or on steps being taken to enforce the security, or as specially stipulated in the deed.

If it becomes necessary to apply to the Court in order to enforce the security, the form of the application will depend on whether the debentures rank pari passu or not. If the former, the action must be taken by the debenture holder on behalf of himself and all the other holders, although he cannot have a personal judgment for more than the amount due to himself. If the securities are not stated to rank pari passu, they rank, as already seen, in order of issue, and the holder taking action will therefore do so on behalf of himself alone. One important power, sometimes conferred on a receiver, should be noticed, as it may affect the debenture holder's security. The Court will, under special circumstances, and for the preservation of the property, empower a receiver to borrow money as a first charge on the undertaking, such charge ranking in priority to the debenture holder's claim. The appointment of a receiver puts an end to the business of the company, and if therefore it is intended that the business shall be carried on until its sale or liquidation, a receiver and manager must be appointed.

Apart from any special powers, the debenture holders have, on the company being wound-up, the same rights as secured creditors have in bankruptey, namely, (1) To rely upon their security, (2) To surrender their security and prove for the entire debt, (3) To value their security and prove for the balance, and (4) To realize their security and prove for the balance. If the third course is adopted, the power to obtain the equity of redemption in the event of the trustee failing to redeem within six months after notice, which is conferred on secured creditors by the Bankruptey Act, 1883, is also available for debenture holders. If the debenture holders elect to rest solely on their security, and therefore do not value it and prove for the balance, they can, on such security proving deficient, prove for the balance of their debt, but no dividend already paid is to be disturbed by their doing so. A holder for value of debentures may, as a rule, prove

for the full amount of principal and interest shown by his deed, though he may have purchased them from the original holder at a considerable discount. This, however, will not hold good when the present holder stands in a fiduciary relation towards the company, as in the case of a director purchasing debentures at a heavy discount, after the commencement of winding-up proceedings.

Before concluding this portion of my paper, I propose to refer to a few miscellaneous matters in reference to debentures in their legal aspects, which may perhaps be of interest. Debentures issued in contemplation of winding-up and in the absence of pressure from the creditors obtaining them, will be invalid as being a fraudulent preference under The Companies Act, 1862, section 164, if a petition for winding-up is presented within three months of the issue of such securities. There will, however, be no fraudulent preference if they are issued under pressure and in order to prevent, if possible, such winding-up.

If debentures are taken up in consequence of statements in the prospectus which ultimately prove to be false, not only has the holder, in many cases, a right to bring an action for deceit or damages, but he may also apply for the rescission of the contract; and in order to succeed in such application, it is not necessary to show that the misrepresentation was wilfully false, it is sufficient to show that it was false in fact. The holder must, however, in order to succeed in setting aside the transaction, promptly repudiate the contract when he discovers the misrepresentations. The transferee of the allottee of debentures taken up through misrepresentation, cannot obtain rescission of the contract. person has contracted to take the debentures of a company and declines to pay the instalments as they become due, specific performance will not be enforced, the remedy being in damages only. In a recent case of this description, the plea that it was not a loan but the sale of a thing called a debenture, did not succeed. The usual remedy for such a state of affairs is to provide that non-payment of any instalments shall involve the forfeiture of the instalments already paid, though this will not meet the case of a man refusing to pay the first and all subsequent instalments.

According to some authorities, it is very doubtful whether debentures simply payable to bearer can, in view of Section 4 of the Statute of Frauds, create a valid charge on the land of the company; and it is stated that, to get over this difficulty, all

such debentures should be made payable to the person originally advancing the money or bearer. Since such a charge is an interest in land, within the meaning of the Statute referred to, a contract for the sale of such debentures cannot be enforced unless in writing.

It sometimes happens that paid-up shares are offered by way of bonus, as an inducement to investors to take up a debenture issue. The result of accepting such an offer will be that, in the event of the company's subsequent liquidation, the holders of such shares will be held liable for their full amount. Such bonuses are, therefore, white elephants of a most objectionable description, since the very fact of the offer of a bonus is an indication that the financial position of the company leaves something to be desired, and therefore suggests possible liquidation.

It occasionally becomes desirable to remove one or more of the trustees for the debenture holders and replace them by others. In view of such a possibility, express provision to that end should be made in the covering deed, as, in the absence of such provision, it will be very difficult to effect a change. The mere fact that a large majority of the debenture holders are in favor of an alteration will not be sufficient.

A power to trustees to invest in "the bonds, debentures, or "debenture stock of any company incorporated by Act of "Parliament" has been held not to authorize trustees to invest in the securities of a company incorporated under the Companies Act, 1862, and this will, of course, apply to an assurance company whose powers of investment are similarly limited.

The position of debenture holders may be affected adversely by the reduction of the capital of the company, or by an alteration in the memorandum of association enabling it to alter the character of its business. Notice of any such proposed change must, however, be given to the debenture holders, who may oppose it if they think fit to do so, but the Court is not bound to give effect to such opposition.

Debentures are frequently redeemable by periodical drawings, and a doubt was once expressed by Sir George Jessel as to whether the redemption of such debentures at a premium did not come within the scope of the Lottery Acts. It has been said that Sir George Jessel's doubts are better than many other people's certainties, but is not customary to give any effect to the doubt thus expressed in the drafting of these debentures. Where,

as sometimes happens, the income available for the holders of such debentures is not sufficient to pay the interest on those outstanding and also to pay off those which have been drawn for payment, the proper course is to pay the interest in full and to suspend the repayment of principal until the income will permit it to be resumed. It may be noted here that the payment of interest out of capital on debentures of any sort is not illegal, as it is in the case of interest on share capital.

Finally, it would perhaps be well to call attention to the possibility, and even the probability, of a change in the law relating to debentures which may adversely affect the position of debenture holders. At the annual meeting of the Incorporated Law Society last year, the President went so far as to advocate the total abolition of floating debentures, not because they failed to protect the debenture holders, but because they protected them too well as compared with the ordinary creditors of the company; and as recently as 9 November of this year, the Lord Chief Justice of England called attention to some of the evils arising in connection with debentures as instanced in the case of the so-called "one man company." This case, as most are aware, refers to certain proceedings which have received the sanction of the House of Lords as being within the terms of the Companies Acts, but which have given rise to much adverse comment on the laws which have made such a state of affairs possible. Briefly stated, the case is one in which a man sold his business to a company in which all the shares but six were held by himself, and these six were held by relatives or friends on his behalf. He received debentures as purchase-money, and, on the company ultimately getting into difficulties, he successfully claimed the assets in his capacity as debenture holder. It is not likely that this case will lead to any such drastic reform as that suggested by the late President of the Law Society, but it seems probable that some limitation of the rights of debenture holders, as compared with other creditors, will be imposed. One somewhat modest reform which meets with a good deal of support is to require debentures conferring a floating charge to be registered in the same way as bills of sale, so that intending creditors may know to what extent the apparent property of the Company is evidence of its solvency. It has also been suggested that the debenture holders' floating charge shall be subject to the just debts of the company, incurred in carrying on the business in the ordinary course; but it seems unlikely that companies would

succeed in borrowing on such terms. Another suggested way of meeting the difficulty to some extent is to considerably limit the borrowing powers of companies with limited liability. Another proposal, and one which is contained in the Companies Bill now under consideration by a Parliamentary Committee, is that no company shall exercise borrowing powers until shares shall have been allotted for an amount not less than the minimum subscription, and until not less than three-fourths of the aggregate amount payable on application and allotment has been received in cash by the company. This provision is, however, to the advantage of the debenture holders rather than to their disadvantage, and the other alterations that have been suggested from time to time, although they may operate to the disadvantage of the holders of debentures to be issued in the future, are not at all likely to be retrospective in their action even if they should become law, and will not therefore affect those who may invest in such securities in the near future,

In concluding this portion of my subject, I must express my great indebtedness to Mr. P. F. Simonson's Treatise on the Law relating to Debentures and Debenture Stock, to Mr. H. Buckley's work on the Companies Acts, and to Mr. F. B. Palmer's Company Law. These, particularly the first named, together with short articles that have appeared from time to time in the Law Journal and the reports of cases relating to Debentures during the past few years, have been my principal sources of information in preparing this part of my paper.

## THE FINANCIAL ASPECTS OF DEBENTURES.

Having discussed the legal aspects of my subject, and shown, as I hope, that the law is able and willing to give an effectual charge over the assets of companies registered under the Act of 1862, it remains to consider the worth or worthlessness of the assets usually offered as security. It is a commonplace to say that the value of a charge depends at least as much on the property charged as on the completeness of the legal charge created. It is hardly worth while to purchase a massive fireproof safe if one has nothing but waste paper to put in it; and it is not much wiser to adopt every possible legal precaution to effectually bind the property, if the latter proves ultimately to be practically valueless. That this is the case with some companies whose debentures are offered to the public is within the knowledge of everyone, but it is easy to form an exaggerated

idea of the number of such companies in existence. One is apt, on reading the reports in connection with the winding-up of rotten companies, to come to the conclusion that these form a considerable proportion of joint-stock companies, and to think that the majority are tainted to a greater or less extent. On this point, I may quote the words of the Lord Chancellor, spoken on the occasion to which I have already referred. He said that " a portion of the evidence, which will no doubt make a great "impression on us, points to the ridiculously small proportion "which what are called fraudulent companies bear to the " enormous mass of boud fide commercial enterprises, boud fide " and profitable in all respects." Perhaps, however, in the case of the great majority of companies for whose debentures the public are invited to subscribe, the assets on which it is proposed to give a charge are real and valuable in their way, but are sometimes of such a nature that any security based on them must be of a speculative character, and would therefore be considered as unsuitable for the investment of life assurance funds.

Perhaps this latter remark needs some amplification, lest I should be misunderstood. In suggesting that a speculative security is unsuitable for the investment of the funds of an assurance company, I do not mean that no risk should be run of fluctuations in the value of the investment. There are, no doubt, advantages about an investment of the nature of a well-secured mortgage where the actual capital advanced will be repaid on giving six months' notice; and if a sufficient number of such investments were available, at a permanent, adequate, rate of interest, it would, perhaps, hardly be necessary to discuss any other form of investment. This is not the ease, however, and life assurance companies have already, by investments in railway debenture stock, to some extent overcome their reluctance to make investments where the repayment of the principal invested depends on the market price of the security at the time of I think, however, that the caution exhibited by assurance companies in this respect might, with advantage to themselves, be still further relaxed, and that the principle of average, which plays so large a part in the actuarial side of their business, might profitably be extended to the investment side. Of course, if it could be shown that, unlike mortality experience, the variations in the value of securities were always, or even mainly, in one direction, and that a downward one, there would be some reason for the caution manifested. As a matter of fact,

however, so far from this being the case, the tendency is for the balance of fluctuations to be in the upward direction, as far as good class securities, at a fixed rate of interest, such as those discussed in this paper, are concerned. By such investments. therefore, not only is a fair rate of interest carned, but there is a reasonable prospect of a profit on realization in addition; and certainly, when the risks are spread over a wide area of carefullyselected securities, there is no more reason to anticipate a loss than a profit, nor, I think, so much. I am aware of the view that the prosperity, and, indeed, the very life, of an assurance company, depends on its being able to meet its liabilities promptly. as they mature, and that therefore no risk whatever should be run in investing the reserve which is to meet those liabilities. Such a view cannot fail to command universal acceptance when interpreted in a reasonable spirit; but I cannot think that it is so interpreted when it is made to mean that, in the case of each individual investment, the exact amount invested must be realized, instead of that result being achieved, approximately. on an average of the investments as a whole. It is difficult to see why, whilst giving the widest play to average in respect to the lives assured, such a principle should be rigorously excluded from other matters to which it also applies. I take it that a mortality experience akin to that of the Black Death would work quite as great disaster to the life assurance institutions of this country as any imaginable depreciation in ordinary investments would do; and even a much less severe mortality experience would be productive of considerable difficulty. Nevertheless, actuaries are unmoved and undisturbed by such disquieting shadows, even when they are accentuated by such events as those which recently occurred at Vienna, and which suggest that the plagues of the Middle Ages are not quite such impossibilities in these days as we sometimes think. They pursue their business, confident in their doctrine of average, and the results justify their confidence, nor does there seem reason to doubt that similar results would follow with regard to investments. I am, of course, far from saying that the probability of an adverse mortality experience, such as that referred to, and the probability of a considerable general depreciation of a temporary character in the price of good class securities, are alike in degree. If, however, the depression is only of a temporary nature, it will be of comparative unimportance to most assurance companies, and the probability of a permanent fall in price of such securities seems

so very remote as to be negligible. Perhaps the explanation of the timidity displayed in respect to the application of the doctrine of average to the question of investments, is to be found in the fact that the actuary of a company has often a good deal more to do with the mortality experience of the office than with its investments, the latter being dealt with by those who do not possess that confidence in the law of average which comes from a constant use of, and reliance upon it. In the subject I have immediately in hand, the foregoing remarks apply mainly to perpetual debentures or debenture stock, as in the case of determinable debentures, whether payable at a fixed date or with no date fixed for their redemption, the full amount invested is repaid, frequently with a premium in addition-assuming, of course, that they were not purchased above par. The remarks will, however, apply to determinable debentures under some circumstances, since it is quite possible that an office investing in such securities may find it necessary or advantageous, on occasion, to sell some of its debentures without waiting for their redemption in the ordinary course. They will also apply to the question of the security offered by the debentures of trust companies. The question is of considerable importance, if the securities now under discussion should be deemed suitable for the investment of assurance funds, from the fact that perpetual or irredeemable debentures find great favour with companies desiring to borrow, and in many cases are equally favoured by investors, who may desire to find comparatively permanent investments.

The assets to which I have referred as being of such a nature as to render debentures secured upon them somewhat speculative, and therefore unsuitable for the purpose under consideration, are those which are essentially wasting in their nature, and those which are somewhat intangible and unsubstantial. In the former category may be placed such securities as mines, concessions, and perhaps also patents, although this latter item contains other speculative elements besides that of wasting, in that there is a delightful uncertainty as to whether the validity of a patent will not be upset by the Courts, or its usefulness destroyed by some other invention. In the second category may be placed such an item as goodwill, which bulks largely in the assets of some companies, and which is of as uncertain a value as the kindred item of extension expenses, with which actuaries are not unfamiliar in connection with life assurance accounts. Where the speculative element in the security simply consists in its wasting nature, the

difficulty may be got over by arranging for the rapid redemption of the debentures by periodical drawings, or by a large sinking fund. Even these precautions, however, will but partially mitigate the unsatisfactory features of such securities, and from the life assurance point of view they are best left alone, more particularly as, after excluding them, there is a very wide field open for investments of a much more satisfactory nature. Where the value of patents forms a large part of the assets of the company, the security is of a still more unsatisfactory nature, and need not be further considered here. The item of goodwill is not necessarily of a wasting nature, and may indeed increase in value, but it is not the kind of asset on which it is wise to place much confidence, and in well-managed companies its value is written down out of profits each year. Where this is done, and the value at which it is placed in the balance-sheet bears only a small proportion to the more tangible assets of the company, it may be fairly considered of some value, though in considering the security offered for the debentures, such an item should be taken at considerably less than the company's own valuation.

Another asset of somewhat doubtful value, which frequently figures in the charge given by debentures, is the uncalled capital. It need hardly be pointed out that the value of this depends on the solveney of those liable for the calls, and it is not easy to see how the investor is to be satisfied on this point, unless indeed the uncalled capital can be satisfactorily insured. It is obvious that a knowledge of the status of the present shareholders will throw but little light upon the matter, seeing that they may have transferred their shares, and be free from all liability in respect to them, before the rest of the capital is called up. In the absence of insurance, therefore, it is advisable to make a very considerable deduction from the amount of the uncalled capital, in considering how much is likely to be realized from it on the debenture holders enforcing their security.

There are other companies whose assets are of a more substantial nature than those to which I have just referred, but which, nevertheless, offer but doubtful security for debentures. I refer to those whose property consists mainly in their stock-intrade. In such a case, a floating charge is usually given, and, as already seen, there is nothing to prevent the company, if in straits, from letting its stock run down to any extent it may choose, and using the money so obtained to relieve its difficulties. The security may thus be greatly depreciated before the debenture

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holders have any idea of what is happening, and when they come to enforce their security, they may find very little left to take possession of. Such considerations as these lead to the conclusion that debentures, whose sole sceurity is a floating charge, are unsuitable for assurance companies to invest in, since, in such a case, with every precaution, it is almost impossible to make reasonably sure that the security will not deteriorate in value, or that subsequent specific charges shall not be created, taking priority of the floating charge. This latter objection is especially true of shipping companies, whose debentures are frequently secured only by a floating charge on all the assets of the company. In such a case, as already shown, if a subsequent legal mortgage of the ships be executed in the statutory form and duly registered, such mortgage will rank before the floating charge, although the holder of the mortgage may have had notice of the debentures. A possible exception to this objection to a mere floating charge may, however, be found in the case of the debentures of trust companies, provided the debenture holders are adequately represented on the board of directors. These considerations suggest that the most suitable form of charge for adequately protecting the debenture holders is by means of a covering deed in favor of trustees which, in addition to creating a floating charge over the whole undertaking including uncalled eapital, gives a first mortgage on such assets as are capable of being dealt with in this way, these assets being approximately equal in value to the amount secured by the debentures. The deed should also confer power on the debenture holders or trustees to appoint a specified proportion of the directors of the company.

Assets of the nature referred to above as being unsuitable as security for debentures, while open to the objections stated, are not, of course, without value, and indeed, may often prove of very considerable value. They do not, I consider, constitute such security as an assurance company should look for in its investments; but when they are combined in the charge with other assets of a more satisfactory nature, they may usefully serve to supply a margin of security where the other assets approximately cover the actual amount advanced.

Having excluded certain classes of securities which seem to be unsuitable for the investment of assurance funds, and in consequence excluded the debentures of those companies which can only offer such security, it remains to consider briefly the nature of the security offered by those companies whose debentures do not seem to be open to the above-mentioned objections. It is not possible in such a paper as this to consider, in detail, all the companies or even all the types of companies, which are accustomed to borrow money by means of debentures, and the peculiar points to be noted regarding the security offered by each. I therefore propose to content myself with a somewhat cursory examination of the assets on which the debentures are secured in a few of the principal classes of commercial undertakings, whose securities appear to be worthy of the attention of those responsible for the investments of life assurance companies.

Among the most important of the companies coming within this description, both with respect to amount of debentures and security offered, are breweries. There are, I am aware, objections urged against this particular class of investment on purely moral grounds. Such objections, however, do not properly come under consideration on such an occasion as this, when I am simply eoneerned with the financial aspects of such securities; and therefore, without expressing any opinion on the moral aspect of the matter, I will content myself with this passing reference to it. In the case of most companies of this description, there is a large ordinary and preference capital coming after the debentures as regards interest, and little anxiety need be entertained on that score. As regards the safety of the principal, there is usually comprised in the charge a sufficient amount of freehold or long leasehold property to amply secure the debentures which, in such a ease, are practically as well secured as an ordinary mortgage, and in some cases even better, seeing that the charge will include, as a rule, the whole undertaking and stock-in-trade. The result of the excellent security offered by breweries is seen in the fact that a comparatively low rate of interest is realized on their debentures, from  $3\frac{1}{2}$  to  $3\frac{3}{4}$  per-cent being a fair average rate on good class securities of this nature. A point that may perhaps call for some notice with regard to the security offered by breweries is the fact that part of this security frequently consists of so-called "tied houses", and that the value of these depends to a great extent on their retaining their licenses. It would, I think, be out of place here to enter into a discussion of the question as to whether legislation is likely, in the near future, to interfere with licensed houses in the direction of taking away such licenses without compensation; but without stating my own views as to the desirability or otherwise of such legislation, I may perhaps be permitted to express my opinion that such legislation seems

very unlikely. It is of course possible, under the existing law, for the license of any individual house to be lost from various causes, but where, as is frequently the case, the tied houses are very numerous and are spread over a wide area, often including several counties, there is little real risk of any appreciable diminution of their number. Even if there were, either under the present law or by reason of new legislation, there need be no risk to the debenture holders therefrom, since it is fairly common, when the property comprised in the trust deed includes licensed houses, to insert a covenant that the borrowers shall keep the licenses insured against forfeiture. In estimating the value of the security offered by breweries for their debentures, it is necessary to take into account the somewhat remarkable inflation that has taken place in the last few years in the value of licensed premises as shown by the prices paid for such property; and it would doubtless be advisable to allow for a possible and even probable reaction in this respect.

Another class of security, resembling in some respects the one just discussed, is that offered by hotel companies. Here, again, the security consists largely of substantial property, usually ample to secure the debenture holders. The question of the license enters here also, but the risk of losing it in the case of an ordinary hotel, such as is here contemplated, is considerably less than in the case of "tied houses", and, moreover, its value will, as a rule, bear a smaller proportion to the total value of the property than in the case of a public-house. The remarks already made as to the insurance of licenses apply here, and it would therefore appear that in most cases, and apart from special circumstances affecting individual companies, the debentures of hotel companies offer unexceptionable investments from a financial point of view, for the funds of life assurance companies. As in the case of breweries, however, it is difficult to obtain more than about  $3\frac{3}{4}$ per-cent in the better class securities of this nature.

Another class of security, somewhat akin to the last mentioned, but into which the question of license does not enter, is that afforded by the debentures of companies owning offices and residential property. The highest class of property of this nature is probably that consisting of city offices; but, unfortunately, debentures secured on these are hardly available for assurance investments, not on account of the absence of sufficient security, but because the presence of ample security has reduced the rate yielded by them to rather less than  $3\frac{1}{4}$  per-cent. On

some of the other securities comprised in this class, however, a rate of interest approximating to  $3\frac{3}{4}$  per-cent may be realized with apparently adequate security.

Another type of company whose debentures seem to be desirable securities from the life assurance point of view, is one that has been somewhat prominently before the public in recent vears. I refer to large retail businesses, some of them concentrated in a single establishment, and others consisting of a large number of branch establishments, in some cases spread over one or two towns, and in other cases over the whole country. securs probable that in the near future all the principal businesses of the country will be in the form of joint stock companies, and if their debentures should be considered as suitable, a very large field will here be opened for the investment of assurance funds. It is true that many companies of this description are largely over-capitalized in order to provide a sufficient margin of profit to satisfy the voracious appetite of the company promoter, and that, where it is a case of amalgamating or transforming existing companies, the capital of such companies is usually watered to a considerable extent. Such facts as these, however, while perhaps seriously affecting the position of the holders of ordinary stock or even of preference stock, do not necessarily render the debentures of such companies unsuitable for investment. It will be found, in the case of the majority of such companies, that they possess assets of a substantial nature, such as freehold and leasehold property, on which a specific charge can be obtained, and which will at least cover the amount of the debenture issue and often considerably exceed it. In addition to this, the debenture holders will obtain, by way of margin, a floating charge over the whole undertaking. circumstances as these, the debentures may be considered safe, both as to principal and interest, in spite of any fluctuations in trade, since the holders will always be able to enforce their securities under the provisions of their deed as soon as the interest falls into arrears for the specified period, and recover such arrears, as well as their principal, out of the amount realized from the sale of the property. Companies of this description, having a large number of branch establishments, are, perhaps, more satisfactory than others from the investor's point of view, as not only have such companies, as a rule, a larger amount of freehold or leasehold property to offer as security, but from the widespread nature of their operations, it is not improbable that a falling-off in the trade in one district may be compensated for by increased trade elsewhere. A slightly better rate of interest can usually be realized on debentures of this type of company than is the case with those previously referred to, the yield from such securities varying from 4 to  $4\frac{1}{2}$  per-cent.

A class of company closely allied to the preceding is that consisting of companies transacting a large wholesale trading or manufacturing business. The nature of the security offered by such companies varies so much with the nature of the business they transact, that it is almost impossible to refer to them in this respect, except in the most general terms. It may, however, be pointed out that, as a rule, the proportion of the assets represented by such items as machinery, goodwill, patents, and stock-in-trade is usually larger in such companies than in the case of the retail companies just referred to, and for that reason their debentures hardly seem to offer such satisfactory security to investors as those of the latter class of company. This, however, is only true when speaking of such companies generally, and when individual wholesale and manufacturing companies are examined, it will be found that many comply with all the conditions that have been suggested as being necessary in any security in which life assurance funds are to be invested. rate realized on these securities varies from 4 to 41 per-cent, and is, perhaps, rather nearer the latter rate than the former.

A totally different class of security for debentures to those already mentioned is that offered by trust companies. I am aware that in referring to these as suitable for the investments of life assurance companies, I am incurring the risk of exciting much adverse criticism, and I should doubtless be reminded, did I not anticipate the remark, that in the ease of one of the most prominent of these institutions, the security offered is so excellent that the shares in it a short time ago actually had a negative value. As a matter of fact, however, in that very company, in spite of the disastrous position of the unfortunate shareholders, the debentures, at present prices, only yield about  $3\frac{3}{4}$  per-cent, an eloquent testimony to the high opinion held as to the security of such debentures, even when a company of this nature has, to put it in the mildest and most polite form possible, been grossly mismanaged. I cannot but think that the evil odour in which trust companies stand in the opinion of many people, does not arise from any reasonable objection to the principle on which such companies are based, but has arisen in consequence of the grossly

dishonest way in which that principle has been applied in the case of certain trust companies now or recently in existence. As most are aware, the theory of a trust company is that it shall borrow a large amount by means of debentures and invest the amount so borrowed, together with such portion of the share capital as may be available for the purpose, in stocks and shares of various descriptions. It is argued that, by reason of possessing a board of management consisting of experts in securities, and by spreading its investments over a very wide area, it is in a position to secure better results for private investors than they would be able to obtain for themselves; the expenses of management and dividend to shareholders coming from the difference between the rate of interest paid to the debenture holders and the rate earned on the securities held by the company.

It is hardly necessary that I should say anything in support of these views, for their truth seems self-evident. Given the honesty and capacity of such a board of experts, and there can be no doubt as to the superiority of opportunity which they will possess for the profitable investment of funds, over private investors. With regard to the advisability of spreading investments over a wide range of securities, and the advantages to be derived from so doing, I feel that I need say nothing to the members of this Institute, more particularly, as I have already endeavoured to show, in an earlier part of this paper, that investments carefully selected and held in large amounts are, to say the least, not more likely to deviate in a downward direction than in an upward one; indeed, where the investments are made by experts, the latter deviation may be considered as distinctly the more probable.

The theory, then, of trust companies may be considered as unexceptionable, but unfortunately the same cannot be said of their practice in recent years. What has really happened has been this. A few large financial firms that have indulged extensively in the underwriting of share issues, have found themselves saddled with large quantities of these shares, which have proved practically unsaleable to the public. A brilliant inspiration has then come to some master minds among them. "Let us form a trust company and sell all our useless shares to it", they have said, and, acting upon the inspiration, the trust company has been duly formed with an imposing looking board of directors, and the public has been invited to take up the debentures of the company, an invitation to which they have, in

many cases, readily responded. The imposing board of directors, imposing in more senses than one, has then purchased the otherwise unsaleable shares from the promoters of the company, using the money borrowed from the debenture holders for the purpose, and the trust company is duly started on the path to prosperity, with the good wishes of its originators. In many cases there is a considerable share capital, as well as money borrowed on debentures, but this fact does not alter the essential nature of the transaction.

I need hardly say that unless there are some adequate means of guarding against such proceedings as these, whether at the inception of the company or during its subsequent career, the debentures of trust companies can hardly be recommended for the investment of life assurance funds. These abuses can, however, be effectually guarded against, and, when such precautions are taken, I venture to think that these debentures form very suitable investments for such funds. Two necessary conditions for the protection of the debenture holders in such companies are (1) that the securities in which the funds of the trust company are at present invested are free from reproach, and (2) that the debenture holders themselves shall have power, by their deed, to appoint such a proportion of the directors as shall give them a controlling voice in the management of the company, or shall possess some power of veto in reference to changes in the investments, and to new investments.

It may well be that these conditions are fulfilled in few or none of the existing companies, but that is no real objection to my proposal. It would be easy, if such debentures should meet with the approval of life assurance companies, to form a trust company for the express purpose of enabling a few assurance companies to invest in the debentures created by it. In such a case, the debentures could be so drawn up as to give the holders the necessary controlling influence over the management of the trust company, and as the directors representing the debenture holders would be selected from the directors or chief officials of all the assurance companies holding the debentures, no difficulty would be experienced in getting together a board of directors, possessing far greater financial ability and influence than the board of any of the individual assurance companies.

The investments of such a trust company as is here suggested would probably consist mainly of preference and perhaps ordinary shares, of undertakings similar to those whose debentures are referred to in this paper, on some of the best of which from  $4\frac{1}{2}$  to  $5\frac{1}{2}$  per-cent can be obtained. A small amount might perhaps also be invested in the shares of companies having a liability in respect to uncalled capital, such as some of the joint stock banks, since one of the objections to such investments on the part of assurance companies, arising from this liability, would not apply under these circumstances. In the somewhat improbable event of the uncalled capital being called up on such shares, the call would be met, as far as possible, out of the reserve fund which should always be accumulated out of profits by trust companies, and even if there were no funds available for meeting such calls, the debenture holders will not be liable, unless they are also shareholders; nor will they run any risk of loss, except in so far as their security is depreciated by the elimination of such shares from the list of assets, since their claim will take precedence of the claim in respect to the calls on such shares. The position of debenture holders in respect to shares of this description will practically amount to the payment of a premium, equal to the difference between the interest received by the trust company on such investments, and that paid on the debentures, to insure against the liability in respect to uncalled capital, the trust company acting as insurer. If the debentures bear interest at 4 per-cent, this would allow, in the case of most of the great joint-stock banks, a premium of ½ per-cent on the sum invested in such shares. The necessary data are not available to enable me to say, with anything like certainty, whether this would be a sufficient premium to cover the risk, but, judging from the experience of American banks on a somewhat similar question, I am inclined to think that such a premium would be ample for the purpose, and would allow a good profit to the insurers in addition.

Such a trust company as is here referred to would require but a very small share capital, such capital being only required to pay the initial expenses and provide a balance in hand, and if the capital is thus kept at a small amount, there can be little doubt that considerable dividends would be paid by the company, in addition to the rapid accumulation of a reserve fund. Under these circumstances, there should be little difficulty in raising the necessary share capital amongst the officials of the assurance companies interested in the concern.

It would probably require the co-operation of several assurance companies to conduct such a trust company on a large scale, but joint action on the part of assurance companies in the matter of investments is no novelty, and is certainly not a line of action to be deprecated. Offices already join each other occasionally in large reversionary transactions, to their mutual profit, and there seems to be no reason against such co-operation on a larger scale, but on the contrary, there is much to commend such action. may perhaps be suggested that if the plan here outlined is an advisable one, it might be adopted by each assurance company for itself, without the intervention of the trust company and the consequent loss of the difference in the rate of interest between that realized on the securities and that paid on the debentures. There are, however, two objections to this. In the first place, most companies are hardly in a position to invest such an amount in ordinary stocks and shares as to make reasonably sure that there shall be no adverse balance of fluctuations on the whole; for it must be remembered that such securities will be likely to show considerably more fluctuation than the debentures of the same companies, seeing that the dividends are not fixed but depend more or less on the profits earned during the year. It will probably be necessary, therefore, to invest several millions in widely-differing securities to secure an average, and this will entail joint action on the part of several companies. In the second place, it frequently happens that the board of an assurance company contains only one or two directors who can be said to be experts in dealing with the shares of companies such as would be invested in by a trust company, and sometimes, perhaps, even one or two is a liberal estimate. A company situated like this may well hesitate to embark in such investments, but if several such companies combine together, and each appoints, as its representative on the board of the trust company, that member of its board of management who is best qualified to deal with such securities, a board of expert directors might be obtained for the trust company that would challenge comparison with that of any similar institution in existence.

In discussing the soundness or otherwise of the security offered for debentures, it may, perhaps, be thought that I should refer at greater length to the amount of ordinary and preference shares of the company issuing them, and the dividends paid thereon, as being some evidence of this. Such facts, however, are not necessarily of much value in enabling one to form an estimate of the real value of the security for the debentures; as it may easily happen that there is a very large

share capital, most of which has been expended in the purchase of such items as goodwill and patents. A business based on such assets may, and often does, yield a good rate of interest to its shareholders, in addition to paying the debenture interest; vet. as already seen, the security for the principal is of an unsatisfactory nature. Where the assets are of a more substantial nature, the share capital should be an indication of the value of the property on which the debentures are secured; but, even then, the indication thus afforded is not a very reliable one in these days of over capitalization. On the other hand, there are companies in existence to-day whose ordinary shares are almost worthless, and vet whose debentures afford first-class security. The only method of estimating the true value of the charge offered appears, therefore, to be to have the actual assets comprised in the security valued by an independent expert, for it need hardly be said that the value placed on the assets in the prospectus is usually a somewhat flattering one.

Attention should be called to the form of debentures considered in relation to the nature of the security offered. In the case of several of the classes of companies already referred to, a part of the security consists of leasehold property. In such a case, and, indeed, wherever the security is of such a nature as that it inevitably depreciates with the lapse of time, the debentures should either be repayable at six months' notice or at a given date. If the latter, the date of redemption must be so fixed that the security shall still be of ample value to secure the debt at the time of redemption. Debentures redeemable only at the option of the company, or on the happening of certain contingencies, are quite unsuitable in such a case from the investor's point of view.

In conclusion, I may be permitted to refer to the fact that 30 years ago a paper was read before this Institute by Mr. J. Coles, in which he advocated the investment by life assurance companies, in railway debenture stock, which was then yielding from 4 to  $4\frac{3}{4}$  per-cent. In the discussion which followed the reading of that paper, some of the ablest members of the Institute expressed the opinion that such investments were unsuitable for assurance funds, not on the grounds upon which they are condemned to-day, namely, the low rate of interest yielded by them, but because they considered the nature of the securities rendered them unfit for the purpose in question. Those, however, who followed Mr. Coles advice have reaped a rich harvest, both

in the realization of a high rate of interest on first-class security, and in a great increase in the value of their principal. I cannot, of course, lay claim to the financial knowledge and foresight possessed by Mr. Coles, which enabled him to forecast so accurately the future of the investments he was advocating, nor, in the nature of things, is it possible for the securities discussed in this paper to have such an experience in the future, as that which has been enjoyed by railway debenture stock during the last 30 years. Owing to the facility with which even the so-called perpetual or irredeemable debentures of trading companies can be redeemed, when it is to the advantage of the company to redeem them, it is hardly possible that, unless under very exceptional circumstances, such debentures will ever rise much above the price at which they can be redeemed. With these limitations, however, it does not seem rash to predict that those companies that avail themselves of the opportunities afforded at the present time, of investing a considerable portion of their funds in the debentures of trading companies, will not only be able to maintain a rate of interest differing but little, if at all, from that realized during the past few years, but will also find themselves possessed of securities that, within certain limits, are steadily appreciating in value.

## Discussion.

The President (Mr. H. W. Manly) thought the question of finding suitable securities for their increasing funds was one of very great importance, and the members must all feel indebted to Mr. Barrand for his clear, comprehensive, and exhaustive treatment of one class of investments to which they had of late turned longing eyes, but which seemed to bristle with as many delicate points as the "quills upon the fretful porcupine." Debentures, as Mr. Barrand said, were of many kinds, and they had often to put to themselves the conundrum-When is a debenture really a debenture? They had seen so many gross and flagrant cases of over-valuation and over-capitalization, that, unless they had some personal knowledge that the undertaking was really sound and honest, they naturally fought shy of the security. He heard of a case during the boom of the mortgage banks in Australia a few years ago when some land was sold for £4,000. The purchaser sold it to a promoter for £40,000, and he sold it to a company for £400,000 on a valuation, and debentures were issued to the amount of £200,000. It was not very difficult to find the value of that security when the bank was wound up. Mr. Barrand had wisely directed attention to the powers to pay off debentures. It was one which he thought occasionally disturbed the holders of that class of security. He was at a meeting of debenture holders not long ago, when it was proposed to reduce the debenture holders' security by

reducing the amount of the uncalled capital. He did not think that it would affect the value of debentures very much, but unfortunately for the promoters of the scheme, it was accompanied by a threat that if it was not accepted the company would go into voluntary liquidation, and the debentures would be paid off at par, and the company would then be re-constructed. The threat of an act of injustice brought out all the fighting qualities in an Englishman, and, needless to say, the scheme was not carried. It was pointed out at the time of the meeting that an act of that kind—namely, the going into liquidation in order to rob the debenture holders—would deal such a fatal blow to commercial morality and honesty in this country that it would be a long time before it recovered from the shock; and the stockbrokers themselves declared that if the company did go into liquidation for such a purpose it would never come out again, and he believed it was perfectly true. They would never find a quotation on the Stock Exchange again. He considered there was something to be said in favour of the scheme which Mr. Barrand had proposed, and the assurance companies should combine together to make a trust company of their own. They would not only secure the advantage of spreading their risks over a wide area of investments, but they could afford to pay a good man to devote himself entirely to selecting the best miscellaneous securities, and of discovering whether any particular debenture was a debenture. A large combination of that kind would practically impose its own conditions on the issue of debentures. because if it became known that this trust declined to deal in any but a certain class of debentures, there would be very few of any other class issued.

Mr. Wm. Hughes was afraid that, as investments, debentures of trading companies were somewhat disappointing. They did not find -at least, such was his opinion after carefully considering Mr. Barrand's paper—that those debentures did in fact offer all the qualities which at first seemed to attract them. Mr. Barrand said that he thought insurance companies had been too conservative and over-cautious. That surely was a very strong assertion to make. It might be that, if they left the secure ground of that class of investments which insurance companies had been accustomed to deal in, for others which were not quite so secure, they might find themselves in an unpleasant position. At all events, the old dictum of De Morgan's, that there was "nothing in the world approaching the security of a well-managed life assurance company", might become a trifle antiquated. The text of Mr. Barrand's sermon appeared to lie in his remark that the debentures and debenture stock of trading companies were to be recommended because "not only could a rate of interest varying from  $3\frac{3}{4}$  to  $4\frac{1}{2}$  per-cent be obtained on good security, but the amount of such security was so large as to render it probable that they would serve for many years to come." There were two or three assertions in that sentence which were the key of the whole argument, and which required some little examination. He had been looking at the London Daily Stock and Share List of the trading companies of the kind which were under discussion, to see how far a mere inspection of the list would offer support for this argument. Of course, a mere inspection could

not prove it—but it would give some idea of how far it seemed to be true. Looking down the list, and considering company after company, and enterprise after enterprise, he thought there were among them a good many that they would not think of considering for a moment. There were others that looked a little better, and some few that looked very good indeed; but when they came to examine them closely they found that the prices of the debentures in these last were such as would yield very little over what they were at present getting on the first-class securities which the insurance companies had been accustomed to deal in. It might perhaps be said that, although those first-class and well-selected debentures might not after all give them a very much higher rate of interest than they had been securing, it was as well to widen the range of their investments. He thought that was a proposition which required considerable examination. If the old investments were secure, and there were sufficient of them, it would require very strong argument indeed to show that there was a necessity for a change. Any other suggestion than a tangible increase of the yield did not therefore appear to exist. He might incidentally refer to one point that Mr. Barrand had mentioned, that companies might go in for new issues. The author had said that that was eminently a thing for the insurance companies to deal with. He (Mr. Hughes), however, thought that the new issues were the last thing an insurance company should look at, if they looked at debentures at all. Then Mr. Barrand had something to say about the application of the doctrine of probabilities to investments, and at first sight that appeared to be a very attractive proposition; but when it was looked at earefully the analogy was seen to be not In life assurance business they had their mortality tables and the earefully examined and analysed experience which it was their business to deal with, but they had no table of mortality of trading companies. On the whole, he was inclined to think that, though the paper contained many valuable suggestions, and was one which would be referred to if they were ever tempted to invest in debentures, nevertheless they might bring in a verdict on the main argument of "not proven."

Mr. Geoffrey Marks dissented as strongly as Mr. Hughes had done from the conclusions of the paper. The object of the paper was apparently to find some means of raising the rate of interest on life offices' funds, and Mr. Barrand suggested, as an effective means of doing so, that some part of those funds might be invested in the debentures of trading companies. Independently of the fact that whatever portion of their funds they did invest in those securities must necessarily be a small one, and so would have little effect on the average rate of interest, he could not help thinking that if Mr. Barrand had pursued his subject somewhat further he would have come to the conclusion at which most of the members had already arrived, that these securities were not in fact, except occasionally, fitted for their investments. In the first place, did these debentures afford perfect security? He ventured to think not. or if they did, the rate of interest which they yielded was too low to be attractive. Independently of the doubts which recent revelations in the Bankruptev Court had east over the promotion of a

great many companies, there were other points which rendered these debentures doubtful investments from the life office point of view. Then again, the legal difficulties appeared very formidable, and he could not help thinking that a careful study of the legal portion of the paper did not tend to commend that form of security to the members any more than did its financial aspects. Mr. Barrand founded his recommendation of debentures as securities for life offices on the idea that they would be able to average their investments. principle of average was responsible for a great many things which were not dreamt of by those who originated it, and he thought Mr. Barrand's suggestion must be one of them. He could not think that they were likely to make one good 4 per-cent security out of two or 20 or 100 inferior 5 per-cent investments. Mr. Hughes had anticipated one point to which he wished to refer, and that was that if those securities were to come up to their standard of safety they vielded such a rate of interest that they were not worth consideration. He thought that was one of the strongest points against the whole of Mr. Barrand's arguments. There was one other practical point. It struck him in reading Mr. Barrand's paper that. if he was going to take in his selection of stocks a certain number as to which there was some doubt, and in regard to which there would be a certain amount of depreciation, how would be deal with them in his annual balance sheet? It was a point of some importance, but especially so to those companies which published a detailed list of their assets at the end of each year. He ventured to think that unless a considerable margin were shown in the valuation of those securities, some of the members would very likely have something to say in regard to them. Running through Mr. Barrand's list of companies or trading concerns which he considered specially suitable for their consideration, he would refer to breweries particularly, as he thought there was no doubt they were the best of the companies which Mr. Barrand had mentioned. There was no doubt that the better breweries in this country were in an exceedingly strong position, and, from enquiries which he had made, he believed them to be in an even stronger position than many imagined. The great object of breweries in the last few years had been to conceal the amount of their profits, and they did so to some extent by giving bonuses on their shares and building up a very large reserve fund, and many breweries were therefore in an extraordinarily secure position. But if legislation were to deal to any large extent with the tied houses, as it very likely might, some portion of their profits, and some portion of the assets on which they relied for security for the debentures, would be swept away. There were other considerations which affected peculiarly prosperous trades, inasmuch as they generally became, sooner or later, the mark of the trades union leaders. In addition to that, there might be difficulties in regard to the raw materials on which the breweries largely depended. The price at which they were able to get their raw materials influenced their profits very much, and if there were any appreciable rise in the price the profits would very soon fall away. He did not, however, lay much stress upon that, because, as was well known, the breweries were mostly accumulating very large reserve funds. The

security mentioned by Mr. Barrand which had astonished him most of all as having his recommendation, was the debentures of hotel companies. He should have thought that most of the members were familiar with the circumstances which had attended the promotion of a great many hotels, and with the fact that a very large number of them, even the best, were the result of processes of re-construction. It might be taken pretty well for granted now-a-days that until an hotel had been re-constructed two or three times it did not pay. The debentures of companies owning offices and buildings was another form of security which Mr. Barrand recommended to their notice. He did not know much about that particular class of investment, because he had not had occasion to look into them except recently. He then had to enquire into two companies, and he found that they were merely associations for relieving over-speculative builders of their liabilities. Mr. Barrand seemed to think very highly of the retail trading companies, at any rate so far as the debentures were concerned, and had said that it would be found in the case of the majority of such companies that they possessed assets of a substantial nature, such as freehold and leasehold property, on which a specific charge could be obtained, and which would at least cover the amount of the debenture issue, and often considerably exceed it. Assuming that no margin on the debenture issue was wanted, he still did not think there was very much in the assets of companies of that kind which could be considered good security. There might be a few large freeholds, but as a rule, the leasehold premises of such companies were held for quite short terms, and would have no value if realized. In addition to that, there was another point which had been mentioned, namely, that if a floating charge was, as it would very likely be, included in the security for the debentures, that extended to the stock in trade and carts, &c., with which those businesses were earried on, and these might be very seriously depreciated before notice was given to the debenture-holders of the difficulties of the company. There was another point to which he thought Mr. Barrand had not attached sufficient weight, and that was that if the company were in such a position that its ordinary and preference shares were practically valueless it would be hampered to such an extent that he did not think the debentures would be of the slightest value on the market. Mr. Barrand said that they probably would be able to recover arrears of interest as well as the principal out of the amount realized from the sale of the property. They would, however, have to sell such a concern to a smaller company or organization, and if it had been damaged in its credit and embarrassed in its general trading by the knowledge that it had been to some extent in difficulties, he did not think that the debenture holders, when they came to sell the business, would get very much out of it. Large wholesale trading companies were in very much the same position. They were the result of a fashion, to some extent, but seemed to have come to stay; but their debentures, as a rule, were high-priced, and yielded very low rates of interest. As regards trusts, it was very likely that many of the members had not had the opportunity of studying the list of assets of some of the trust companies. If they were to do so he was sure that what some of those companies invested

in would startle them. He was in opposition to Mr. Hughes on one point which he had mentioned, and that was the question of Mr. Hughes seemed to think that the company would be better advised to invest in the securities of old-established companies than in those of a new company. Of course there was this to be said about old-established securities, that they had stood the test of practical experience; but in regard to new companies, his opinion was that the original allottee of debentures was generally in a far better position than a subsequent transferee to judge of the real value of the security which was offered to him, and, moreover, made his investment more cheaply. If they could, in addition, obtain an underwriting commission, there seemed to be no reason why they should not more advantageously invest in new securities than in those of old-established companies. There was one other point to which he must refer, because Mr. Barrand had not done so, and that was that trading companies depended for their prosperity very largely on management. It was often seen, in fact it was continually manifest to anybody who took an interest in these securities, that a business which had been built up by the energy and perseverance of one man, and was then sold to a company, did not always meet with the same success as in the one-man days. He thought it was tolerably obvious that, however successful those companies might be on their first inception, a time would come, owing to deaths or other causes, when the management would pass into other hands than those of the founders, and their success would not continue. Of course that was equally true of railway, water, and gas companies, or any other Parliamentary company. Still, although Parliamentary companies depended largely for their success on good management, in addition to that it must be remembered that they had large tangible assets, and also had given to them by Act of Parliament what was a practical monopoly in the district covered by their operations, and consequently the value attaching to their goodwill was extremely large. The same thing did not apply to the companies under discussion, and there was a very great distinction between Parliamentary companies and ordinary trading companies. With regard to the foundation of an insurance trust company, which Mr. Barrand suggested, and the appointment of a board of directors, he thought that if each company chose one good director—they knew how those directors were chosen very often-in a year or two the board of the trust company would merely be an average life assurance board, so that it would be better to keep their own investments in their own hands.

Mr. J. E. Faulks thought the legal portion of Mr. Barrand's paper merited a word or two of reference. In the first place, a word of caution—and he had rather hoped that the President would have given it from the presidential chair—might be useful in connection with the paper, and to some extent in connection with all legal papers. He could not help thinking that there was a certain danger in legal papers, because they might lead some of the members to think that they knew all about the law with reference to a particular point—debentures, for instance—whereas, as he was sure Mr. Barrand would admit, the paper, and other papers too, furnished only a very brief résumé of the subject. English law was

very largely a matter of case law, and it required a man skilled in the general principles of the law, and subsequently in the application of those principles to individual cases, to thoroughly understand the matter. He thought, for instance, that the pages in Mr. Barrand's paper which dealt with the remedies of debenture holders were, if he might say so without seeming ungracious, rather superfluous. What actuary or manager would take upon himself to advise his board of directors as to the remedies to be pursued in the event of his company having investments in debentures, when there was a solicitor to the company to advise the directors directly—a man with far greater knowledge of the remedies than the actuary himself could possibly have? Yet he thought there must admittedly be in connection with debentures some legal points which it was necessary for actuaries and insurance officials to know. One of those points to which Mr. Barrand had referred was the question of so-called perpetual or irredeemable debentures or debenture stock. That question was clearly explained by Mr. Barrand. Some of the members might remember a few years back—he thought it was in 1894—there was a great outcry about a certain company that went into voluntary liquidation in order to redeem debentures which were professedly irredeemable. Another question with which Mr. Barrand had fully dealt, and which was of great importance, was that of the stamping of debentures, more particularly with reference to the decision in Rowell and Sons' case. Mr. Barrand stated that the stamp duty on registered debentures was half-a-crown per-cent, but that in 1885 the stamp duty was raised to 10s. per-cent on debentures payable to bearer, and that where the debenture was redeemable at a premium the stamp duty must be paid, not only on the par value of the debenture but also on the premium. For some years after the Stamp Act of 1891 the view was entirely otherwise; in fact, in the Fourth Edition of Alpe's Stamp Duties, it was specially laid down that in such a case the Stamp Duty was not payable on the premium but on the principal sum secured only. Then the case of Rowell and Sons occurred in 1897. That was a case of registered debentures, and it was held that the Stamp Duty must be charged on the premium as well as on the principal sum secured. He thought Mr. Barrand might now add a footnote referring to a case which was decided last week of Knight's Deep Debentures, a case of bearer debentures, where exactly the same point arose. The Queen's Bench Division held that the Stamp Duty, as the debentures were redeemable at any time after 1900 at 103, must be 11s., and not 10s. as they were originally stamped. The wording of Mr. Barrand's statement at the end of that clause, "It should be pointed out that the decision only applies to those debentures that are redeemable at a premium in any event, and not to those which are only so redeemable if the redemption takes place before some particular date", might be perhaps, a little improved, although he was not prepared to suggest wording to improve it at that moment. Originally, when the Inland Revenue authorities, after Rowell's case, issued their circular of June, 1897, referring to the case, they used words similar to those of Mr. Barrand. But he believed it was then the opinion of a committee of the Council of the Incorporated Law Society that where the company simply had an option to redeem, whether before or after a fixed date, the claim for additional Stamp Duty did not arise. That appeared to be opposed to Knight's Deep Debenture case of last week, and he thought under those circumstances a little difference in the wording might be made. That the point was not altogether without interest to those who had occasion to look into bearer securities might be instanced by a fact that came to his notice less than a fortnight ago. Some debentures, issued in 1895, were purchased and presented stamped only with a half-a-crown stamp although they were redeemable at 102. Of course they were treated as bad delivery and had to be stamped—with an additional stamp as they were without penalty, by the Inland Revenue. Therefore, to anybody who invested in debentures the question of stamping was decidedly important, and of course it was a question that could not be well referred to a solicitor but must be dealt with on the spot. Mr. Barrand referred to another legal point in terms rather of commendation, though somewhat slight commendation, namely, the subject of blank transfers. He could not think that blank transfers formed in any case a satisfactory security. There were so many points to be considered. In the first place, in certain companies a transfer had to be made by deed. That was not a point referred to in this portion of Mr. Barrand's paper, but if a transfer had to be made by deed Mr. Barrand would probably admit that a blank transfer would not fulfil that stipulation, which was one of the imperative conditions referred to later on when dealing with the Articles of Association, and therefore the blank transfer would not be valid in that case, as it would not contain the names of the parties in the ordinary way of a deed. But apart from that, there were other disadvantages attached to blank transfers which he would not refer to further. Mr. Barrand, in discussing the question of floating charges, dealt with a case which was easily identified as Castell and Brown's case. That paragraph of the paper might with advantage be modified, because the particular clause referred to by the author was inserted in Castell and Brown's debentures which were subordinated to the bank's charge. As to the question of floating charge, it seemed to him that the law had nibbled at the security of debenture holders from start to finish. Originally practically nothing came before a floating charge. Then Vice-Chancellor Malins gave a decision which enabled the company to charge its property, in spite of the floating charge of the debenture holders, in order to pay wages. That decision had been extended in various ways. There was a case in 1885 where the floating charge was postponed to a legal mortgage. In 1898 there was another case where the floating charge was postponed to an equitable mortgage. Therefore there did not seem very much left for debenture holders who were secured only by a floating charge, except priority over a mere creditor. respect to that point, he thought the author might do well, in describing the case in question, to draw attention to the fact that the judge laid very great stress on the question of notice, although the author did not seem to think very much of that question, and stated that, even if the bank had had notice, the case might have been decided as it was decided.

Mr. J. B. Hart said that one point of view of this question had occurred to him while listening to the gentlemen who objected to Mr. Barrand's recommendation. Many industrial undertakings, now in the hands of companies, were formerly carried on by private individuals. When the proprietor of the undertaking required a loan, he would bring a mortgage to the office, which probably would be accepted. But companies found it more convenient to raise their loans by way of debentures. He imagined that the gentlemen who objected to debentures would not object to a loan in this way, simply because it was a matter of form in carrying it out.

Mr. J. Burn was inclined to think that several of the suggestions in the paper would meet with more appreciation at some future time, when the effect of the falling rate of interest earned by the funds of assurance companies became more evident. Some of the objections raised against Mr. Barrand's paper were not altogether warranted. The question of whether debentures of a trading company did yield a higher rate of interest was not one which should be endeavoured to be proved by referring to special cases, but one that might be absolutely proved by means of statistics. By taking a large number of trading companies, it could be proved that during the past years they had yielded on the average—including also the losses which had occurred in certain cases—a considerably higher rate of interest than had been obtained on several other classes of investments which had been more favourably considered by assurance companies.

Mr. Tennant admitted that he shared fully the opinion that security was essentially the thing at which life assurance companies must aim in the investing of their funds, but he thought the

circumstances which the author had put before them must not be overlooked. They were faced with the fact that interest was falling, that funds were increasing, and that they were all competing for the comparatively narrow field of the absolutely unimpeachable trustees' investment. If there was a field outside that, which was in some eircumstances reasonably safe, it was well that they should get to know as much as they could about that field, and see whether it was one that might reasonably be cultivated. The fact that companies were being formed every day, and that the forming of them was a rage, and a somewhat unhealthy aspect of the present time, was, after all, another matter. Businesses were continually being turned into limited companies; it was the spirit of the age to do it, and therefore it was being done in regard to good businesses as well as in regard to doubtful ones. It was the duty of responsible officers of life assurance companies to discern which were the good and which were the bad companies, and endeavour to reasonably invest in the wider field of investment, and so relieve the pressure which came from over-competition in the narrower field. One clause which was seen sometimes on debenture prospectuses had its value, namely, "That the debentures shall immediately become payable if the value of the assets at any particular time is shown by the accounts to have fallen below a certain point", which might be 50 per-cent more than the debenture issue. That, taken with due caution and with a mind

wisely exercised upon it, might help the members somewhat.

The President asked the members to pass a hearty vote of thanks

to Mr. Barrand for his very interesting paper, and this resolution was carried by acclamation.

Mr. A. R. BARRAND, in reply, said he need hardly tell the members that he expected a good many criticisms upon his paper, and expected them to be unfavourable. He had, however, been very much disappointed in some of the remarks, and particularly those of the two referees, many of whose statements had been altogether beside the point. First of all, Mr. Hughes remarked that it was impossible to obtain a higher rate of interest in some of the better class of investments mentioned in the paper than the rate of interest at present realized. That was hardly correct. There were examples of companies whose debentures had been offered for subscription in the last two or three weeks at 4 per-cent interest, which afforded unimpeachable security, and it was not every insurance company that was earning 4 per-cent on its new investments now-a-days with such security as that. A remark had been made with regard to his advocating investments in new issues. It seemed to him that if the securities he had advocated were sound, and the precautions he had suggested were adopted, namely, examining the assets and finding that they were such as could be mortgaged, and were sufficient to cover the whole amount advanced, and if in addition they had a floating charge, which was no mean security in itself, then there was as good a security in the new issue that was first offered to the public as in many of the investments which they heard of at the present time. He had heard of mortgages where the insurance company had advanced the money and where the transaction had not turned out a very favourable one. The margin had turned out to be a vanishing quantity, and they had found themselves with a considerable loss on the transaction. If they took the same precaution in valuing the property on which the debentures were secured as they did in valuing property for mortgages, the debentures would turn out quite as satisfactorily. Mr. Marks had argued at great length about the difficulties in some companies. He had shown how they had been over-capitalized, how much depended upon the management, and that when the present management ceased the company might come to grief. But Mr. Marks had apparently overlooked the fact that the paper did not advocate investing in ordinary shares but in debentures, and only in those when properly selected after adopting the precautions he (Mr. Barrand) had suggested. Debentures did not depend for their security upon the management of the company, but upon the value of the assets which should have been previously valued by the assurance company's own valuer. If the company should come to grief, it seemed to him that the debentures at least were well secured. An instance of this had occurred only a few days ago in the case of a most unfortunate company that had its eapital reduced. The shareholders had come off very badly, but the debenture holders had assets equal to more than double the value of their debt. If the members examined carefully even into the hotel companies, with which Mr. Marks dealt so severely, he thought it would be found that at any rate the security for the first mortgage debenture debt, even in some of the most hopeless of the companies, was amply secured. Mr. Marks, when he quoted the figures, lumped

together the first and second mortgage debt, but of course no one would invest in second mortgage debentures, Mr. Faulks, in speaking about blank transfers, suggested that he had spoken of them with somewhat faint praise. He (Mr. Barrand) should perhaps have been a little more explicit, but if Mr. Faulks would read the clause again, he would find that "praise" was hardly the proper term to apply to his remarks. He would like to point out that when insurance companies refused to advance on debenture issues they were refusing to advance on a security which a few years ago they would have been willing to advance upon. If the owners of those businesses which had now been turned into limited companies had come to them and asked for an advance of money they would not have been refused. But when the businesses were turned into companies, and the companies asked for a loan on mortgage in the shape of debentures, the insurance companies turned away, and would have nothing to do with them. In the matter of such investments, therefore, insurance companies were becoming more cautious rather than less cautious as years went on. When insurance companies complained of the security offered in the case of debentures, they pointed with pride and satisfaction to the securities which they already possessed; but he might call attention to the fact that many of the investments which life assurance companies held at the present day were secured to nothing like the extent to which investments in debentures were. One had heard of such things as doubtful investments in life assurance companies. With regard to railway debenture stock, for example, the provisions of an Act passed in 1889 had been overlooked in many instances. That Act enacted, among other things, that where the Board of Trade requires a railway company to execute any works that could be put down to capital account, then they could empower such company to raise money by way of debenture stock; and such charge should, if necessary, take precedence over all the debenture stock of that company then in existence. That, perhaps, would not materially affect a large railway company, but it might swamp the ordinary debenture holders in a small one. He thanked the members for the great kindness and courtesy which they had shown him, and trusted that the remarks he had made, if not bearing much fruit that evening, would bear fruit in the future.

Some Considerations in reference to the Fall in the Rate of Interest experienced in the Past, and the Probability of its Continuance. By Joseph Burn, F.I.A.

[Read before the Institute, 30 January 1899.]

"THE rate of interest has fallen." "The rate of interest is falling." "The rate of interest will probably continue to fall." These sentences, and others of a similar nature, we have read so often, that I fear it is quite possible "familiarity" may, to some extent, have had its usual effect. The fact remains, however, that any further fall in the interest yield is of vital importance to insurance companies with large funds to invest, and bound by contracts, the fulfilment of which depends, to a large extent, on the rate of interest obtainable.

British assurance companies have earned a reputation for security which we are proud to think they justly deserve; this reputation we are all concerned in maintaining, and, in order to do so, we must examine all possible elements of danger.

It must, in the first place, be clearly recognized that assurance companies are in a unique position as regards the nature of their contracts, since in many cases these will not be determined for fifty, sixty, or even a greater number of years, and during that time undreamed of changes may occur, and the rate of interest may fall to a level which some would now consider impossible. Mr. Bailey, in his paper read before this Institute in 1862 (see J.I.A., vol. x, p.143), calls attention to this point when he says—" Life assurance societies . . . . engage to pay fixed sums " of money at periods generally long distant from the time when " the contracts are entered into: in mercantile phrascology, they " may be said to accept bills drawn at very long dates."

Our profession teaches us to study the experience of the past in order to obtain guidance for the future. This, certainly, is sound doctrine, and is applicable in an investigation of the interest yield, as well as in obtaining what is generally known as a Mortality Table. The way, however, in which we regard our data must be completely different. As to Mortality, we have no good reason to think that the average lifetime of man has greatly varied since we were told that "the days of our years are three score years and ten." We have then the great advantage in setting out on an investigation into mortality rates of some rough guide as to the limits of variation. There does not appear to be any such guide available for the subject now in hand. When investigating interest rates, we must study all our data with constant attention to "cause and effect." Some phenomena may be interesting and useful to us as general experience, but the causes of such phenomena may have permanently disappeared. In other cases, however, it is, I think, possible to recognize causes which are still in operation, and are likely to continue to produce similar results.

I have in the present paper attempted to make some investigations in a direction which appears to me most likely to aid in any attempt to forecast the future course of the rate of interest.

I am supremely conscious of the fact that there are many here who are far better qualified to deal with so weighty a matter; the subject, however, has interested me considerably, and has prompted me to offer the results of my endeavours for the consideration of this Institute. My intentions are, I fear, somewhat selfish, as I have greater hope of increasing my own knowledge by means of the discussion and criticism than I have of being able to add materially to the knowledge of others.

For want of any more reliable measure, if we refer to the rate yielded by Consols, we find from statements made in the Report of the Commissioners for the Reduction of the National Debt, that in 1749 the Three-and-a-half per-cents were above par, and Mr. Pelham therefore proposed that the Four per-cents should be reduced to Three-and-a-half per-cent, with a further reduction to Three per-cent at the expiration of seven years; which proposal was successfully carried out. Other examples of the price of Consols, during a considerable part of last century, could be given, which would show that the rate yielded was comparatively low.

Adam Smith, in his Wealth of Nations (published 1776), remarks in Book I, Chapter ix, "For some time after the "conclusion of the last war, not only private people of the best "credit but some of the greatest companies in London, " commonly borrowed at 5 per-cent, who, before that, had not "been used to pay more than 4 and 41 per-cent." And in the same chapter, "The Province of Holland, on the other hand, in " proportion to the extent of its territory and the number of its "people, is a richer country than England. The Government "there borrow at 2 per-cent, and private people of good credit at "3." Again, in Book II, chapter iv, he remarks, "In a country " such as Great Britain, where money is lent to Government at "3 per-cent, and to private people, upon good security, at 4 and " $4\frac{1}{2}$ , the present legal rate of 5 per-cent is perhaps as proper as "any." And a little later on, speaking of the market price of land, he says, "The market rate of interest is higher in France "than in England, and the common price of land is lower. "England it commonly sells at 30, in France at 20, years' " purchase."

From these examples we see that in the last century there were considerable variations in the rate of interest in the same country at different times, and in different countries at the same time, and the reason of this, when the differing degrees of risk

are allowed for, appears to be found in the fluctuating ratio of supply and demand. Thus, Professor Marshall in his Principles of Economics, in discussing the "Influence of Progress on Value", remarks that from the 17th century onwards "we find a "constant and nearly steady increase in the amount of accu-"mulated wealth per head of the population", and goes on to say 'p. 770, 3rd edition): "This increase of capital per head "tended to diminish its marginal utility, and therefore the rate " of interest on new investments; but not uniformly, because "there were meanwhile great variations in the demand for "capital both for political and military and for industrial "purposes. Thus the rate of interest, which was vaguely re-" ported to be 10 per-cent during the Middle Ages, had sunk " to 3 per-cent in the earlier part of the 18th century; but the "immense industrial and political demand for capital raised it "again, and it was relatively high during the great war. It "fell as soon as the political drain had ceased, but it rose again "in the middle of this century, when railways and the develop-" ment of new countries made a great new demand for capital. These new demands have not slackened, but the rate of "interest is again falling fast, in consequence of the great " recent accumulation of wealth in England, on the Continent, " and, above all, in America."

I have thought that it might possibly prove interesting and instructive to give a graphic representation of the average price of Consols from 1787 to the present time. A glance at this will enable us to see at once the principal features of the fluctuations. I may mention that the lowest point touched by 3 per-cent Consols was  $47\frac{1}{2}$  in 1797, at the time of the mutiny at the Nore, and the highest  $103\frac{3}{4}$  in 1887.

The high rate yielded by Consols during the latter part of the last century and the beginning of this, is sufficiently explained by the troublous times which so far reduced our national credit as to necessitate the borrowing of huge sums at the onerous rates of  $5\frac{1}{2}$  per-cent and over. Thus we find that in 1797 £112, 10s, of 5 per-cent stock was offered for £100 cash. Again, £175 of 3 per-cent Consols, £20 of 4 per-cent Consols, and an annuity-certain of 6s, per annum for 63 years was given for £100 cash. Also, in another case, £219 of 3 per-cent Consols was given for £100 cash. The actual yield obtained by investors who purchased Consols at the above rate is not, I think, any very reliable measure of the current rate of interest obtainable at the time, as

the probability of a substantial rise in capital value was undoubtedly a great attraction. Lord Rosebery, in his admirable essay on Pitt, refers to this in the following words:-"It has "been said that he (i.e., Pitt) should have borrowed in a stock " of a denomination more nearly corresponding to the actual "credit of the State. . . . This much may, however, be said, that Pitt had no choice. He was borrowing on a scale "unknown in the history of the world, and he had to borrow, "not in accordance with his own views, but with those of the " lenders. He made repeated attempts to borrow at 5 and 4 "per-cent, but met with no response . . . in 1796 he " offered options in 3, 4, and 5 per-cent stock. According to the "market price, the option in 5 per-cents was the most favourable "to the lender, but 85 per-cent was taken in 3 per-cent stock, " and only 11 per-cent in 5 per-cent stock . . . had he not " offered the temptation of stock which was certain to rise sooner " or later in capital value he could not have secured the requisite "means for carrying on the war."

From 1801 to 1815 £480,000,000 was paid into the Exchequer in return for-

> £572,000,000 Three per-cents. £37,000,000 Four per-cents. £99,000,000 Five per-cents. £316.529 Terminable Annuities.

Following on the conclusion of the Napoleonic wars, we notice that the price tends on the whole to increase, although there are occasional severe downward movements, which may generally be accounted for by the events of the times. Thus the effects of the panics of 1825 and 1847 seem to be particularly evident. After 1852 the fall is the result of the Crimean War. In 1866 the failure of Overend, Gurney and Co. is marked by the lowest average price of Consols since 1848. The price, quickly recovering, remains very steady from 1867 to 1874, when the gradual appreciation commences, which continues until the year preceding Mr. Childers' conversion. In 1884 and 1885 we have slightly lower prices, and then again a rise until the year of Mr. Goschen's great conversion.

In order to appreciate the significance of the various fluctuations of the prices of securities it is necessary to recall the principal financial events of the present century, and for convenience of reference I give below a short list of the more important :—

1825, 1837, 1847, 1857, 1866 Panics. Bank losses of £360,000 (Fauntleroy forgeries).

1831 Reform agitation. 1835, 1845-6 Railway manias.

1844 Bank Act.

1848-53 Large gold finds in Australia and California, which led to a belief in a general reduction in the value of money. In February, 1853, the rate of interest on Exchequer Bills was reduced from  $1\frac{1}{2}d$ , to 1d, without causing any offers for repayment. Consols were at par, and Mr. Gladstone proposed a conversion to  $2\frac{1}{2}$  per cent, the failure of which was largely attributable to the commencement of Russian troubles.

1854-56 Crimean War.

Limited Liability Act. 1855

Increase in Trustee Securities (Lord St. Leonard's Act). 1560

1865 End of American War.

1867 Companies Act.

Albert Failure for £5,000,000. 1569

1871 Treaty of Paris-Franco-Prussian War-indemnity of £200,000,000 was paid 1871-1873.

1880 - 5Agrarian trouble in Ireland. Mr. Childers' Conversion. Mr. Goschen's Conversion. 1854 1888

1558-9 Large increase in list of Trustee Securities.

Baring Collapse. 1890

1593 Australian Banking Failures.

As regards other securities than Consols, there appears to be no advantage in attempting to obtain a record of prices previous to 1870; since, even if it were readily obtainable, I do not think it would materially assist us to form any idea of the probable future decline in the rate of interest. During the years immediately preceding and succeeding 1870 the whole condition of trade and commerce was revolutionized owing to the sudden growth of railways, telegraphs, &c., and the opening up of large tracts of country in America and Southern Europe for the production of food supplies. Thus, as Mr. Noves points out in his interesting work, "Thirty Years of American Finance", "The Russian rail-" way system grew during this period from something like 2000 " miles to upwards of 13,000. In Austro-Hungary the per-" centage of increase was almost equally large. All of these new "transportation lines, like our own New Grainger Railway, were " at once engaged in carrying to the seaboard supplies of grain, "which never before had reached an export market. . . . . "The problem of an earlier generation had been how to feed the "constantly increasing population; a wholly new problem was "presently to arise, based on the question how to find a ready and " profitable market for the year's output of bread stuffs. Prices,

"in short, which rose almost continuously throughout the world during the period of slack production from 1858 to 1873, "receded almost as continuously in the ensuing generation." In this country it is true there had been considerable activity in promotion of railways at an earlier period than that referred to by Mr. Noyes, but the enormous growth of railways in Great Britain during the latter part of this century may be gathered from the fact that in 1850 the mileage was only 6,621, and in 1897 it was 24,487.

Mr. Van Oss considers that the long depression in trade experienced in England during the seventies was the natural result of overtrading and speculation, which had been directly fostered by the various inventions which at first acted as trade stimulants. In 1881 there was a slight recovery, but as regards general trade a further decline was witnessed in 1884-5, after which Mr. Van Oss has shown, in a most interesting and convincing manner (see his "Decade of Finance"), that the financial world witnessed a complete cycle; trading activity increased steadily from 1885 to 1890, when the climax was reached, after which it steadily declined. It is well known that trade is subject to these cycles of alternating activity and depression, and the prices of a great number of securities move in a similar manner, but the securities which are suitable for investment by Life Assurance Companies, being invariably chosen more particularly with a view to their safety, are not, for the most part, subject to these periodical inflations; in fact, it is after the climax of speculation has been reached, and the inevitable "slump" has occurred, that the some-time speculators develop an unwonted degree of caution, and insist on investing only in securities whose monotonously regular payments of the same low rate of interest places them far out of reach of "booms" and "slumps." Thus we find that in years of slack or declining trade those securities in which the rate of interest is fixed will tend to appreciate in value, whilst other securities which will feel the effect of stagnation in the form of decreased dividends, will fall in price more or less rapidly, according as their previous rise has been warranted to a less or greater extent.

It is, I think, very important to clearly recognize that the rate of interest on first-class securities has been for a very considerable period, tending to fall *steadily*, and any very rapid depression will, I think, be capable of some explanation which will be found to be quite independent of the steady fall.

The rate of interest obtainable in younger and less settled countries is necessarily higher, but speaking very generally, it would seem that the rate obtainable on first-class securities (where the risk is small) must necessarily tend to settle down to an universal level. Thus, even allowing for the undoubted patriotism of our American cousins, it is surely a significant fact that the war loan of £40,000,000 raised last year was applied for seven times over.

Referring to this country alone, it is generally understood that the fall in interest rates has resulted from a recognition of the first-class nature of the securities offered and the constantly increasing competition of investors due to the rapidly accumulating wealth of the Nation. At the present time, the annual savings of this country are said to amount to something like £200,000,000, in face of which fact Mr. Mackenzie's forecast of the total amount of the funds of assurance companies in the future does not appear in any way exaggerated. In the November number of the Bankers' Magazine, Mr. Mackenzie points out that, according to the returns issued in 1872 and referring to the financial year 1870, the invested funds of assurance companies in Great Britian amounted to £106,000,000, whereas the total amount in 1896 was £234,000,000; and, estimating the future amounts by means of the increase shown of the last few years, Mr. Mackenzie considers that they will be £500,000,000 in 1921 and £1,000,000,000 in 1946. It may be argued that the large number of endowment assurances which will mature in future years, will possibly result in a lower total of invested funds than the above estimate, but it may be pointed out that several offices have found that the sums due on matured endowments are to a large extent offered as purchase money for annuities, and we know that agents, in general, look forward to the maturing of endowment assurances as an unanswerable argument for further insurance.

The fall in the rate of interest, however, has at least one redceming feature, and that is the fact that it causes all fixed interest securities to appreciate in value. If a fixed rate of interest could be obtained for all future years on a given sum invested now, it follows theoretically that for every fall in the general rate of interest obtainable on similar securities, the value of the investment should appreciate to the extent of a perpetuity of the amount of the decline. Thus, for example, if I can to-day purchase perpetual debenture stock to yield £2. 15s. per-cent per annum on the amount invested, then when the general rate

obtainable on such security shall have fallen to  $2\frac{1}{2}$  per-cent, the value of each £100 previously invested should be

£100 + 
$$\frac{.25}{.025}$$

that is

In other words, the increased rate obtained by an investor who bought when the general rate of interest was at a higher level, is at once discounted in perpetuity and added to the market value of the security. Similarly, in the case of securities the interest on which is guaranteed at a fixed rate for a certain number of years, an increase in the capital value due to a fall in the general rate should, theoretically, be exactly equal to an annuity-certain of the amount of the fall. If, therefore, an investment has been made at the time when the interest yield was higher, and after several years it is desired to realize on the investment, it is of course perfectly allowable to consider the increased capital value as an additional interest yield; in fact, if it is only desired to ascertain which particular class of investment would have proved the most remunerative, I think this is the most simple method of doing so. But the labour necessary for making an exact calculation of this kind would be considerable and I have, therefore, appended an example of a table which may prove useful in this direction. Some explanations of the formulæ used and the way in which they were obtained will be found in the Appendix.

How should this increase in the capital value of invested funds be treated by assurance companies? Some offices have carried the increase in value into the Revenue Account. By far the more usual method, however, is to allow the securities to remain at cost price. In this way it is possible for a large reserve fund to be accumulated, the necessity for which, although probably quite evident to the directors of the company, could not possibly be fully appreciated by the general public if presented in another form. This practice has the effect of concealing the full extent of the fall in the rate of interest.

Offices which had large funds many years ago will in most cases have made large profits from the increased value of their investments, and, by still entering them at their original cost, they are able to show a comparatively high rate of interest. It appears to me, however, that if this method is adopted, it must, nevertheless, be necessary to make a very thorough examination of the actual position. If the business be a rapidly-increasing one there will be large annual amounts to invest at a lower rate

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of interest, and the absolute necessity for some re-adjustment of the premiums charged may not be so apparent as it would be in the case of a younger Company. It would of course be a difficult question to decide to whom the profit arising from increased value of investments actually belongs, but it certainly would seem fairly evident that the greater part of it cannot belong to the more recent members, although they in their turn may become entitled to a similar profit if the interest yield continues to fall.

As already stated, the contracts of Life Assurance are calculated on the basis of a uniform rate of interest being obtainable. And if there is any danger of the rate on general investments falling, it would seem all the more important to secure as large a proportion as possible of those investments on which the rate of interest cannot be reduced for a very long period, if at all. Even when a large amount of the funds is invested in this way, the effect of a fall in the rate of interest must be considerable in its effect on Assurance Companies, owing to the fact that the premiums for the most part are payable annually, and consequently fresh investments become necessary as the reserves increase. When interest has fallen to a certain level, it must become advisable to lower the standard rate of valuation, and to do so an increase in the reserve becomes necessary. How is this increase to be provided? I think that the increase in the value of the investments is the natural provision for making the necessary alteration, and it should therefore be strictly guarded and used only for this purpose; certainly none of it should be used to provide bonus. In the new assurances to be effected in the future (assuming the interest to fall to a still lower level) it will apparently be necessary either to re-adjust the scale of premiums or encroach on the profit margin. In assurances already in force, any alteration of the premium will be impossible, and the importance of some provision for a change in the standard rate of valuation should be all the more carefully considered.

It must be remembered that in the past there were probably a large number of investments open to Assurance Companies in which the certainty of an increasing value could be set against the declining rate obtainable on first-class securities, and many offices have obtained larger profits in this way than they could have done had the rate remained uniform. At the present time it is so generally believed that the rate will fall, that it is becoming more and more usual, in the case of fresh issues of Debentures, &c., to provide for some fixed date for repayment, so that it is probable that the profit obtainable from the increased value of securities will be less in the future than it has been in the past.

If it were possible to forecast the rapidity with which the rate of interest will fall in the future, it would be much more easy to provide for all contingencies. In other countries, where the rate is now comparatively high, the risk of a rapid decline is necessarily greater, and one is almost inclined to think that it may be found necessary to issue Policies with some special clause providing for the contingency of interest obtainable in future years.

A reference to the table at the end of this paper will, I think, conclusively demonstrate how much more profitable Railway Debentures and Preference Stock, and other similar investments, have proved in past years than mortgages on real property, which have always been regarded as such a desirable form of investment. In the past, the rate obtainable on mortgages was much higher, but as the general rate fell the lenders have merely had the option of having the amount repaid or reducing the rate.

It is extremely difficult to obtain any exact information as to the rate of interest obtained during the last few years on mortgages on freehold property, as the rate necessarily is so largely dependent on the nature of the property and the amount of the loan. It would seem that in 1880 the rate varied from about 4 to 5 per-cent (the latter only in cases of small advances); the rates obtainable, however, steadily declined, and by 1893 applications for reduction in the rate were becoming fairly numerous, and on large amounts the request was generally granted,  $3\frac{1}{2}$  per-cent being a very usual rate. At the present time many large mortgages are arranged at  $3\frac{1}{5}$  to  $3\frac{1}{4}$  per-cent, and in some cases at even a lower rate. When granting a reduction of the original interest rate, it is often stipulated that the mortgagor shall agree not to pay for some fixed period, usually 6 to 10 years, and this would certainly seem to be a very advisable arrangement, and one which the Court would uphold. Thus, I find in Mr. Ashburner's Treatise on Mortgages that cases of 7 and 9 years have been allowed, and presumably would be again, although an agreement not to repay for 30 years was not allowed, neither was another for 20 years; the last ease, however, was possibly influenced by the fact that the parties concerned were Solicitor and Client.

On leasehold property a rather higher rate can be obtained, owing, to a very large extent, to its not being admitted as a trustce investment; and, as in many cases, the security is unquestionable, mortgages on this class of property are, I think, rather more favourably considered by some offices now than was formerly the case.

In the larger towns of the United States 4 to  $4\frac{1}{4}$  per-cent can, I believe, still be obtained on first mortgages of freeholds, but although the rate is attractive the initial expenses must be considerable, and difficulties may arise in the event of a foreclosure becoming necessary, when, according to local laws, the lender possesses no control over the property unless it is vested in the names of citizens.

In Melbourne, Sydney, Wellington, Montreal, and other large colonial towns,  $1\frac{1}{2}$  per-cent can still be obtained on first-class freeholds; but recently some transactions at 4 per-cent have been heard of. Rates on the Continent are in many cases considerably higher than in England, although it is certain that a substantial fall has been experienced during the last 25 years of peace; thus I find, from information supplied by gentlemen whose experience in the matter is exceptional, that in Holland the rates obtainable in mortgages are estimated as follows:

At the present time, where  $3\frac{1}{2}$  per-cent is agreed to, the nature of the security must be absolutely first-class, with a very large margin of surplus value.

In this class of investment there is, of course, no possibility of any appreciation of the amount invested; and speaking of mortgages on freeholds in England, only a very moderate decline in the general rate of interest is necessary to result in a higher actual gain arising from an investment in even the highest class of Trustee Security, yielding now, say, £2. 15s. per-cent, than would result from a mortgage at  $3\frac{1}{8}$  per-cent; since the mortgage will almost certainly be offered for repayment when the rate has fallen a further fraction. There is, of course, the possibility of a rise in the rate of interest, and the consequent depreciation of marketable securities; but this is for the most part considered so improbable that it may practically be left out of consideration in any extended view of the future.

44 91 Dec 1904

I have stated above that in some cases where the fall in the rate has been peculiarly rapid, it was advisable to look for some further explanation than the gradual fall of the general rate of interest on first-class securities. This is, I think, the case with Consols. In past years they were undoubtedly an excellent form of investment, but at the present time I fail entirely to understand why such large amounts are held by assurance companies. By reference to the annual returns of assurance companies, it is impossible to find the exact amounts held, owing to the method of grouping all British Government Securities together, but we are able, by reference to the separate accounts, to form some idea of the amounts, and I have therefore extracted the following particulars:

| At 31 Dec. 1894—   |   |
|--|---|
| 40 Offices doing Life Business only (in- cluding Industrial) | B. Gov. Securities £3,259,509<br>Bank Stock 485,943   |
| 16 Offices doing Life  | B. Gov. Securities 1,034,218 Bank Stock   |
| held l<br>Total ar   | nount of B. Gov. Sees. stated to be } £4,293,727  by Life Depts., as above }  count of Bank Stock stated to be }  by Life Depts., as above }  661,642   |
| At 31 Dec. 1895—   |   |
| 38 Offices doing Life Business only (in- cluding Industrial) | B. Gov. Securities 3,186,515<br>Bank Stock 543,619  |
| 17 Offices doing Fire  | B. Gov. Securities 1,034,233 Bank Stock . 180,319 epts. of B. Gov. Securities £779,318 fices Bank Stock . 44,450  |
| held<br>Total A  | mount of B. Gov. Secs. stated to be by Life Depts., as above  |
| At 31 Dec. 1896—   |   |
| 36 Offices doing Life Business only (in- cluding Iudustrial) | B. Gov. Securities £3,456,408<br>Bank Stock 603,166   |
| 16 Offices doing Fire and Life Business Fire D 6 Off         | B. Gov. Securities 1,100,461 Bank Stock 180,319 epts. of \ B. Gov. Securities £801,880 fices \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \  |
| Office doing Life and \( \) Accident Business \( \)          | B. Gov. Securities 8,126  |
|  | mount of B. Gov. Secs. stated to be £4,556,869 by Life Depts., as above }   |
| Total A  | mount of Bank Stock stated to be to |

Thus it will be noticed that British Life Offices are known to have held British Government Securities (mostly Consols) at the end of the three years 1894, 1895, 1896 to the extent of £4,293,727, £4,220,748, and £4,556,869, which amounts include sums held by various offices whose accounts do not permit of our separating the figures relating to other departments.

I find also that 26 of the British Life Offices held no Consols at 31 December in each of the years referred to as above. Notwithstanding the increase in the total amount of Consols held at 31 December 1896, it will be found by reference that several offices took advantage of the high prices of 1896 and decreased their holdings. It must also be remembered that in some cases Consols are held merely as a temporary investment for money which it is found impossible to invest more favourably at the moment.

Now turning to the graphic representation of the average prices of Consols, we find that the high levels have mostly occurred at times of low Bank Rate, the reason being that at such times Consols are largely sought by investors with large balances which it is difficult to invest in any other way, and also that at times of low Bank Rate it is possible to buy Consols with borrowed money and make a fair margin of profit. The same remark applies, to a certain extent, to other securities, but Consols are peculiarly suitable owing to the very small expense involved in transfer. From 1891 to 1896 the rise in the price of Consols was phenomenal. Undoubtedly the general level of prices of firstclass securities was rising, but I think a glance at the "spirelike" representation of the rise should convince us that there were special causes at work. True, in face of the absorption of such large amounts by Government Departments, and the decrease of the total outstanding amount due to the operation of sinking funds, it might perhaps have been expected that the rise in price would be more rapid than that of other Trustee Securities; but even after making all allowance, the rise is, I think, abnormal. In the April 1896 number of the Nineteenth Century Mr. Van Oss pointed out that the high price of Consols was due mainly to the following causes:

- I.—Temporary. (a) Timidity of Investors.
  - (3) Cheapness of Money.
- II.—Permanent and Natural. The gradual decline in the Loan Value of Capital.

- III.—Artificial. (a) The diminished supply of Consols due to operations of Sinking Funds.
  - $(\beta)$  Increased demand for Consols by Government Departments and Trustees.

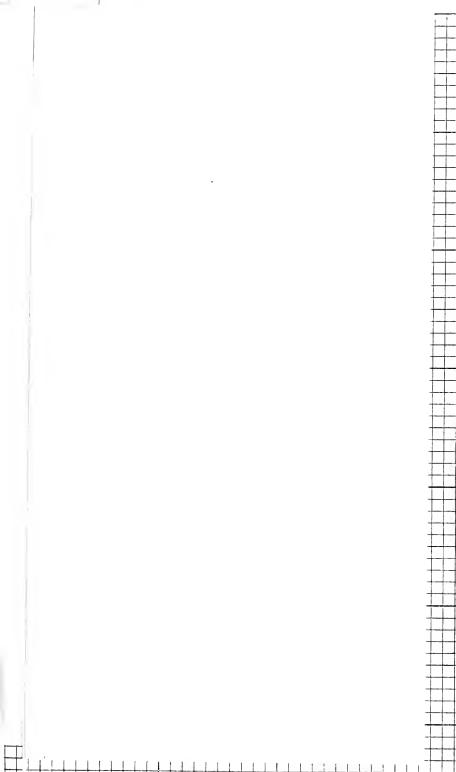
As regards the first cause, namely, the timidity of investors, this began with the Baring Collapse, and was increased by the Silver Crisis in America and the Banking Crisis in Australia. These causes led to distrust of Foreign and Colonial investments, and consequently an abnormal demand for Home Securities. Consols were resorted to for temporary investment, since the Banks gave next to nothing for deposits. Then, again, the Bank Rate had been at 2 per-cent for over two years, "Call Money" was lent at  $\frac{1}{4}$  to  $\frac{1}{2}$  per-cent per annum, and Bills were being discounted at  $\frac{3}{4}$  per-cent per annum and less. This state of affairs necessarily favoured operations in Consols. Thus, for example, a person with £10,000 could purchase £100,000 nominal Consols, which at £110 would cost £110,000, since he could borrow £100,000 on this at about 1 per-cent.

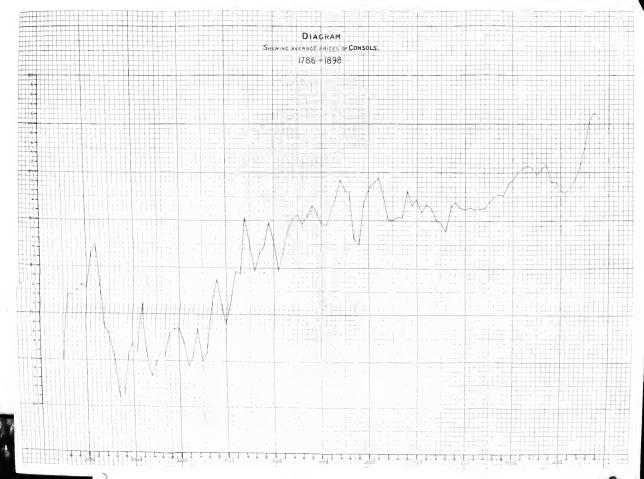
Thus he paid £1,000 interest, The Consols yielded £2,750,

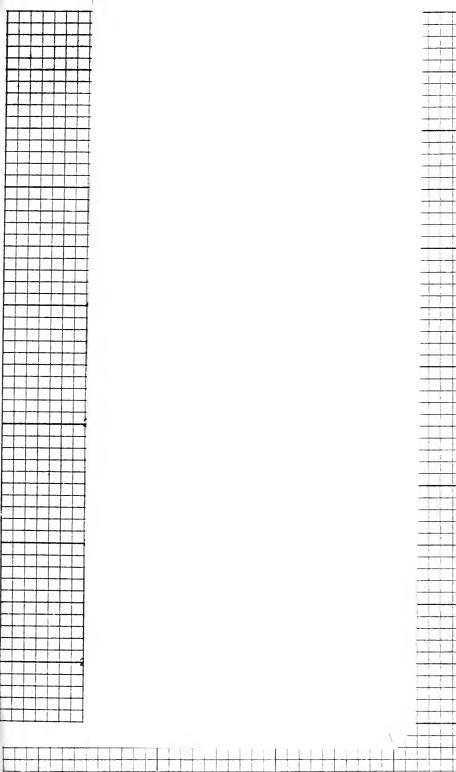
and he therefore made £1,750 on his £10,000, that is,  $17\frac{1}{2}$  percent. Of course, he ran the risk of depreciation, but he could well afford to run *some* risk, and he also had a good chance of making a further gain if the price appreciated.

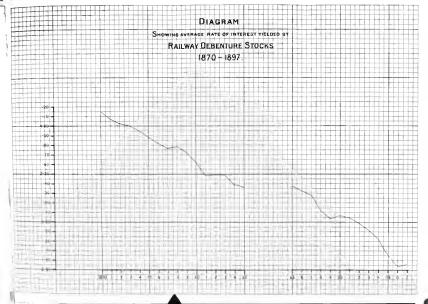
Under these circumstances it seems surprising that so few offices took the opportunity of selling in 1896, especially when we consider that Consols bought at the average price of 1891 and sold out at the average price of 1896 would be equivalent to an investment for five years at about  $5\frac{1}{2}$  per-cent compound interest. At the present time I fail to see any reason for holding Consols unless it is with the intention of selling out at a favourable opportunity, which may very possibly again occur. As an investment to be held until the date of redemption in 1923 they will yield only about 2 per-cent. It is sometimes argued that they are held because of the ease with which they can be converted into cash at a moment's notice, but the necessity for such a characteristic is surely less important than is often imagined.

It appears to me that Consols are unsuited for permanent investment of the funds of a Life Office for three main reasons:



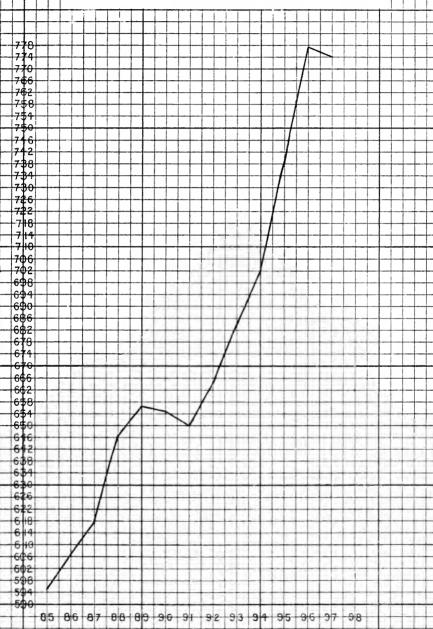






## DIAGRAM REPRESENTING THE SUM OF THE PRICES OF SIX CORPORATION STOCKS,

YIELDING A TOTAL OF \$20.10, INTEREST PER ANNUM.





- (1) Their fluctuations are considerable, and, in the event of a sale being necessary, it may possibly result in considerable loss.
- (2) They are unnecessarily dear, because they are sought after by a class of investors to whom the small expense of buying and selling is of great importance, whereas this is of very little moment to a Life Office.
- (3) The larger amounts of Consols are held by those Offices whose annual income is very large, and whose funds are increasing so rapidly that they are most unlikely to be called upon to meet a sudden and unexpected amount of claims.

I have given in Diagram 1 a representation of the capital increase from 1885 in six corporation stocks, namely:—

Metropolitan 3 per-cent Redeemable 1941.

Birmingham 3½ per-cent ,, 1946.

Hull 3½ per-cent Irredeemable.

Liverpool 3½ per-cent ,,

Manchester 4 per-cent ,,

Notts 3 per-cent ,,

These six stocks have been chosen as representative of the rise in value of stocks which have a long term, for which a fixed rate of interest is guaranteed. The irredeemable stocks are, of course, redeemed only by means of purchase in the open market; they are all trustee securities.

The price of each stock has been taken as the mean of the highest and lowest for each year, and does not therefore show the full extent of any rise or fall; thus, for example, in 1896 the sum of the highest prices of the six stocks was £818, and for 1897, £789\frac{3}{8}. The effect of the Baring crisis on the prices of 1890 and 1891 is very plainly seen, but the main feature of the diagram is the sudden rise from 1891 to 1896. It is well known that the prices of 1896 were unduly inflated; thus, at one time the Metropolitan 2\frac{1}{2} per-cent Stock stood at £107. Many smaller corporations, whose stock is not available for trustees, were encouraged by the high prices to float their loans publicly. These stocks necessarily yield a higher rate of interest than the larger corporation stocks, not only because trustees cannot invest in them, but also because they consist, as a rule, of smaller amounts, and consequently have a less free market.

Since 1896 the prices have fallen considerably. The prices of the immediate future will depend largely on the state of the money market; but the average prices of the future will necessarily be mainly governed by the general level of interest on first-class securities.

I think that the most striking representation of the steady fall in the rate of interest on first-class securities is obtained by a comparison of the rates vielded on Railway Debenture I was particularly anxious to obtain some reliable information as to the rates obtained since 1870, and for this purpose I made many enquiries for a list showing the highest and lowest prices for each year. Whether such a list is in existence, I do not know; but, if it is, I was unable to find it. I referred to Mathieson's lists, and found, to my astonishment, that for 1873 to 1878 the lists contained prices of 188 securities, amongst which not a single railway debenture was included. I therefore was forced to turn to the daily lists published in the Times, and, in order to obtain an average price, I decided to take three quotations for each year. To avoid any difficulties arising from the dividends being deducted from the price, I thought that 1 February, 25 June, and 1 December would be the most suitable dates, and most of the prices were taken at about those times, although in some eases the date was a few weeks earlier or later, owing to the fact that the stocks were not quoted for several weeks together. Misprints were by no means uncommon, but, I think, by a careful comparison of the prices, I succeeded in avoiding these. The stocks taken were-

Great Northern 4 per-cent.
Great Western 4 per-cent.
London and Brighton 4½ per-cent.
London and North-Western 4 per-cent.
London and South-Western 4 per-cent.
Midland 4 per-cent.

The first part of Diagram 2 representing the rates of interest obtainable from 1870 to 1885 inclusive, was based upon the average prices as derived from entries in the *Times* lists. The second part of the diagram was obtained by taking the mean of the highest and lowest recorded prices for each year, and where the stock was subsequently exchanged for its equivalent of stock yielding a lower rate of interest, the proportionate amount of new stock was taken.

I think it will be found that the first method will show a slightly lower rate than the other, and I have therefore presented the two separately. For the year 1885 it will be noticed that the rate shown is 3:365 per-cent by the one, and 3:375 per-cent by the other, and I do not think we shall be falling into any very great error by considering the return upon Railway Debenture Stocks from 1870 to 1896 as fairly represented by the two diagrams.

On the 21st of December 1868, Mr. Coles drew attention to several points in favour of Railway Debentures as investments for the funds of Life Assurance Companies. He pointed out that they were amply secured and could be bought to yield from 4 to  $4\frac{1}{2}$  per-cent; those Companies who followed Mr. Coles' advice have reaped a rich reward. For instance, Midland 4 per-cent Debenture Stock could have been purchased in 1870 for £98; whereas its present market value is about £150 (that is taking the equivalent of  $2\frac{1}{2}$  per-cent stock). Again, London Brighton 4 per-cent Debenture Stock could have been purchased in 1870 for £90, and its present value is about £150.

Following the graphic representation of the rate of interest yielded by Railway Debentures, we notice that the fall is almost continuous, but is very much more rapid after 1885. It must be remembered that a fall of a shilling is much more important when the rate is 3 per-cent than when it was 4 per cent, and the rapid decline after 1885 is therefore greater in its effect than would be perhaps at first imagined. Midland 4 per-cent Stock in 1885 could be bought for £120, which shows a rise of £20 for a fall in the yield from £4 per-cent to £3 $\frac{1}{3}$  per-cent, whereas a further fall of £ $\frac{2}{3}$  per-cent results in a further rise of £30, and a still further fall of £ $\frac{2}{3}$  per-cent would mean a further rise of £50.

We must therefore regard the fall since 1885 as very severe compared with the fall from 1870 to 1885.

From 1870 to 1885, Railway Debenture Stocks were, I think, very steadily absorbed by many financial undertakings having large and accumulating funds which it was necessary to invest in first-class securities. Also during this time it was very usual to so word the "investment clause" in settlements, that Railway Debenture Stocks should be available, and in eases where only Railway Bonds were mentioned, the Trustees were empowered by the 1871 Debenture Stock Act, to invest in Debenture Stock.

I think, however, that Mr. Childers' conversion in 1885, started a general shifting of Trustee Investments, and Railway

Debenture Stocks were soon recognized as offering decided advantages. The movement was accelerated by the great conversion of 1888, as also by the 1888 Order of Court and the 1889 Trustee Act, which distinctly recognized Railway Debenture Stocks as Trustee Investments. Then again in 1890 came the Baring collapse, to be closely followed by the Australian Banking crisis; and all this time there was a continuous demand for first-class securities, to an extent which in previous years would have seemed impossible. These causes are, I think, sufficient to explain the rapid rise in prices which culminated in 1896.

The record of interest yields on other classes of securities although instructive and interesting, I am forced for the present to leave, owing to the time at my disposal having been unexpectedly curtailed. They would, however, show very much the same results, namely, that the rate of interest on first-class securities has been continuously declining for many years, but has shown a very rapid fall during the years 1890 to 1896.

In January 1858, Mr. Samuel Brown read a most instructive paper before this Institute (vol. ii, p. 142), in the course of which he remarked, "Some of the old companies not having yet "considered the new sources of investment which have of late "years opened up, are still realizing  $3\frac{1}{2}$  per-cent on the average, "on their whole funds; whilst others of equal standing have, by "changes and by skill and attention to this business, secured " $4\frac{1}{2}$  per-cent on the whole. It will probably be found that "4 per-cent is very near the average interest on capital of all the "existing companies, including their productive and non-"productive funds."

In 1872, Dr. Sprague, in stating his opinion as to the proper rate of interest to be used by the Australian Mutual Provident Society, said that he believed in England Life Assurance Companies could safely reckon on 4 per-cent as likely to be realized during the next 30 years. Again, in November 1884 (J.I.A., vol. xxv, p. 81), he stated that "For, say the next 30 " years, the present average rate may be maintained by those " offices which are ready to move with the times." Unfortunately the latter forecast at all events seems unlikely to be realized.

The extent of the fall in the rate of interest realized on the funds of life insurance companies has been investigated by several writers. Mr. Deuchar discussed the question in a paper printed in the *Journal* (vol. xxviii, p. 442), and gave figures based upon the answers returned by the Companies to question 9 of the

6th Schedule of the Life Assurance Companies Act, of 1870, but, as Mr. Mackenzie pointed out in his paper read before the Institute just eight years ago (see vol. xxix, p. 185), this was, for many reasons, not a satisfactory basis. Mr. Mackenzie took the summaries of the Revenue Accounts, given as an appendix to the Board of Trade Returns published in 1883 and subsequent years, and calculated the effective rate of interest on the total life funds by Mr. Hardy's formula.

Making a similar calculation on the basis of Dr. Sprague's Summary of the Board of Trade Returns published in 1872, he was able to compare the rates obtained in the years 1881 to 1888 with the rate obtained in 1870, the conclusion being that the rate of interest had practically fallen in 18 years from  $4\frac{1}{2}$  to  $4\frac{1}{8}$ 

per-cent.

Mr. Paulin, in a paper read before the Actuarial Society of Edinburgh in 1895, used a similar method, and extended the table of interest earnings to 1893, adding also the rate for 1875, based upon a special examination of the Blue Book referring to that year. He introduced a correction in several years on account of offices making returns for broken periods.

It may be questioned, however, whether the basis adopted by Mr. Mackenzie and Mr. Paulin gives entirely satisfactory results. It is an obvious weakness that while we have, on the one hand, the rates for a series of consecutive years, commencing with 1880, on the other hand the rate for the year 1870, which is taken as the standard of comparison, stands by itself; and we cannot use the rates of neighbouring years to assist us in forming any judgment as to whether the rate for that year was modified by exceptional circumstances or not.

That there were exceptional circumstances affecting the figures for 1870 becomes apparent by reference to Dr. Sprague's Treatise on Life Assurance Accounts.

On p. 20 he says, referring to "Interest and Dividends", ". . . this item is overstated by several companies in "consequence of their including in it the interest received in the "year and the interest outstanding at the end of the year."

On p. 21 he says:-

"An examination of the accounts of the Companies shows "that it is becoming more common to take credit for the current "interest. Of course, in the year when it is for the first time "brought into the accounts the interest for the year is unduly "swelled."

(At least two Companies brought accrued interest into account for the first time in 1870.)

Again, on p. 122: "Some companies . . . . . have "carried the whole of the interest received to the Life Account, "the effect of which is that, unless the accounts are carefully "looked into, the whole of the interest received by each of these "companies appears to have been produced by its Life Funds; "whereas it is, of course, partly produced by the investments "representing the Fire Fund and the Shareholders' Paid-up "Capital."

And finally, on p. 140: "In the cases where the interest "received is divided between different accounts, information is "seldom given as to the principle on which it is so divided."

I have examined the accounts of the Blue Book in the light of these cautions, and after making corrections in cases where they are obviously required, the accounts of 12 companies being thereby affected, I find that the average rate of interest derived from Dr. Sprague's summary is reduced from 4.512 per-cent to 4.415 per-cent.

Dr. Sprague's warnings have to be borne in mind in examining the accounts of all subsequent years, particularly in reference to the item of accrued interest when brought into account for the first time, (an instance of this occurred as lately as 1892, but it is of course the earlier years that are chiefly affected by this correction). It must be remembered also that in the case of companies doing other classes of insurance in addition to "Life," the rate of interest passed to the Life Account is in many cases arbitrarily fixed, and as our object is to ascertain what rate of interest was earned by the Life Funds, and not merely what rate was apportioned to them, there is danger of introducing an appreciable error if we include such companies. In view of all these considerations, I have felt that the best course was to take a limited number of companies and make a critical examination of the accounts of each for every year from 1870 to the latest year for which a Blue Book has been issued. In order to avoid disturbing elements as far as possible, 12 of the leading companies, whose business is confined to ordinary life assurance, have accordingly been taken, and the effective rate of interest on the combined funds has been calculated for each of the years 1870 to 1896 by Mr. Hardy's formula. I may mention that the combined funds of these companies increased from 27 millions to 66 millions in the period.

The rate is given after deduction of Income Tax, and the effect of profit or loss on investments is shown separately. The average rate for each quinquennial period is also shown, this being found by adding together the mean interest-bearing funds of the five years of a quinquennium, and comparing with the total interest received in the same period; and a column has been added giving the rate of Income Tax for each year.

This Table shows that, for the companies in question, the highest rate was obtained in 1875. From that year, there was a steady fall at an average rate of about 6d. per annum until 1892, after which, the rate of decline suddenly increased to 1s. per annum.

| Year<br>ending<br>31 December | Rate of<br>Interest<br>per-cent | Profit or<br>Loss on<br>Investments<br>per-cent | Income<br>Tax from<br>5 April<br>of each<br>year | Year<br>ending<br>31 December | Rate of<br>Interest<br>per-cent | Profit or<br>Loss on<br>Investments<br>per-cent | Income<br>Tax from<br>& April<br>of each<br>year |
|-------------------------------|---------------------------------|---|--|-------------------------------|---------------------------------|---|--|
|                               |                                 |   | d.   |                               |                                 | -   | d,   |
| 1870                          | 4.280                           | 012   | 4  | 1885                          | 1.147                           | + 101   | 8  |
| 1871                          | 4.272                           | +.057   | 6  | 1886                          | 1:115                           | + .023  | $\mathbf{s}$                                     |
| 1872                          | 4.303                           | 006   | 4  | 1887                          | -4.121                          | + .078  | 7  |
| 1873                          | 4.389                           | + .060  | 3  | 1888                          | 4.094                           | + .017  | 6  |
| 1874                          | 4.408                           | + 114   | 2  | 1889                          | 4.117                           | - :092  | 6  |
| 1870-1874                     | 4.348                           | + .041  |  | 1885-1889                     | 4.119                           | + '028  |  |
| 1875                          | 4.458                           | + 181   | 2  | 1890                          | 4.139                           | 140   | 6  |
| 1876                          | 4.371                           | +.060   | 3  | 1891                          | 4.123                           | + .036  | 6  |
| 1877                          | 4.337                           | + 123   | 3  | 1892                          | 4.101                           | + .039  | 6  |
| 1878                          | 4.328                           | + .053  | 5  | 1893                          | 4.047                           | + .020  | 7  |
| 1879                          | 4.342                           | + 188   | 5  | 1894                          | 3.961                           | 040   | 8  |
| 1875-1879                     | 4.365                           | + '122  |  | 1890-1894                     | 4.071                           | 010   |  |
| 1880                          | 4:312                           | + .055  | 6  | 1895                          | 3.878                           | + 082   | 8  |
| 1881                          | 4.224                           | + 069   | 5  | 1896                          | 3.837                           | + 023   | 8  |
| 1882                          | 4.233                           | + 174   | $6\frac{1}{2}$                                   | 1300                          | 0 307                           | , 020   | 0  |
| 1883                          | 4.172                           | + 064   | 5  | 1                             |                                 | I .   |  |
| 1884                          | 4.197                           | + .052  | 6  |                               |                                 |   |  |
| 1880–1884                     | 4.226                           | +.082   |  |                               |                                 |   |  |

The rates earned by individual companies ranged, in 1870, from 3.803 per-cent to 4.496 per-cent, and in 1896 from 3.537 per-cent to 4.032 per-cent.

What conclusion are we to draw from an investigation of the

rates of interest yielded in the past? I think there should be little doubt on this head. We see from past experience how much more profitable some investments have proved than others, and we see that a correct anticipation of the probable future course of interest would have enabled many Insurance Companies to have greatly improved their position. At the present time Assurance Companies hold larger funds than at any previous date, and it is still more important that they should be able to take advantage of any compensation which may arise concurrently with a further fall in the average interest yield.

The greatest authorities are agreed that the rate of interest will fall still lower in the future, and to some extent I believe it would be possible to estimate the speed at which the fall is likely to continue. If the fall is rapid, then many securities which are now thought to be unsuitable on account of the low rate yielded, may, nevertheless, be the source of considerable profit, owing to their increased capital value. Temporary investments which yield only a slightly higher rate of interest than the fixed interest securities can easily be shown to be only superior in the event of the general rate of interest rising or only falling by very slow degrees. Thus mortgages on first-class security arranged eight or nine years since have not proved so profitable as other investments which, although yielding a low rate of interest, had the advantage of a fixed rate for an indefinite time.

If the rate of interest continues to fall, many offices will be under the necessity of lowering their standard rate of valuation, thereby necessitating a considerable increase in their reserves, and as already explained, I am strongly of opinion that the increase in the capital value of our investments is the natural provision for this necessity of the future. I am fully aware of the increasing difficulty of investing large funds, and that it is impossible to limit ourselves to investments on which the rate of interest will remain fixed, but I am persuaded that many of the securities which are now most eagerly sought after will not prove to be the most profitable and advisable, unless the forecasts of the future rate of interest prove to be incorrect. It is said by some "a great war will completely change the present conditions; undoubtedly we have read of such wars in modern magazines, but do we believe that such will ever happen? In any case the question is beyond the scope of the present paper. Trust moneys will continue to increase, and will eventually necessitate an increase in the list of "Trustee Investments", with the result that those added will

almost certainly increase in value, more particularly if the Stocks are irredeemable, or are to be redeemed at some distant period. Mr. Denny Urlin, in his "Handy Book on the Investment of Trust Funds", remarks: "The best Colonial Securities have for "some time been gaining favour with testators and settlors; and "many investment clauses have included 'the government "securities of the Dominion of Canada', as also some of the "Australian Colonies."

Some of the better class Colonial Corporation Stocks, where well secured and irredeemable for a lengthy period, seem likely to be of more value at a future date, when the total amount of Trust and other analogous funds may possibly have increased to such an extent as to make the figures of to-day appear small by comparison.

However, any attempt to recommend particular securities is certainly not my intention; I shall be more than satisfied if I am able to awaken some enquiry as to the probable future rate of interest, by means of which it may be possible to consider the subject of the investment of assurance funds from a correct theoretical standpoint as well as a practical and business-like one.

I feel bound, before closing the paper, to acknowledge the very great assistance which my friend, Mr. Duncan C. Fraser, of the Royal Insurance Company, has given me, and I have also to thank several other gentlemen who have very kindly supplied me with information which was otherwise unattainable.

## APPENDIX.

A Method of finding the Actual Yield on Stock sold at a premium, including the increase in Capital Value.

[The following note has been supplied by Duncan C. Fraser, M.A., F.I.A.]

Take a Perpetual Stock paying i per annum on each nominal unit.

Suppose the Stock purchased now at price  $1+p_0$  and sold at the end of n years at price  $1+p_n$ ; then if all interest earned in the meantime is invested in the same Stock, what is the actual yield; that is, what uniform rate of compound interest would produce the same result at the end of n years?

Let the price of the Stock at the end of each year be  $1+p_1$ ,  $1+p_2$ ,  $1+p_3$ , ...  $1+p_n$ .

Then 
$$1 \begin{Bmatrix} \text{invested at the commencement} \\ \text{commencement} \\ \text{of the 1st year} \end{Bmatrix} \begin{Bmatrix} \text{will amount} \\ \text{at the end of} \\ \text{the 1st year} \end{Bmatrix} \text{ to } \frac{1+p_1+i}{1+p_0}$$

$$1 \quad \text{,, 2nd year} \qquad \text{,, 2nd year to } \frac{1+p_2+i}{1+p_1}$$

$$1 \quad \text{,, 3rd year} \qquad \text{,, 3rd year to } \frac{1+p_3+i}{1+p_2}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$1 \quad \text{,, nth year} \qquad \text{,, nth year to } \frac{1+p_n+i}{1+p_n}$$

 $\therefore$  if 1 be invested in the Stock now and the annual interest invested in the same Stock, the amount at the end of n years will be

$$\frac{1+p_1+i}{1+p_0} \times \frac{1+p_2+i}{1+p_1} \times \frac{1+p_3+i}{1+p_2} \times \dots \times \frac{1+p_n+i}{1+p_{n-1}}$$

$$= \frac{1+p_n}{1+p_0} \left(1+\frac{i}{1+p_1}\right) \left(1+\frac{i}{1+p_2}\right) \left(1+\frac{i}{1+p_3}\right) \dots \left(1+\frac{i}{1+p_n}\right)$$

And we have to find x such that—

$$(1+x)^{n} = \frac{1+p_{n}}{1+p_{0}} \left(1 + \frac{i}{1+p_{1}}\right) \left(1 + \frac{i}{1+p_{2}}\right) \left(1 + \frac{i}{1+p_{3}}\right) \dots$$

$$\left(1 + \frac{i}{1+p_{n}}\right)$$

Taking logarithms

$$n\log(1+x) = \log\frac{1+p_n}{1+p_0} + \log\left(1+\frac{i}{1+p_1}\right) + \log\left(1+\frac{i}{1+p_2}\right) + \dots + \log\left(1+\frac{i}{1+p_n}\right)$$

$$\therefore \log(1+x) = \frac{1}{n} \log \frac{1+p_n}{1+p_0} + \frac{1}{n} \log \left(1 + \frac{i}{1+p_1}\right) + \&c.$$

$$\therefore \log_{10}(1+x) = \frac{1}{n}\log_{10}\frac{1+p_n}{1+p_0} + \frac{M}{n} \cdot i \cdot \left\{\frac{1}{1+p_1} + \frac{1}{1+p_2} + \dots + \frac{1}{1+p_n}\right\}$$

$$-\frac{M}{n} \cdot \frac{i^{2}}{2} \cdot \left\{ \frac{1}{(1+p_{1})^{2}} + \frac{1}{(1+p_{2})^{2}} + \dots + \frac{1}{(1+p_{n})^{2}} \right\} + \frac{M}{n} \cdot \frac{i^{3}}{3} \cdot \left\{ \frac{1}{(1+p_{1})^{3}} + \frac{1}{(1+p_{1})^{3}} + \dots + \frac{1}{(1+p_{n})^{3}} \right\}$$

- &c.

where M is the modulus of the common system of logarithms.

So far no assumption has been made as to the relations between the quantities  $p_0, p_1, p_2, \ldots p_n$ . In order to obtain a basis for numerical calculation, assume that the series

$$\frac{i}{1+p_0}$$
,  $\frac{i}{1+p_1}$ ,  $\frac{i}{1+p_2}$ ,  $\ldots$   $\frac{i}{1+p_n}$ ,

forms a Geometrical Progression. We may therefore put

$$\frac{1+p_1}{1+p_0} = \frac{1+p_2}{1+p_1} = \frac{1+p_3}{1+p_2} = \dots = \frac{1+p_n}{1+p_{n-1}} = 1+a$$
so that
$$1+p_1 = (1+p_0)(1+a)$$

$$1+p_2 = (1+p_0)(1+a)^2$$

$$1+p_3 = (1+p_0)(1+a)^3$$

$$\dots$$

$$1+p_n = (1+p_0)(1+a)^n$$

Making these substitutions,

$$\log_{10}(1+x) = \frac{1}{n}\log_{10}(1+a)^{n} + \frac{M}{n} \cdot \frac{i}{1+p_{0}} \cdot \frac{1}{(1+a)} + \frac{1}{(1+a)^{2}} + \dots + \frac{1}{(1+a)^{n}} \cdot \frac{1}{(1+a)^{2}} \cdot \frac{1}{(1+a)^{2}} + \frac{1}{(1+a)^{4}} + \dots + \frac{1}{(1+a)^{2n}} \cdot \frac{1}{(1+a)^{3}} \cdot \frac{1}{(1+a)^{3}} \cdot \frac{1}{(1+a)^{3}} \cdot \frac{1}{(1+a)^{3}} \cdot \frac{1}{(1+a)^{3}} \cdot \frac{1}{(1+a)^{3n}} \cdot \frac{1}{(1+a)$$

$$\log_{10}(1+x) = \log_{10}(1+a) + \frac{M}{n} \cdot \frac{i}{1+p_0} \cdot a' = -\frac{M}{2n} \cdot \left(\frac{i}{1+p_0}\right)^2 \cdot a''_n$$

$$+ \frac{M}{3n} \cdot \left(\frac{i}{1+p_0}\right)^3 \cdot a'''$$

$$- &c. \qquad (1)$$

where  $M = 4342945 \dots$ 

and the annuity  $a'_{\bar{n}}$  is to be calculated at rate a

and so on.

If  $p_0$  and a be positive, the second term is less than  $\frac{1}{2}i$ ; since M is less than  $\frac{1}{2}$ , and  $\frac{a'\frac{1}{n}}{n}$  less than unity. And similarly the 3rd term is less than  $\frac{1}{4}i^2$ , the 4th term less than  $\frac{1}{6}i^3$ , and so on. It will therefore be seldom necessary to take more than three terms of the formula. (Note.—The error in x will be about double the error in the logarithm.)

Putting  $p_0=0$ , that is, taking the purchase price as the par value, and retaining only three terms, the formula becomes—

$$\log_{10}(1+x) = \log_{10}(1+a) + \frac{M}{n} \cdot i \cdot a'_{n} - \frac{M}{2n} \cdot i^{2} \cdot a''_{n} \quad . \quad . \quad (2)$$

Again, since

$$\log_{10}\left(1+i\cdot\frac{a'_{|\tilde{n}|}}{n}\right) = \frac{M}{n}\cdot i\cdot a'_{|\tilde{n}|} - \frac{M}{2n}\cdot i^2\cdot \left(\frac{a'_{|\tilde{n}|}}{n}\right)^2$$

we may write

$$\log_{10} (1+x) = \log_{10}(1+\alpha) + \log_{10}\left(1 + i \cdot \frac{a'_{n}}{n}\right) \quad . \quad . \quad (3)$$

with an error in the logarithm, if we neglect higher powers of i than the second, of  $\frac{M}{2} \cdot i^2 \left\{ \binom{a'_n}{n}^2 - \frac{a''_n}{n} \right\}$ . The part of this expression within brackets is the difference between two positive fractions, each of which is less than unity,\* and therefore the approximation is better than if we took merely the first two terms in formula (2). From (3) we deduce

$$(1+x) = (1+a)\left(1+i\cdot\frac{a'_{\overline{n}}}{n}\right) \quad . \quad . \quad . \quad (4)$$

which gives a good approximation to the value of x.

A specimen table follows which shows the actual yield on a Stock held for 15 years and then sold at a premium on the purchase price. The table is calculated by formula (2).

\* The two fractions  $\left(\frac{a'_{\overline{n}}}{n}\right)^2$  and  $\frac{a''_{\overline{n}}}{n}$  are nearly equal.  $a'_{\overline{n}}$  is calculated at rate a, which may be assumed to be a low rate of interest.  $\therefore \frac{a'_{\overline{n}}}{n} = 1 - c$ , where  $c = \frac{1}{2}a(n+1)$  approximately. Also  $a''_{\overline{n}}$  is calculated at rate  $2a + a^2$ , or, approximately, 2a; and therefore we shall have, approximately, if n is not greater than say 20,  $\frac{a''_{\overline{n}}}{n} = 1 - 2c$ . But since c is small  $\left(\frac{a'_{\overline{n}}}{n}\right)^2 = (1 - c)^2 = 1 - 2c$  approximately.

Specimen Table, showing the Yield on a Stock sold at the end of 15 years at a Premium on the Price of Purchase.

| 500.                                    | 1.0777   | Sold Actual<br>fo Yield Yield          | 1020-28   | 1080-11  | 1180-02                                 | 0280-00  | 0880 69   | 0380-82   | 61 EO- 88   | 6980- 26   | 6980- 90   | 8280. 21                                | 55 0388   |
|---|--|--|---|--|---|--|---|---|---|--|--|---|---|
| 500                                     | 1.0939   | Sold Actual Syridd Yield Yield         | 1020+ 2820+ 0080+1620 8080+ 9220+ 2180+ 2220+ 9120+ 3120+ 9120+ 9120+ 3180+ 3120+ 0080+ 6280+ 6080+ 920+ 920+ 920+ 920+ 920+ 920+ 920+ 92 | 1920   1920   6080   8120   8180   1820   9240   1820   2720   2720   1720 | 030-030-030-030-030-030-030-030-030-030 | 0230-0120-0230-9520-2530-2530-6120-6120-6120-6120-6120-6120-6120-612 | 0330   0340   0500   0338   0500   0330   | 0180 8180 4120 9101 0001 0020 0020 0020 0020 0020 0 | 01.80 - 01.00 - 02.00 - 01.00 | 6250- 7020- 7020- 2020- 2020- 8750- 8850- 8850- 1820- 1820- 0820- 0820- 0820- 5720- 7010- 2720- 6110- 8520- 8210- 1520- 1520- 1810- 0520- 6810- 5520- 280- | 033 026 010 026 026 013 026 026 013 026 013 026 010 026 010 028 010 028 010 028 010 026 036 036 036 036 036 036 036 036 036 03 | 8780 8780 1880 1880 1880 1880 1880 1880 | -0850 -0850 -0466 -0450 -0466 |
| 200.                                    | 1-1103   | old Actual<br>fo Yield<br>ield Yield   | 0225 0308   | 0231 0318  | 0213 0327                               | 7880 2520  | 0261 0346   | 0220 0270   | 0220 0365   | 1 2780 8820  | F 1880- 2650   | 0300 -0391                              | 0315 0103 0   |
| Š                                       | 0251-1   | Sold Actual<br>to<br>Yield Yield       | . 2180- 2220-   | 0231 0326  | 02 10 0335                              | 0219 70315   | 0258 0351   | 1980, 9950  | 8280, 2270  | ESE0. 1820   | 2080- 8020-  | 2010: 2080                              | 0311 0411   |
| . 600                                   | 1-1-130  | Sold Actua<br>for Vield Vield          | 0219 (0325  | 0227 -0334   | 0236 0311                               | 0215 0353  | 0251 0362   | 5280- 5950  | 1880, 1270  | 1680. osao   | 0010- 8820   | 6010- 2620                              | 0306 -0119  |
| 010.                                    | 1.1610   | Sold Vetua<br>to<br>Vield Vield        | 0216 -0333  | 0221 -0343   | 0233 0352                               | 1980-1170  | 0250 0371   | 0820 6220   | 0267 -0389  | 0320 9250  | S010- 1820   | 2110 8550                               | 0301 0427   |
| ======================================= | 1.1783   | Sold Actual<br>10 Yield<br>Yield Yield | 212 03 12   | 0221 (0351   | 0980-0550                               | 0238 30370   | 6280- 9170  | 9255 -0388  | 2680 -8950  | 7010- 2720   | 9HO 0850   | 2210. 6820                              | 1810 782  |
| 21 9                                    | 1-1959   | Sold Actual<br>to<br>Yield Yield       | 0200 -0320  | 9217 0359  | 9556 -0369                              | 231 0378   | 4880- 5150  | 251 -0396   | 5010- 652   | 268 0115   | 1210 922   | 881 o 188                               | 293 -01 12  |
| 2 E I O                                 | 1-2138   | Sold Actual<br>to Yield Yield          | 6280- 9020  | 8980. 1120   | 222 0377                                | 231 -0386  | 680-685   | 1010-2150   | 1110- 252   | 0.000 1970   | 252 0132   | P 1110- 0850                            | 0010. 885   |
| 110                                     | 1-2319   | Sold Actual<br>to Yield<br>Yield       | 2980-807  | 9280-1176  | 9150 0385 4                             | . 1680- 2220   | 1010- 222   | 213 -0113   | ) 5510. 555d  | ) 1810. 09Z  | 0.0110-897   | 2010-922                                | SS10-1850   |
| a = .015                                | $ \begin{array}{l} (1 + p_{15}) \\ = (1 + \alpha)^{15} \\ = 1.2502 \end{array} $ | Sold Actual<br>to Yield<br>Yield       | 9280. 0020  | 026 0208 0385 0211 0376 0214 0368 0217 0359 0221 0351                      | 1680-9170                               | 0. 1010. 1220  | 029 0222 0214 0255 1010 0356 0356 0356 0357 0357 0356 0356 0357 0357 0357 0357 0357 | 030 0170 0170                                       | 0810. 8170  | 0256 0139  | S110-1970  | . 2210. 2220                            | 9910-0820   |
|   | Bought of  |  | 520.  | 920.   | . 270.                                  | S20.   | 620.  | 080-  | 180.  | 250  | £50.   |   | -035  |

## Discussion.

The President (Mr. H. W. Manly) said that Mr. Burn had chosen for his first essay one of those subjects which, although old. had a perennial freshness, and had produced a very interesting paper, which he had no doubt would lead to an important discussion. Members who had large funds to manage, and not only to manage but to keep invested for long periods in order that they might meet their long-deferred contracts without difficulty, naturally were very much troubled about the future rate of interest; and he thought they would all agree with Mr. Burn when he said that "if it were possible to forecast the rapidity with which the rate of interest would fall in the future, it would be much more easy to provide for all contingencies." The power was not given them to forecast what was going to happen in the future. So long as savings increased without a sufficient outlet, so long, he was afraid, would the rate of interest fall; but whether the time was coming when some great enterprise would absorb any of those large funds, or some gigantic war would destroy accumulations of capital, they were unable to foretell, and he was afraid they could not look ahead with certainty beyond a few years. Their hope might be that something would occur, such as a great development of electricity, which would employ large funds, similar to what had been done in the past by the railways; but it

could only, he was afraid, be a pious hope.

Mr. A. J. Finlaison remarked that the considerations in reference to the fall in the rate of interest, to which Mr. Burn had invited the attention of the Institute that afternoon, related to a subject of the greatest importance. Members might study the movements in the rate of interest in past years, but the conclusion arrived at by Mr. Burn would be forced upon them, namely, that they could discover no such guide to the rate of interest in the future from the events of past years, as was afforded by an investigation into the rate of mortality in respect of the future death rate. The rate of interest was largely dependent upon the ever-changing transactions and dispositions of mankind, while the rate of mortality, during the period of life with which life assurance was interested, was subject to little change, and that little change they hoped was in a favourable direction. The interest received on capital had been well said to be divisible into two portions, the first of which was a payment for the use of the money, and the second was a payment for the risk of its partial or total loss. The amount of both of those payments was affected by the ordinary operations of supply and demand. A large proportion of the gain arising from the loan of capital came from the skill and discrimination with which money was lent, so as to obtain as much as possible for the risk of loss and to sustain as little loss as might be. The gain that had been obtained in past years from investments skilfully made in railway debentures was a proof of that, and the rise in their market value was a general recognition of what was apparent to a comparatively small number of far-seeing men years ago. Mr. Burn showed that the rate of interest for money lent on good security had been for a considerable period of time tending to fall steadily. Was not that due to an

increase of knowledge of the best securities available, and had not that knowledge been obtained by a search for them, owing to the increase in the demand exceeding the supply? The limit of the fall in the rate of interest in the best class of security was to be sought in the payment which the owners of money would be content to accept in return for its use, to the practical exclusion of any allowance for the risk of loss. There did not appear to be much scope left for a further decline in the rate. That consideration did not point to any large gain in the future from an additional rise in price which might act as a compensation for investments now made at the present high prices. When a rise in value in a security had already taken place approaching the ultimate limit so as to almost attain it, there was no room for repetition. For that reason great caution should be exercised in dealing in stock in the manner indicated in the paper. If money could be borrowed for a length of time at a much lower rate of interest than was obtainable from the stock itself, and the price of it increased or was even maintained during the term of the operation, a gain might be anticipated. In respect of consols, however, the bank rate was now 3½ per-cent. It had recently for some time been at 4 per-cent. The average price of consols through 1898 had been at least 12 per-cent less than it was throughout 1897, so that a loss would have been sustained instead of a gain being made by such a transaction as that suggested in consols between 1897 and 1898. The price of consols, and consequently the rate of interest yielded by investments in that stock, had been largely influenced by the risk of loss on a realization of capital in past years. It was also largely influenced in the period delineated on Mr. Burn's diagram, in the period between 1797 and about 1820, by the depreciation in the currency, which much affected the amount of money raised by the Government during that time. From 1820 to about 1887, the rate of interest obtainable from investments in consols appeared to be a very fair criterion for the rate of money in England. Since 1887, the redemption of the National Debt had been accomplished to so large an extent that the price of consols had hardly continued to be a trustworthy standard for the measurement of the rate of interest, although the price of consols seemed to continue to influence the price of other securities. The redemption of the Debt was likely to continue, for between £36,000,000 and £37,000,000 of stock became redeemable, at the price of £100, in January 1905. In anticipation of that date about £6,000,000 a year of terminable annuities would cease, generally between 1901 and 1904, so that that annual amount would be liberated for the service of the Debt. Mr. Burn alluded in the course of his paper to the large amount of stock absorbed by Government departments, but he had given no figures in relation to that part of his subject. It might interest the meeting to learn the amounts from returns made each year to the House of Commons: on 31 March 1897, £231,000,000 odd of Government securities were held by Government departments, and on the 31 March 1898, £238,000,000 odd, which was an increase of no less than £7,000,000 in the course of the vear.

Mr. James Sorley said he was very pleased indeed that Mr. Finlaison had been able to open the discussion with such weighty

remarks upon the subject of Mr. Burn's very interesting paper. Referring to the diagram representing the price of consols, Mr. Finlaison had pointed out a fact, which he had also noted, namely, that the diagram omitted to give any weight to the depreciation of the currency occasioned by the suspension of cash payments by the Bank of England in the year 1797, which continued for some twenty odd years. At that time the Government, although the Bank would have been able to meet its obligations in cash, would not allow it to do so, the cash in hand being reserved in order to subsidize German States and others who were fighting our battles, but ultimately the Bank, finding large profits derived from the paper money, issued it far in excess of the legitimate requirements of the country, with the result that the value of the currency became very considerably depreciated. He had in his hand a book by one Robert Mushet, published in 1821, which gave an estimate of the depreciation of the currency in the successive years from 1800 to Commencing in 1800, with the currency at par, namely, when £3. 17s. 103d. of currency purchased an ounce of gold, it traced its value through the subsequent years till it reached par again In the intermediate period the currency price of gold was, in 1821. for instance, £4, 5s, per ounce in 1801, £4, 10s, in 1810, £5, 4s, in 1814—this being the year of greatest depreciation. If due weight were given to the aberrations, a very considerable alteration in the diagrammatic representation of the fluctuations in the price of consols would result in the period referred to. At another period also Mr. Burn's curve of consols failed adequately to represent the money market, namely, the period when the three per-cents went above par. There was no doubt that they would have gone much higher than 103, but for the fact that they were repayable at par on one year's notice—as was ultimately done. He had also in his hand another interesting book by James Vansommer, published in 1834 by request of the General Purposes Committee of the Stock Exchange, and exhibiting in a series of diagrams the monthly fluctuations in consols in each year from 1789 to 1833. An inspection of these diagrams showed that gigantic fluctuations occurred in individual years, and frequently several times over in the course of one single year. Truly an economic interpretation of history. At the end of the book was given a table of the highest and lowest prices in each year, and he found that, for instance, in 1792 the lowest price was 74 and the highest  $97\frac{1}{8}$ , a fluctuation of 23. In 1803 there was also a fluctuation of 23, from  $50\frac{1}{4}$  to  $73\frac{5}{8}$ , in 1817 a fluctuation of 22, and in 1830 a fluctuation of 18. Stated otherwise, if a company had invested its capital in consols in January 1803, it would have had the melancholy satisfaction of seeing it depreciated by about one-third by the following July; and even so late as 1830 the capital shrinkage between 1 January and 8 November was not far short of one-fifth. That seemed rather dangerous business for institutions that had to provide for posterity, but on the whole investments in consols had been wonderfully profitable in the past. Whether they would be so in the future was another matter, and he agreed with Mr. Burn that consols were not now a suitable investment for life offices. He had that day had the opportunity of turning up some old records of

purchases of consols. The lowest price given for consols was stated by Mr. Burn to have been  $47\frac{1}{2}$  in the year 1797. He found, however, an actual purchase of £10,000 3 per-cent Imperial Annuities (as they were described) at 46½ in February 1798, a price that might be relied upon, and which was lower than he had found recorded in any of the books of reference. In the previous year £10.000 4 per-cents were purchased at  $62\frac{3}{4}$  cum. div., and £5.000 5 per-cents at  $86\frac{1}{4}$ . It was important to bear in mind that in those olden days there were only two classes of investment available—consols or Government securities and mortgages. The rate of interest on mortgages had not had such tremendous fluctuations. It had gone up as well as gone down. In the old records of a company which the president knew a great deal about, he found that strong pressure was put upon the board of directors to lend money on mortgages at  $3\frac{1}{2}$  per-cent, and that this was declined only on the ground that it might necessitate the reduction of interest on existing mortgages. Mr. Burn was, he thought, a little too confident in his predictions of a steady fall in the future rate of interest. He could not approve of low-rated investments being procured for the bulk of life office funds in the hope of appreciations of capital. It was not desirable to write up investments at the end of each quinquennium, and vet, if low rates were taken and there were no writing up, where were their bonuses to come from? There was one view stated by Mr Burn which he was inclined very thoroughly to endorse, that was, in regard to the necessity in view of the rates of interest now realisable, of some re-adjustment of the without-profit premiums now charged. Some rates of premium—e.g., single premiums now current—were neither fair to the company itself nor to the existing policyholders. For example, if a company holds that it is not justified in assuming a higher rate than 3 per-cent in calculating its liabilities, why should it offer to issue policies at single premiums much less than the 3 per-cent reserve? He was not without hope that when the new mortality investigation was in their hands, it might be found practicable for all the leading companies to combine to charge uniform rates not only for annuities but for assurances without profits, as, he understood, the French companies did. If Mr. Burn, in addition to giving the actual price of consols, could put in his chart two other curves showing the highest prices of each year and the lowest prices each year, it would give an interesting representation of what the fluctuations were, and would add to the interest of the chart.

Mr. A. G. Mackenzie remarked that the question of investments had been for many years absent from the discussions of the Institute. It was said that that country was happy that had no history, and certainly actuaries of old were fortunate in not having to give the question of investments such unremitting attention. The first two papers of the session, after the presidential address, had been devoted to that subject, and he thought it was not inappropriate that the authors of both those papers were connected with that great institution that had not only the largest amount of funds at present, but was accumulating further funds with a rapidity that was the envy and admiration of all. He thought, when he read the account of the proceedings that took place on the last occasion, that in the vast

acreage of Furnival's Inn, in those palatial buildings and that spacious desert now awaiting architectural development, there was room for just a little friendly difference of opinion. Such difference of opinion must always occur when a question was approached by men of various minds, moods, and temperaments. It was not desirable that practical unanimity should exist on this question. They had each to endeavour to learn from one another, so that they might see the other side of the shield of truth. On the last occasion the paper related to investments which had not as a rule in past years been thought desirable for insurance companies. On the present occasion they were considering securities which had for a long time appeared in their balance sheets. With a great deal of what the author said he felt himself in great sympathy. Upon one or two points, which might not be of the greatest importance, he would venture to make one or two remarks, not in any spirit of captiousness. The point to which he would first direct his attention was the question of the realization of profits on investments. He understood that the author considered that by far the most usual practice for offices at present was to carry forward their investments at cost price. Although that was conspicuously the ease with some offices, a perusal of the Blue Books for the last few years would scarcely induce him to hold this opinion. He noticed that very large appreciations of investments had been earried to revenue account, and doubtless had been done so to a great extent in order to strengthen the valuation reserves and provide for the assumption of lower rates of interest. When Mr. Burn said that he thought the only purpose, for which any increase in the value of securities ought to be used, was to increase the reserves, he should beg to differ from him. Such an opinion seemed to him of too sweeping a character. It must be remembered that in life assurance there were many interests, some of which were, to a certain extent, conflicting. There were in some offices a body of individuals, who might not be altogether approved of, but whose existence had to be taken into account. Shareholders in proprietary companies were entitled to fair treatment, and, as a rule, would see that they got it. When there was a large profit arising from investments, it was quite possible they might consider they had a fair claim to some share of the profits. He was well aware that, in most of such companies, shareholders might also consider that it was to their highest interests to strengthen the reserves, and not to touch a certain portion of the profits, in order that more new insurants might join, because the larger the amount of business the larger the amount of profit which the shareholders would probably earn, owing to the rule in those companies that they take a fixed proportion of the total profits. But with regard to policyholders, and particularly the older policyholders, men who had been paying their premiums for many years, whose interest in the concern was of a strictly limited character, and who might expect their representatives to draw out the full amount of their money with bonuses in a very few years, it seemed rather a serious view that the profits, which had been realized, were all to go to swell the strength of the office as an attraction to new entrants and as a bonus provision for younger policyholders. It had been stated that the question of rates was an

anxious one, and that it might be necessary in future to increase the rates of insurance companies owing to the fall in the rate of interest. He thought there would be very considerable difficulty in doing so. He did not think himself that the rate of interest would in any way affect the face value of the contracts of insurance companies. He thought that for non-profit policies the rates at present were adequate, and if they were compared with the rates now being charged by the Post Office for insurances of from £25 to £100, it would be found there was very little difference indeed. The Post Office did not look for the rate of interest insurance companies were earning on their funds, and he thought that the competition between offices in future would be principally for business which participated in profits, and of course unanimity among offices, as to charging the same rate for profit policies, would be absurd, looking to the different prospects of profits in different offices. He did not think the amount of consols held by the companies, mentioned by the author as being £4.000,000, was at all out of the way. It represented about 2 per-cent of the invested funds of the companies. For many reasons consols might form suitable occasional investments, as in the case of an office waiting to find a more remunerative outlet. He thought, however, that care should be exercised with regard to the price, because there were considerable fluctuations even in consols. The instance of the gentleman, who made 17; per-cent on an investment of £10,000, was enough to make the mouth of impecunious mortals water; but on a little examination it would be found that he would be a very daring spirit indeed, because a depreciation of 2 per-cent in consols would do away with his profits and make the transaction result in a loss. Mr. Burn had given a very interesting account of an investigation he had made as to the fall in the rate of interest in certain offices since 1870. It differed by but a shilling or two from that of other writers, and it was important as showing that the trend was unnistakably one of decline in the rate of interest. That was open to no doubt whatever, and the fall was really much greater than appeared, on account of the large number of offices carrying forward their investments at cost price at the present time. The suggestion had been made in the paper that, if only offices could get hold of long term debentures realizing a small rate of interest, they might, with the appreciation which those investments would obtain owing to the pressure of capital, get a satisfactory result. If, by putting money into trustees' securities, a rate of interest could be eventually obtained of the amount desired, it would be most comforting. But he did not believe that any such result was possible. If it was attempted, they would defeat their own purpose. There was but a limited number of those securities in the market, and if they were all buying them, debentures now yielding about £2. 15s. per-cent would quickly appreciate to a price, which there seems some prospect of their attaining, yielding only £2, 10s. There was a probability of much greater appreciation in good securities now paying 31 or 4 per-cent, because, when railway debentures came to yield only 21, these securities might go up to a 3 per-cent basis. With regard to the nature of the securities

desirable for investment, if it could only be ascertained what securities were likely to be authorized by Government when any fresh extension of powers was given to trustees, those investments would form very suitable securities. But it was a very difficult matter. Colonial securities were now only paying 3 per-cent, and, if they were included in trustee securities, there would not be very much appreciation, because they had only limited terms to run before they were redeemed. The other securities which might be authorized would, he supposed, be, as a rule, limited to the United Kingdom, and offices were finding some good investments now beyond our shores. British railway ordinary stock was not supposed to be a security which would be ever approved by Government for trustees, as the fluctuations both in interest and price were considerable, which would not suit either the life tenant or reversioner. What insurance offices must look to was to obtain suitable securities paying a fair rate of interest, and, by watching closely the price, take their profits at the right time, and repeat the operation as often as possible. As to what was to be done with the realisation of securities, that was a question which he thought every office would have to determine for itself according to its particular accessities, and with due regard to the equity of all parties interested. In fact, offices would have to work out their own salvation in the way of investments on the rules and in the methods most suited to their special circumstances.

Mr. A. H. Bailey considered the value of the diagrams very great. Mr. Finlaison had pointed out that the diagram on the price of consols was not a source from which the prevailing rate of interest could be derived. A little more attention might be given to the next diagram, showing the average rate of interest yielded by railway debenture stock from 1870 to 1897. The curve ran almost uniformly in one direction, and, as there were none of those circumstances, which Mr. Finlaison had pointed out, affecting the value of railway debenture stocks, the diagram afforded good evidence of the fluctuations in the rate of interest. Judged by this standard in those twenty-seven years, the rate of interest had fallen  $1\frac{1}{2}$  per-cent. He thought perhaps the author was a little too wise after the event, and he did not think anyone could have foreseen such a remarkable fall in the rate of interest in recent years. If they could, what a prodigious blunder the railway companies had made when they turned their debentures into debenture stock! It could not be supposed that the financiers of railway companies were absolute fools. But they converted the debentures into 4 per-cent debenture stock, whereas now they certainly could have been 3 per-cent, or less, and those unhappy people who were holders of ordinary railway stocks would have a better dividend than they were receiving at the present time.

The President asked the members to pass a vote of thanks to Mr. Burn for his very able and interesting paper. He was sure Mr. Burn would feel pleased with the reception that it had received and with the very important discussion which had followed its reading.

Mr. Burn, in reply, thanked the members for the very kind way in which they had received his paper. He was certainly much

interested in several of the facts which had been mentioned. It had been suggested that a diagram might be given representing the highest and lowest price of consols in each year; this would undoubtedly be of interest, and would, he thought, serve to emphasize still further the necessity for recognizing the violent fluctations to which they were subject, which was one of the reasons why he considered that they were unsuitable for investment of the funds of life offices. Mr. Sorley was of opinion that the depreciated condition of the currency should have been mentioned in connection with the low price of consols at the end of last century. This was, of course. true, but he (Mr. Burn) had mentioned that he did not consider there was a great deal to be learnt (for purposes of estimating the probable future rate of interest) from a consideration of such points, which might now be regarded an "ancient history." The main point seemed to be that consols, although fluctuating violently, had tended to rise in value for many years, and he believed that it was partly the anticipation of this rise which had induced investors to buy at times when prices were very low. In reference to the profit arising from the sale of securities which had appreciated in price, if, as he held, the rise in the value of securities was the natural provision for a lowering of the rate of interest used in valuation, then it would seem evident that the proper course to adopt would be to carry the profit to a reserve fund set aside for that purpose. Mr. Mackenzie had mentioned that the possibility of obtaining 17½ per-cent by means of consols was enough to make one's "mouth water." but he hoped he had not appeared to be recommending this kind of speculation, he had merely mentioned it as an example of one of the causes of the sudden rise in securities similar to consols; it also explained, to some extent the downward tendency which was noticeable when money became dearer. From 1890 to 1896 the rise in the price of securities had been peculiarly rapid, and it was quite possible that it had to a certain extent discounted the rise of the immediate future, but he could not help thinking that the cheek was merely temporary. It appeared to him that everything pointed to a further fall in the rate of interest, and that those who held the contrary opinion should show reasons which would prevent this fall. In some securities the elimination of the risk may have been one of the causes of the rise in value, but if all the risk were eliminated the price might still continue to rise owing to the constant accumulation of wealth. Some securities may have more nearly approached their limiting price, and undoubtedly there were still many securities which would be recognized as perfectly safe and well secured, and would therefore eventually attain the same level as those which were now thought to be of a higher class. Mr. Bailey had mentioned that it was easy to be wise after the event. He would however, point out that in the past there had been some who were wise before the event, and he hoped that this would again prove to be the case in the future.

Further Remarks on the Valuation of Endowment Assurances in Groups. By George J. Lidstone, F.I.A.

In a paper by the present writer, entitled Some Remarks on the Valuation of Endowment Assurances in Groups, which appears in the current volume of the Journal, pp. 61-84, a new method of valuing Endowment Assurances, based on Makeham's formula for  $\mu_x$  and  $l_x$ , was described, and it was shewn that very good results were obtained by the use of the method in the case of a fairly large Endowment Assurance business, particulars of which were summarized in the paper.

The same mode of valuation has been recently applied to a very much larger volume of business with results which are also entirely satisfactory, and it is thought that these results may be of sufficient general interest to justify the publication of particulars in the pages of the *Journal*.

The business in question embraced the whole of the Endowment Assurance Policies of a large and expanding Office,\* with the exception of a few Policies which it was considered desirable to value separately, namely, (a) Policies maturing at ages higher than 70, if assuring upwards of £1,000, (b) Policies subject to Limited Premiums, and other special Policies. The assurances scheduled in Table II, J.I.A., xxxiv, p. 65, were included if still in force after an interval of five years, but all such cases were five years nearer maturity than at the date of the valuation to which that Table related, so that the group n=5-9 now appears as 0-1, and so on. In addition, the valuation included the large amount of Endowment Assurance business transacted during the quinquennium.

Owing to the form in which the rates of premium are published the bulk of the assurances are payable at one of the ages 50, 55, 60 or 65, but there was a fair number of Policies maturing at other ages, all of which (with the few special exceptions referred to above) were included, so that many of the groups contained Policies maturing at from 10 to 20 different ages.

For each group the mean maturity age (on which depends the mean valuation age) was found correct to one decimal place

<sup>\*</sup> The Policies of a transferred Office were excluded.

by means of Table A, J.I.A., xxxiv, p. 80–81, which gives the values of the special function  $Z_M$ , where M is the maturity age. It should be borne in mind that the Table is based on a common ratio equal to 1.09561, i.e., the value of Makcham's constant c used in the Text-Book Graduation of the  $H^M$  experience. This ratio being greater than that which was employed in the previous application of the method, it follows that the valuation may be expected to produce a slightly excessive reserve. See articles 18–20 of the Paper above referred to.

The following Schedules—pp. 512, 513—give particulars of the assurances in each group, with the results of the group valuation and the corresponding deviation from the exact figures. The following is a summary of the results:

| Total Sums Assured and Bonuses                             | £1.984,030   |
|--|--------------|
| Premiums   | €70,869      |
| Estimated value of Sums Assured and                        | 61.165.610   |
| Bonuses  | \$1.197,946  |
| Error in ditto .   | + £597       |
| Estimated value of Net Premiums less Reductions of Premium | £\$40,929    |
| Error in ditto .   | <u>-£165</u> |
| Estimated Total Liability                                  | £357,017     |
| Error in ditto .   | + £762       |

The final net error of £762 represents 213 per-cent of the total liability, or 039 per-cent of the total sums assured and bonuses, a result which must be considered quite satisfactory.

It is not easy to say whether the error would be greater or less in the case of a business which included a smaller proportion of recently effected Policies. Probably there would not be any great variation in either direction, and it will be seen that the error would have to be considerably increased before it would have any practical importance in relation to the total reserves, especially as the latter become greater in proportion to the sums assured as the average Policy-duration increases.

The error might be reduced by employing a smaller ratio than c=1.09561 in the calculation of the function Z (as was done in the case of the previous application of the method), but this course can hardly be considered desirable in view of the

## SUMMARY OF PARTICULARS.

| Number of                        |                       | IARI OF               | 1 ARTIC CHARG                                    |                                   | 1   |
|----------------------------------|-----------------------|-----------------------|--|-----------------------------------|---|
| Years Premiums<br>remaining<br>n | Sums Assured          | Bonus<br>Additions    | Sums Assured<br>plus<br>Bonus Additions          | Annual<br>Reduction<br>of Premium | Net Annual<br>Premiums<br>H <sup>M</sup> 3 per cent |
|                                  | £                     | £                     | £  | ŧ                                 | £   |
| 0                                | 6.550                 | 209                   | 6,759  |                                   | 1.000   |
| $\frac{1}{2}$                    | $14,460 \\ 15,700$    | $1,773 \\ 150$        | 16,233 $15,850$                                  | 13                                | 1,093<br>636  |
| 3                                | 15.100                | 917                   | 16,017   | 13                                | 826   |
| 4                                | 8,250                 | 991                   | 9,241  | 1                                 | 406   |
| 0-1                              | 60,063                | 1,040                 | 64,100   | 26                                | 2,961   |
| 5                                | 40,800                | 1,585                 | 42,385   | 15                                | 1,963   |
| 6                                | 14.550                | 1,341                 | 15,891   | 12                                | 949   |
| 7                                | 12,950                | 682                   | 13,632   |                                   | 706   |
| 8                                | 28,550                | 883                   | 29,433   | 13                                | 1,599   |
| 9                                | 38.161                | 2,252                 | 40,413   | 11                                | 2,150   |
| 5-9                              | 135,011               | 6.743                 | 141,754  | 51                                | 7.367   |
| 10                               | 57,000                | 1,054                 | 58,054   | 2                                 | 2,788   |
| 11                               | $32,\!817$            | 1,535                 | 34,352   | 2                                 | 1,505   |
| 12                               | 106,008               | 1,513                 | 107,521  | 6                                 | 5,390   |
| 13                               | 73,900                | 1,607                 | 75,507   | 6                                 | 3,575   |
| 1.1                              | 53,567                | 943                   | 54.510   | 1                                 | 2,559   |
| 10-14                            | 323,292               | 6,652                 | 329,944  | 17                                | 15.817  |
| 15                               | 71,825                | 1,354                 | 73,179   | 3                                 | 3,082   |
| 16                               | 56,850                | 1,539                 | 58,389   | 10                                | 2,308   |
| 17<br>18                         | 125.279               | 2,398                 | 127,677  | 3                                 | 4,893   |
| 19                               | 98,652 $98,150$       | 1,652 $1,490$         | 100,304<br>99,640                                | $\frac{9}{4}$                     | 3,816<br>3,676                                      |
| 15-19                            | 450,756               | S,433                 |  | 29                                | 17,775  |
| 20                               |                       |                       | 459,189  |                                   |   |
| 21                               | 90,933 $116,133$      | $\frac{2,058}{1,103}$ | $\begin{array}{c} 92.991 \\ 117.236 \end{array}$ | $\frac{6}{4}$                     | 3,133<br>3,984                                      |
| 22                               | 53,400                | 1,135                 | 84,535   | 3                                 | 2,674   |
| 23                               | 64,450                | 1,189                 | 65,639   | 2                                 | 1,951   |
| 24                               | 76,778                | 1,254                 | 78,032   | 3                                 | 2,223   |
| 20-24                            | 431,694               | 6,739                 | 438,433  | 18                                | 13,965  |
| 25                               | 58,450                | 1,046                 | 59,196   | 3                                 | 1,626   |
| 26                               | 70.283                | 760                   | 71,043   | 1                                 | 1,922   |
| 27                               | 56,700                | 383                   | 57.083   | 7,                                | 1,468   |
| 28<br>29                         | 61,733<br>51,783      | 815<br>747            | 62,548 $52,530$                                  | 1 2                               | 1,554 $1,249$                                       |
| 25-29                            | 298,949               | 3.751                 |  | 17                                | -   |
|                                  |                       | -, -                  | 302.700  |                                   | 7,819   |
| $\frac{30}{31}$                  | 49.750 $40.500$       | 564<br>256            | 50,314   | 1 0.2                             | $\frac{1,177}{943}$                                 |
| 32                               | 35,100                | 136                   | 40,756 $35,236$                                  | 1                                 | 781   |
| 33                               | 28,533                | 149                   | 28,652   | li                                | 617   |
| 34                               | 27,050                | 249                   | 27,299   |                                   | 566   |
| 30-34                            | 180,933               | 1,354                 | 182,287  | 3                                 | 4.084   |
| 35                               | 21,850                | 37                    | 21.887   | 0.2                               | 428   |
| 36                               | 15,650                | 124                   | 15,774   |                                   | 301   |
| 37                               | 8,750                 |                       | 8,750  |                                   | 169   |
| 38                               | 8.767                 | 7.1                   | 8.841  |                                   | 160   |
| 39                               | 3,300                 | 21                    | 3,321  |                                   | 59  |
| 35-39                            | 58,317                | 256                   | 58,573   |                                   | 1,120   |
| $-\frac{1}{40}$                  | 2,350                 |                       | 2,350  |                                   | 43  |
| -11                              | 2,200                 |                       | 2,200  |                                   | 38  |
| 42                               | 1,400                 |                       | 1,400  |                                   | 23  |
| 43                               | 100                   |                       | 100  |                                   | 2   |
| 44<br>40-44                      | $=-\frac{400}{6,450}$ | ·                     | 400  |                                   | 7   |
|                                  |                       |                       | 6,450  |                                   | 113   |
| 45                               | 200<br>200            |                       | 200  |                                   | 3   |
| 46<br>47                         | 200<br>100            |                       | 200<br>100                                       |                                   | 3 2   |
| 18                               | 100                   |                       | 100  |                                   | 1 1   |
| 49                               |                       |                       | 100  |                                   |   |
| 45-49                            | 600                   |                       | 600  | -                                 | 9   |
| Grand                            | 1,946,062             | 37,968                | 1,984,030  | 161                               | 71,030  |
| Total                            |                       | 1 3,,000              | 1 2,000,000                                      | 1 -0-                             | 1,500   |

|   | ]            |   |            |   |                |   | o up |                                  | 0.1             |   |
|---|--------------|---|------------|---|----------------|---|------|----------------------------------|-----------------|---|
| [H <sub>M</sub> 3                               | 7.           |   | St         | MMART                                       | OF V           | LUATION   |      |                                  | [H <sup>M</sup> | 3 ° <sub>2</sub> ].                       |
| Value of<br>Sums Assure i<br>by Group<br>Method | Error        | Value of<br>Bonus<br>Additions by<br>Group Mathod | Error      | Value of Net<br>Premiums by<br>Group Methos | Err er         | Value of<br>Reduction of<br>Premium by<br>Fronty Method | Err  | Lichility<br>by tyroup<br>Method | Error           | Number o<br>Years<br>Premium<br>remainin. |
| £   | £            | £   | 6          | £   | £              | f.  | ŧ    | ŧ                                | 6               | "   |
| 6,350   |              | 203   |            |   |                |   |      | 6,562                            |                 | (1  |
| 13,639  | - 1          | 1.672   |            | 1,064                                       | - 1            | _   |      | 14.247                           | - 2             | 1   |
| 14.397  |              | 138   |            | 1.208                                       |                | 25  |      | 13,352                           |                 | 2   |
| 13.468<br>7.161                                 | - 1          | 515<br>559  | - 1        | 2,295                                       | - 1            | 32<br>3   |      | 12.020                           | - 1             | 3   |
|   |              | 3,690   | _ 2        | 6,043                                       |                |   |      | 6,550                            | -               | 1   |
| 55,024  |              |   |            |   | + 1            | 60  |      | 52.731                           | - 3             | ()~ }                                     |
| 34,440<br>11,965                                | - 1          | 1,338<br>1,103                                    | - I        | 5.751<br>43957                              | + 10           | 67<br>65  |      | 27.093                           | - 11            | 5   |
| 10,365  | - +          | 546   | - 2        | 4,225                                       | - 6            | ****  |      | 5,176                            | - 6<br>+ 8      | 6   |
| 22,291  | - 3          | 650   | - 1        | 10,690                                      | - 0            | ~~  | + 3  | 12.378                           | + 14            | 7   |
| 29.111  | - +          | 1.715   | - 3        | 15.746                                      | - 2            | <b>S</b> 3  | + 1  | 15,166                           | - 6             | 11  |
| 108,175   | - 12         | 5.394   | 3          | 44,369                                      | - 10           | 303   | + 4  | 69,503                           | - 23            | 5-9                                       |
| 42,401  | - 7          | 754   | - +        | 22,257                                      | 0              | 20  |      | 20,951                           | ~ 2             | 10  |
| 23.771  | - 10         | 1.112   | 1          | 13,045                                      | - 27           | 20  | - 1  | 11.858                           | - 37            | 11  |
| 74.410  | - +          | 1,062   | ;          | 50,867                                      | - E6           | 58  | - 3  | 24.663                           | + 12            | 12  |
| 50,950  | - 6          | 1.108   | - 2        | 35,351                                      | - 75           | 65  |      | 16.775                           | - 67            | 1:3                                       |
| 36,055  | - +          | 635   |            | 26.776                                      | - 20           | 15  | - 1  | 0.1921                           | - 21            | 1 /                                       |
| 227,590   | - 2,3        | 4.701   | - 2        | 148,296                                     | + 67           | 151   | 5    | 54.176                           | - 37            | In-11                                     |
| 17.075  | + [          | 557   | - 1        | 34,135                                      | + 9            | 31  |      | 13,861                           | - 9             | 1.5                                       |
| 36,358<br>78,165                                | - 3          | $\frac{985}{1,496}$                               | - 1        | 26.545<br>59.574                            | - 12           | 115   | - 1  | 10.613                           | - 17            | 16  |
| 60,259  | + 14<br>+ 23 | 1,000   | - 11       | 18.217                                      | - 19           | 111   | + 1  | 20.121 $13.162$                  | - 2,3           | 17  |
| 58.971  | - 21         | 895   | - 2        | 17.771                                      | - 1            | 56  | + 1  | 12,148                           | - 15            | 15  |
| 280,831   | + 62         | 5.272   |            | 216,548                                     | + 12           | 350   | + [  | 69,005                           | + 24            |   |
| 52,882  | + 42         | 1,197   | - 1        | 12.778                                      | - 10           | 82  | - 1  | 11,383                           | + 44            | 15-19                                     |
| 66,716  | - 60         | 635   | + ;        | 55,361                                      | - 45           | 55  | - 1  | 12,122                           | + 2;            | 20<br>21                                  |
| 46,565  | + 45         | 634   | - 2        | 35,705                                      | - 22           | 10  |      | 8.531                            | + 65            | 22  |
| 35,055  | + +7         | 617   | 2          | 29.221                                      | - 18           | 30  | - 1  | 6.511                            | - (12           | 23  |
| 10,553  | - 55         | 668   | - 5        | 34.180                                      | - 68           | 43  |      | 7.414                            | +148            | 24  |
| 242.178   | + 288        | 3,751   | 3          | 200,248                                     | - 79           | 250   | - 2  | 15.961                           | + 362           | 20-21                                     |
| 30,523  | + 33         | 546   | - 3        | 25.591                                      | - 21)          | 11  |      | 5,519                            | - 50            | 25  |
| 35.787  | + 10         | 357   | - 3        | 31.110                                      | - 34           | 17  |      | 5.051                            | + 71            | 26  |
| 25,309  | ± 32         | 191   | - 1        | 24,277<br>26,284                            | - 2,3          | 115   | - 2  | 4.33                             | - 56            | 27  |
| 30,154<br>21,792                                | + 26<br>+ 22 | 398<br>357  | - 3<br>- 3 | 21,541                                      | -18 $-12$      | 67<br>34  | - I  | 4.335                            | - 40            | 25  |
| 149,565   | -153         | 1,879   | - 13       | 125,503                                     | -116           | 274   | _ '  | 3,612                            | + 30            | 29  |
| 23,491  |              | 266   | . '.       | 20,579                                      |                |   |      | 22.915                           | + 256           | 25-29                                     |
| 18,661  | + 16         | 115   |            | 16,861                                      | 3              | 15<br>3   |      | 3.193                            | - 18            | 30  |
| 15,795  | - 3          | 61  | - 1        | 14.251                                      | - 2            | 20  | •    | $\frac{1.921}{1.625}$            | + 23<br>+ 1     | 31<br>32                                  |
| 12,621  | - 6          | 66  |            | 11,422                                      | - 4            | 10  |      | 1.278                            | - 10            | 33  |
| 11.795  | - 8          | 109   |            | 10.605                                      | - 3            |   |      | 1.296                            | - 11            | 31  |
| \$2,366   | ± 48         | 620   | — ı        | 73.721                                      | - 19           | 15  |      | 9,313                            | - 66            | 30-31                                     |
| 9,235   | ± 5          | 16  |            | 5,213                                       | - 2            | 3   |      | 1,041                            | - 7             | 35  |
| 6.510   | ÷ 7          | 51  |            | 5.917                                       | <del>-</del> 2 | * * *   |      | 644                              | - 6             | 36  |
| 3,560   | + 11         | 30  |            | 3,334                                       | - 5            |   |      | 226                              | + 16            | 37  |
| 3,512<br>1,299                                  | + 11         | 50  |            | 3,198                                       | - 3            |   |      | 344                              | - 14            | 35  |
|   |              | 105   |            | 21,862                                      |                |   |      | 107                              | T               | 39  |
| 24.119  | 35           | 100   |            | 570   | - 12           | 3   |      | 2,365                            | - 47            | 35-39                                     |
| 834   | - 2          | * * *   |            | 753   |                |   |      | 46                               | - 1             | 40  |
| 520   | + 1          |   |            | 483   | - 1            |   |      | 51<br>37                         | - 2             | 41<br>42                                  |
| 36  |              |   |            | 35  |                |   |      | 1                                |                 | 43  |
| 146   |              |   |            | 135   |                |   |      | 5                                | - 1             | 44  |
| 2,452   | - 3          |   |            | 2,309                                       | - 1            |   |      | 143                              | - 1             | 19-14                                     |
| 7()   |              |   |            | 67  |                |   |      | 3                                | - 7             | 45  |
| 66  |              |   |            | 65  |                |   |      | 1                                | [               | 46  |
| 34  |              |   |            | 31  |                |   |      |                                  |                 | 47  |
| 34  |              | ***   |            | 33  |                |   |      | 1                                | 211             | 45  |
|   |              |   |            | 160   |                | •••   |      |                                  |                 | 49  |
| 204   |              |   |            | 199   |                |   |      | 5                                |                 | 45-49                                     |
| 1,172,504                                       | -624         | 25,442  | - 27       | 542,398                                     | -157           | 1,469   | + 8: | 357,017                          | + 762           | Grand                                     |

smallness of the error and of the fact that the use of the ratio c leads to a very simple method of calculating expected claims. It will probably be admitted, moreover, that a small systematic error in the direction of *increased* reserves can but varely be regarded as objectionable.

It would be of the greatest interest to make a similar comparison of the true and approximate results in the case of business consisting more largely of Endowment Assurances maturing at the expiration of 10, 15, 20 . . . . . years from entry, instead of Assurances maturing principally at quinquennial ages (50, 55, &c.). It is to be hoped that some Actuary who is in possession of the necessary data will make such a comparison and publish the results, so that members of the profession may be able to judge of the effect of the method in such a case. In view of the theoretical basis of the method there seems no reason to think that the result would be other than satisfactory, but it would be very desirable to actually test the matter.

On Extra Risks in relation to Double Endowment Assurances.
By A. Levine, M.A., F.I.A.

THERE appears to be a very general belief that every life may be accepted under the Double Endowment Assurance Table at the same rate, provided the term is fixed. One office, at least, grants such assurances without medical examination; another has accepted publicans under this table without the usual trade extra; and medical officers have been known to recommend acceptance at tabular rates for a Double Endowment after having declined the life out-right for an ordinary Endowment Assurance. It may be useful, perhaps, to ascertain how far this practice is justified by theory; and it is hoped, therefore, that a short note on the subject may not be without interest.

Following the classification suggested by Messrs. White and Whittall (J.I.A., xxiv, 385), we may adopt three hypotheses as to the incidence of the extra risk:

I.—The extra risk may be continuously increasing throughout life. This is usually met by the addition of a certain number of years to the ages at death and maturity. Taking Select 3 per-cent rates, we obtain the following table of pure premiums for a Double Endowment Assurance:

TABLE I.

| ge<br>at |          |          | Тепм     |             |           |
|----------|----------|----------|----------|-------------|-----------|
| itry     | 10 Years | 15 Years | 20 Years | 25 Years    | ::0 Years |
|          |          |          | € s. d.  |             | € s. d.   |
| 25       | 16 17 S% | IO 9 5%  |          | -5 - 9 - 0% | 1 4 9%    |
| 30       | 16 16 11 | 10 8 10  | 7 5 6    | 5 8 4 .,    | 1 1 1,    |
| 35       | 16 16 9  | 10 5 2   | 7 1 9    | 5 7 9 .,    | 138,      |
| (1)      | 46 15 S  | 10 7 3   | 7 1 2    | 5 7 3       | 1 3 10 .  |
| 1.5      | 16 14 6  | 10 6 4   | 7 3 6    | 5 7 6       | 4 5 3,    |

It seems clear, then, that for this kind of extra risk, the ordinary practice is safe.

II.—The additional risk may be distributed uniformly over the entire period of life. This is the case of the extra for residence abroad, or for hazardons occupation, &c. Assuming that the extra for a whole-life assurance would be 15s. per-cent, the corresponding premiums for the Double Endowment Assurance may be obtained approximately by entering a 3 per-cent Conversion Table with the 4 per-cent temporary annuity, and adding the premium for the pure endowment at 4 per-cent. Thus we get the following table:

TABLE II.

| Age         |   |          | TERM                                    |  |                         |
|-------------|---|----------|---|--|-------------------------|
| at<br>Entry | 10 Years                                    | 15 Years | 20 Years                                | 25 Years                               | 30 Years                |
|             | 16 18 8 %<br>16 17 10<br>16 17 7<br>16 16 7 | 10 11 9  | 7 10 7 %<br>7 10 2<br>7 9 7<br>7 9 5 ,. | 5 15 0 %<br>5 14 5<br>5 14 3<br>5 14 5 | 4 12 0 ,,<br>4 11 11 ,, |

Comparing these rates with those of the previous table, it would appear that, when the term does not exceed 15 years, the

extra is not large. For longer terms, however, it does not seem safe to neglect the extra—as, e.y., for a term of 30 years it is about three-fifths of the whole-life extra.

III.—The extra risk may be very great in the early years of the assurance, and may continuously diminish to zero.

One purely hypothetical form of this kind of extra is obtained by assuming that the extra diminishes at the same rate as that at which the rate of mortality increases with the age. For example, assume that a life now aged 30 will be subject, during the next 25 years, to a constant rate of mortality = 0.22187 (the H<sup>M(5)</sup> rate at age 55), and thereafter to the ordinary H<sup>M(5)</sup> rate; this would entail for a whole-life assurance an extra of 15s. per-cent. At 3 per-cent, the Double Endowment premium will be £5. 19s. 1d., as compared with £5. 8s. 4d. in Table I. Similarly, for age 40 and a term of 20 years, we obtain £7. 19s. 1d., as compared with £7. 4s. 2d.

A more practical assumption is that Case II is a mean, so to speak, between Cases I and III. This will require an extra which is a very considerable proportion of the whole-life extra, and cannot with safety be neglected.

On the whole, then, it appears that only when the extra risk is an increasing one is it correct to grant Double Endowment Assurances at tabular rates. The point is not at present of much importance, as very few offices include such a table in their prospectus. This form of assurance may, however, in time become more popular; and when it does, it is to be hoped that some attempt will be made to bring the practice of offices, with regard to the rates of premium, more in line with theory.

## JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

Some Notes on Life Assurance in Greater Britain, particularly with reference to the Work and Development of the Native Offices. By Arthur Wyndham Tarn, F.1.A., of the Westminster and General Life Assurance Association.

[Read before the Institute, 27 February 1899.]

 $\operatorname{ALTHOUGH}$  a great deal of what might be fairly comprised under the heading to this paper came under discussion last year at the International Congress of Actuaries, I vet venture to submit that, upon so important and comprehensive a subject as the one I have chosen, there is a considerable amount of information, statistical and actuarial, which at that great gathering was not even touched upon. In the first place, the contributions to the Congress, valuable though they were, dealt Object of Paper. only with the experience of individual Colonies or groups of Colonies, and the observations of the contributors were limited to particular branches of their subject. Secondly, some of the principal British Possessions, such as India, Canada, and the West Indies, appear to have been entirely unrepresented as regards life assurance among the subjects discussed. therefore, with the object, partly of filling up a few of the gaps left vacant at the Congress, but chiefly of surveying the subject as affecting not merely individual Colonies or Dependencies, but the whole of the British Empire outside our own country, that I have collected and set down these notes. In one respect,

perhaps, a paper on Life Assurance in Greater Britain may be considered as not altogether inappropriate in a year when an exhibition of the products of our Possessions in all parts of the world is to be opened at Earl's Court. Thirteen years ago, it will be remembered, a similar exhibition was held in London, and the attention that it was the means of drawing to the Colonies, and especially to Australasia, was ascribed as being one of the main reasons for a life office from that part of our Dominions deciding to settle in this country, and to compete for business side by side with institutions purely British. Another Australasian office had indeed previously announced its intention of opening a branch in that year. The announcement, however, was London in premature, the society being prevented from taking this step by the opposition of the Colonists themselves, who, so Mr. Archibald Day told us in the course of his Presidential Address, "expressed "themselves strongly on the impolicy of diluting their greater "vitality with the exhausted stamina of the old country." Since that date the pioneer of 1886 has been followed in its enterprise by a Canadian and two more Australasian companies, and it is whispered that the above-mentioned society is no longer content with the undiluted vitality of its supporters at the Antipodes, and will at no distant date join its compeers in Great Britain, where it will no doubt receive the same welcome that has been accorded to its predecessors.

The history of life assurance in the Colonies is necessarily of a modern character-at any rate as regards native offices, which for the most part have sprung into being within the last 30 years. The oldest existing Colonial life office is the Barbadoes Mutual Life Assurance Society, which was established in 1840, and has ever since enjoyed uninterrupted prosperity. The office next in seniority is the South African Mutual Life Assurance Society, instituted in 1845, and till 1888 known as the "Mutual Life Assurance Society of the Cape of Good Hope." In 1847 the Canada Life Assurance Company was established, and two Earliest Colonial years later saw the birth of the Australian Mutual Provident Life Assurance Society. Each of these institutions, representing life assurance in four of the principal sections of the British Empire, has therefore been actively engaged in the promotion of thrift in the Colonics for 50 years and upwards; and each, it may be added, has done good service to actuarial science by publishing its mortality experience throughout the greater part of its existence. In other important possessions of the Empire, such as India and New Zealand, life assurance has been conducted on lines entirely distinct from those prevailing either in this country or in the four groups of Colonies already referred to. In the former country native prejudices and other obstacles have had to be contended against, so that it is only within the last quarter of a century that societies of this nature have met with any appreciable favour. In the latter Colony native life assurance is entirely monopolized by the Government.

In connection with the establishment of native offices in the Colonies, it should be mentioned that a considerable impetus was given to life assurance in different parts of the Empire by an office founded at Edinburgh in 1846, and called the Colonial Life Assurance Company, the main purpose of its promoters being that of extending to the Colonies of Great Britain and to India the full benefit of life assurance. Being conducted on sound business principles, this company proved eminently successful, and to its example may be traced the popularity of life assurance in the Colonies. After an active existence of 20 years this company was transferred to the Standard Life Office.

Having shown the origin of native life offices in Greater Britain, I will now proceed to give a brief account of their progress and development. Commencing with the group of Colonies where life assurance dates furthest back, namely, the West Indies, I may observe that here the Barbadoes Mutual, which confines its business to the assurance of lives resident in these islands, reigns supreme. To the members of this Institute the work of this society is not unfamiliar, since on two occasions within comparatively recent years its operations have been deemed of sufficient interest to form the subject of a paper read at our sessional meetings.

In South Africa, up to the year 1891, when the Southern Life Assurance Society was established, the South African Mutual was the sole native office. Of late years, however, branches have been formed in this rapidly-developing part of the world by offices from Great Britain, the United States, and Australia. In 1897 the numbers of such offices were 21 British, 2 American, and 2 Australian. The competition and enterprise thus engendered has had a beneficial effect upon the native

J.I.A., xxiv, 373, and xxvii, 161.

offices, which have extended their operations beyond the British Colonies into the Orange Free State and the Transvaal Republic. In the mortality experience published by the South African Mutual last year, the rapid extension of the business of that office is strikingly illustrated by the circumstance that during the last 16 years covered by the experience the number of lives under observation had increased to nearly four times that of the first 34 years. To reach the scattered outlying districts of these parts, a system of canvassing is employed which was imported from Australia, and of which a description will be given when speaking of that group of Colonies. In 1894, an Industrial Life Office was established. heing worked on lines somewhat similar to those of institutions of the kind in this country. It may be mentioned that, at the last annual meeting of one of the native offices, the chairman stated that not more than three policies had been drawn from that part of South Africa known as Rhodesia, and that the policy of the office for the present was to restrict business to persons living in the principal towns and in settled occupations, and then only at a substantial extra premium. Hence miners, prospectors, police, transport riders, and such people who have no settled homes, have been excluded from the risks of this society. From the Annual Reports of the Government Actuary at the Cape, it would appear that new business, both as regards numbers of policies and sums assured, is about equally divided between the native and foreign companies.1 The average sum assured per head in South Africa is over £25, and the average amount per policy is about £400.

In the Dominion of Canada, from the year 1847 to 1866, life assurance business was of a very limited character, there being only two offices actively working in that Colony, namely, the Canada Life and the Colonial of Edinburgh. In the latter year, however, Canada was invaded by offices from the United States, which infused into the business an amount of enterprise never before experienced, and within a few years other native offices sprang into existence. In 1877 an Act of the Dominion Parliament was passed, containing provisions drawn with a view to protect native companies. As a result of this Act, many foreign companies either withdrew altogether from the Dominion

<sup>&</sup>lt;sup>1</sup> The figures for 1895 were:

Foreign Companies, £5.762,008 under 13,184 policies; Native , 5,044,026 , 14,637 ,,

or ceased to transact new business, and the annual reports of the Insurance Superintendent show a steady increase in the numbers of Canadian companies, as well as in the amount of new business transacted, and the proportion which it bears to that of British and American offices. This increase is best illustrated by a comparison between the new business obtained by Canadian, British, and American companies during the years 1876, 1886, and 1896.

| Year | Canadian<br>Companie√ | British<br>Companies | American<br>Companies |
|------|-----------------------|----------------------|-----------------------|
|      | 8                     | 8                    | 8                     |
| 1876 | 5.465.966             | 1,683,357            | 6.740,804             |
| 1886 | 19,289,694            | 4,054,279            | 11,827,375            |
| 1896 | 26,171,530            | 2,869,971            | 13,582,769            |

As regards relative numbers of companies in active work in the Dominion, it may be stated that in 1876 there In Canada. were 37 offices, of which 7 were Canadian, 17 British, and 13 American; while in 1896 there were 39 offices, of which 11 were Canadian, 8 British, and 10 American. Since then 4 more Canadian offices have been licensed to carry on business in the Dominion, thus bringing up the number to 15.1 Of these 15 companies, 12 have their head office in the Province of Ontario, 2 in that of Quebec, and 1 in Manitoba. The reason of so large a number having been established in Ontario is that, not only does that Province consist almost entirely of a population of British origin, whereas the inhabitants of Quebec are mostly of French descent, and hence not so familiar with the principles of life assurance as their neighbours, but the great centres of trade, such as Toronto, Hamilton, and London, are situated in Ontario, and this circumstance greatly facilitates the transaction of life business. It may be added that all life assurance companies in Canada are under the supervision of the Superintendent of Insurance for the Dominion, who year by year issues an elaborate publication containing detailed statistics relating to the various companies, which, after being duly licensed, are required to make an annual return to the Government. The Insurance Department, on its part, institutes a most searching investigation into the accounts and securities

<sup>1</sup> All these companies, with one exception, are proprietary.

of each company, which, upon the officials being satisfied that everything connected with the company's business is as it should be, receives an annual license. Notwithstanding, however, the unquestionable security possessed by Canadian offices, life assurance appears to be less general in the Dominion than in other parts of the British Empire, the average amount per head being only about £12, or about half that of South Africa. This low average is, no doubt, due in great measure to the fact that nearly one-third of the population of Canada is of French extraction, while of the remainder a large proportion are of mixed nationality.

In Australasia life assurance for 20 years was practically monopolized by the Australian Mutual Provident Society, the other native offices then in existence being purely local in character. In the year 1869, however, the marvellous success of this office appears to have induced three of these Colonies to establish other societies of a similar kind. Accordingly, in July of that year, a rival office to the Australian Mutual Provident in New South Wales, called the Mutual Life Association of Australasia, came into existence, and in a few years justified the comprehensiveness of its title by establishing branches in five of the other six Australasian Colonies. month later, not to be outdone in this direction by the sister Colony, Victoria was able to boast of possessing a life office with an equally comprehensive title, namely, the National Mutual Life Association of Australasia, which has also branches throughout the Australasian Colonies, and which owed its foundation mainly

to the efforts of its present Managing Director, Colonles. Colonel Templeton, C.M.G. The third Colony to establish a life office of this type was New Zealand. This democratic Colony, however, determined that, as security was the most important element in life assurance, the office should possess a Government guarantee. It was established at the end of 1869, and commenced business in the following March. It would appear that up to that date there was no branch of any life office in the Colony, and hence the start was made under the most favourable conditions. An attempt was made to conduct the business by means of a Board partly elected by the policyholders, but, this experiment proving unsatisfactory,

 $<sup>^{\</sup>rm 1}$  This year witnessed the failure of the Albert Life Office in England, which caused such a sensation throughout the insurance world.

the management became exclusively that of Government officials. The progress of the Department may be best judged from the fact that, at the end of 1897, it had on its books 36,174 policies, insuring, with bonus additions, close upon ten millions sterling. Its work, however, which is strictly confined to New Zealand, is so well known to members of this Institute, both from repeated references to it in the Journal and the insurance papers, and also from the very graphic account of the details of the management given at the Congress by its head, Mr. J. H. Richardson, that it is unnecessary for me to dwell on it. While the Government has, by means of repeated legislation, taken steps to make it impossible for any native office to be established in the Colony, the latest returns show that branches of six Australian, one British, and two American offices have been formed there. At the end of 1895 a comparison between the total business in force and the European population in this Colony brings out an average sum assured per policy of £267, and an average amount per head of £26. 14s. 1d., this latter sum being the highest in any Colony.

On the Continent of Australia the progress of life assurance has been rapid, especially since the year 1869, when it received a fresh impetus by the formation of the two offices already referred to. During the last 30 years a number of societies have been established, and there are now 11 native offices actively at work, besides branches of the three principal American Companies. Six of these offices have their headquarters in Victoria, four in New South Wales, and one in South Australia. Since the total population of Australia is only four millions, it may be imagined that competition in this part of the world is exceptionally severe. In spite of this, however, the native offices succeeded in 1896 in obtaining over seven millions in new business. In the course of an interview by a leading American insurance

paper a few years ago, Mr. Richard Teece, the Well-Business in Australia. Actuary, gave the following description of the method adopted in the interior of Australia by enterprising offices in search of new business. He says:—"When a trip of this nature is proposed, the agent "secures his buggy and horses, which he pays for himself, and also a black boy to ride the spare horses, for, as he may

 $<sup>^{1}</sup>$  A summary of the revenue accounts and new business of these offices, for the year 1886, appears in J.I.A., xxvi, 476. For their present position, see list in Appendix to this Paper.

"be going to drive for several days right across the Continent, he has to take along about thirty horses. He notifies the company of his intentions, and they send a doctor with him, whose expenses they pay, but the agent is not allowed a cent for expenses. The agent starts off, driving four horses in his buggy, and they go out into the country to get applications. A man is seen ploughing in the field; the agent goes to him, talks to him, and perhaps gets his application. The doctor is at hand, and makes an examination. The agent takes the premium, issues a binding receipt for it, and then they drive to the next person in sight; and so they keep on to the end of the trip, covering the country from station to station, and driving often over 100 miles a day."

The principal trouble, however, in Australia is not so much the obtaining of new business as the keeping it on the books when once it is secured. It is a constant source of complaint that some individuals make it a practice of becoming assured over and over again, taking out policies and dropping them after a year or two, since they find that they can be assured as often as they please with but little trouble. A serious obstacle in the way of maintaining a permanent business by offices is the facility with which loans are granted on policies, the tendency of which is undoubtedly to encourage lapses. A striking illustration of the high rate of lapses in the Colonies during the early years of assurance is given in the recently-published Mortality Experience of the Canada Life Office. Here the percentage of discontinuances in each year of assurance is traced in various experiences, from which it appears that whereas during the years of assurance 0 and 1 the percentage to the total in the HM experience is only 9.7, that the Canada and A.M.P. Experiences is 17.8 and 25.9 respectively.

The financial crisis which took place in Australia during the spring of 1893, when 14 banks in New South Wales, Victoria, and Queensland either failed or temporarily suspended payment, naturally gave a severe blow to life offices in those Colonies, from the effects of which they are only now recovering. For some length of time before this date all forms of life business had shown evidence of great expansion. During the two years succeeding the crisis, however, the offices had a severe struggle to maintain the position they had already acquired, the lapses and surrenders exceeding the new business.

It was, I believe, to some extent owing to the depression caused by this crisis, that towards the end of 1896¹ two of the Victorian life offices decided to join forces, and after a short delay the sanction of the Supreme Court was obtained for the amalgamation, which took place at the beginning of the following year, the business, assets, and liabilities of the smaller society being transferred to the larger, which in consequence was placed in a much stronger position than before.

Life assurance in Australia is more prevalent among the masses than is the case in most other countries. The average sum assured per policy is certainly less than in either Great Britain or Canada, being about £290. This average, it may be added, is highest in Western Australia and lowest in South Australia. The number of policies in proportion to the population, however, is much greater than elsewhere, the ratio being 66 per 1,000 of the population as against 32 in the United Kingdom and 38 in Canada, and the average sum assured per head being £21.2

The last of our Possessions that calls for notice with regard to the progress of life assurance is the Empire of India. Here, owing to the entirely different conditions of life prevailing in that vast country, life assurance has been placed in a position totally distinct from that of the Colonies of which we have been speaking. It would appear, however, that so far back as 1833 the establishment of a life assurance office was contemplated by the Government of Bengal, which was to be of an official character, but on political grounds the scheme was successfully opposed by the East India Company.<sup>3</sup> During the last 60 years a few native offices have been started in India, the oldest still in existence being the Madras Equitable, founded in 1842, which assures the lives of Europeans, East Indians, Eurasians, and Parsees. The business transacted by this Society, however, is of a very small

<sup>&</sup>lt;sup>1</sup> Shortly before this date, another Victorian company took over the business of one of the two existing South Australian offices.

<sup>&</sup>lt;sup>2</sup> Wealth and Progress of New South Wales, 1896-7.

<sup>&</sup>lt;sup>3</sup> Slater—Rise and Progress of Native Life Assurance in India. In his work on India in 1880, Sir Richard Temple observes: "It has sometimes been strongly recommended that the Government in India should undertake life insurance to a moderate and limited extent. The measure was designed for the benefit of the natives, as being calculated to teach them habits of thrifty forethought, and to form ties of the happiest kind between them and the State. Owing to various practical difficulties, and also to objections against interference with private enterprise in this respect, the Government has not yet seen its way to undertaking business of this nature, however desirable that may be on many grounds."

character, the average number of policies effected Early Life Assurance in India. during the last three years being only 27. In the eourse of a report on this Society, made in 18811, Mr. A. H. Bailey observes that the general rate of mortality experienced in the Society coincided almost exactly with that of the Uncovenanted Service Fund, published by Mr. A. J. Finlaison in 1874, which accordingly was adopted as the basis of the valuation. In an article which appeared in the Calcutta Review, in 1855, and which was reprinted in vol. vi of the Journal, the writer complains that the rates of premium for residents in India were too high, and suggests that an addition of three years to the age for the extra risk was quite sufficient. The following rates were at that time charged for an assurance of Rs. 1,000 for civil and military officials respectively by the native offices:2

| $_{ m Age}$          | Pri   | EMIUM    |     | Premiem |          |  |
|----------------------|-------|----------|-----|---------|----------|--|
| Age                  | Civil | Military | Age | Civil   | Military |  |
|                      | Rs.   | Rs.      |     | Rs.     | Rs.      |  |
| 20<br>25<br>30<br>35 | 28    | 31       | 45  | 45      | 53       |  |
| 25                   | 30    | 34       | 50  | 50      | 58       |  |
| 30                   | 33    | 38       | 55  | 59      | 68       |  |
| 35                   | 37    | 42       | 60  | 72      | 84       |  |
| 40                   | 41    | 47       | l   |         |          |  |

On comparing the above rates with those contained in Mr. Bailey's report just referred to, I find that the latter, which are described as being a reduced scale, are considerably higher throughout the Table.

As an illustration of native ideas of life assurance in India, the following remarks are of interest: "It is certain that our "remote ancestors had no idea of life assurance. Their joint "family system afforded all the comforts, conveniences, and "assurance they needed. English ideas have, however, of late "undermined the system, and the necessity for a "substitute in an artificial contrivance like life "assurance is being daily more and more felt. A quarter of a "century ago an attempt was made by some Bengali gentlemen "to establish a regular Indian Life Office, but it failed. The

<sup>&</sup>lt;sup>1</sup> J.I.A., xxiii, 52.

<sup>&</sup>lt;sup>2</sup> All these offices, it may be noted, have long since passed out of existence.

"late Albert Life Office next opened its doors for the admission of Indian lives, and several Bengali gentlemen eagerly subscribed, but the loss which its unfortunate—not to say disgraceful—failure brought on, cast a damper from which the Indian mind has not yet fully recovered."

In their modern form the principles of life assurance may be said to have been first introduced into India 25 years ago, when Mr. D. M. Slater founded the Oriental Life Office at Establishment of Bombay. The policy of this Company was then, as it still continues to be, that of assuring the lives of every race in India, whether European, Parsee, Hindoo, Mohammedan, Cingalese, Burmese, or Chinese, at precisely the same rates of premium, provided, of course, that they are able to pass a satisfactory medical examination. From various pamphlets and criticisms which have been published regarding this policy, it would appear that the management of a native life assurance company in India is not altogether a bed of roses. In a recent monograph which Mr. Slater has written upon his experiences upon the subject, however, he assures us that "no "other single measure undertaken by private enterprise has "conferred greater social blessings upon both the European " and native populations."2

Some interesting information with regard to the modern growth of life assurance in India may also be found in the evidence of Mr. Slater and of various medical men resident in that country, given before the Opium Commission in 1893. From this evidence we learn that the practice of life assurance among the natives is considerably increasing, and that assurance companies will accept a proposal from a man who takes opium as readily as if he did not, if in other respects his life is an eligible one. During the first decade of the history of the Oriental it was considered expedient to make a small extra for opium consumers in moderate quantities, more, however, as an extra precautionary measure than on account of any proved ill effects of the opium habit. At the end of this period an investigation was made into the matter, and it was then ascertained that out of 167 deaths not a single one was due either directly or indirectly

<sup>&</sup>lt;sup>1</sup> Hindoo Patriot, 14 June 1886.

<sup>&</sup>lt;sup>2</sup> Rise and Progress of Native Life Assurance in India. It may be here mentioned that, on last New Year's Day, the King of Portugal conferred on Mr. Slater the honour of a Knight of the Order of Christ, in consideration of his long and laborious services in the cause of life assurance in India.

to opium. The ratio experienced for every 100 deaths computed by the HM Table was-British 154, Eurasian 82, native 103. Hence, while British lives experienced 54 per-cent greater mortality than what was the case in this country, that of the aggregate Indian lives had been precisely the same as that shown by the HM Table. The result of this investigation modified the practice of imposing extras, and it was stated that it

sometimes happened that when a life proved exceptionally healthy no extra at all was made for opium A further investigation took place six years later, the smoking. number of deaths being 779, when it was found that the mortality of British lives in India was 52 per-cent greater than that of the HM Table, while that of the aggregate native lives was only 16 per-cent greater. Here, again, it appeared that no death was due to opium, and in view of the additional light thrown upon the subject by this experience, the management decided in future to charge no extra whatever to opium eaters. It may be added that two years ago another native life office under British management was established in India with the object of conducting business on lines similar to those of the Oriental. While, however, the policy of accepting native lives on the same lines as those of Europeans has been a successful one in Mr. Slater's office, it by no means follows that it would be equally so when pursued by British offices, which, indeed, have been very chary as to accepting this class of risk, and have limited operations of this kind to men of prominent position, and hence easily traced. It has been stated that the natives of India are to a great extent devoid of the moral sense of truth, regarding the business of life assurance as a mere game of skill, and any fraud in connection with it as a Another difficulty, as regards the general meritorious act. population, is that of identifying a particular life on a claim arising, owing to the similarity in names borne by the natives.

The foregoing description of the origin, progress, and development of the native offices in Greater Britain, though necessarily of the briefest character, will, I think, be considered sufficient to illustrate the growth of life assurance amongst our kindred in different parts of the Empire. I will now proceed to examine in a general way the position of these offices as regards the nature of their assets, the decline that has taken place in the rate of interest vielded by their funds, the stringency observed in the calculation of their reserves, and the rate of expenditure with which their management is conducted. In doing so, it will, of

course, be necessary before making comparison with life offices in this country to take into consideration the very different circumstances surrounding societies in such comparatively newly-formed countries as our possessions in Canada, Australasia, and South Africa. These different circumstances are perhaps seen most clearly when we look at the first of the items I have mentioned, namely, the assets set down in the balance-sheet, and hence this subject seems not unnaturally to form the

next most appropriate for our notice.

In Canada, life offices claim to possess absolute Government security, since, as we have seen, every item in their accounts is carefully investigated by, and has to receive the approval of, the Insurance Department of the Dominion. From the summary of the assets of all the Canadian companies tabulated in the report of the Insurance Superintendent for the year 1896, I have calculated the percentages of the various classes of securities as follows:

 Security
 Percentage

 Real Estate
 9·5

 Loans on Real Estate
 37·3

 "Collaterals
 7·1

 Cash Loans and Premium Obligations on Policies
 10·5

 Stocks, Bonds, and Debentures
 27

 Cash in Hand and in Banks
 2·5

 Other Assets
 6·1

100

If we compare these percentages with those relating to securities of a corresponding nature in the assets of British companies, we find a very marked similarity, especially as regards the two principal items—" Loans on Real Estate", which is very little higher than our "Loans on Mortgage", and "Stocks, Bonds, and Debentures", which is rather lower than that in British companies.

When we turn to the balance-sheet of Australian offices, we find that a much greater contrast in the character of the assets on the Australian prevails here than in the Canadian offices, when the percentages of those of British offices are placed alongside. This contrast will be readily seen in the following Table, which is taken from a very interesting paper on

<sup>&</sup>lt;sup>1</sup> The Assets marked thus (\*) were, in 1897, 48·6, 20·4, and 10·3, respectively per-cent of the total.

"Australian Investments", read three years ago by Mr. W. R. Day before the Insurance Institute of New South Wales.

| Security                            |   | Perc    | ENTAGES      |
|-------------------------------------|---|---------|--------------|
| security                            |   | British | Australasian |
| Mortgages                           |   | 37.2    | 50.7*        |
| Loans on Policies                   | . | 4.7     | 20.7*        |
| Government and Municipal Securities | . | 21.6    | 7.6*         |
| Property owned                      |   | 6.8     | 10.9         |
| Stocks, Shares, and Debentures .    |   | 20.3    | .6           |
| Cash in Hand and at Bankers' .      |   | 3.4     | 6.2          |
| Other Items                         |   | 6       | 3.3          |
|                                     |   | 100     | 100          |

The most striking features in the percentages of the Australian companies' assets are the very large proportion borne by "Loans on Policies" and the very insignificant one represented by "Stocks, Shares, and Debentures." As regards the former item, the explanation is to be found in the operation of the non-forfeiture system. This system, which in its present form was founded some thirty years ago in the Colony of Victoria, has been universal throughout Australasia for many years, and, it has been estimated by Mr. W. R. Dovey, accounts for three-quarters of the total amount set down against this particular asset. The extremely small proportion borne by the latter item is due to the fact that this class of securities scarcely exists in Australasia-at least, in a form suitable for life office investments—on account of Government control prevailing to so great an extent in commercial undertakings. In fact, the favourite securities of Australian life offices may be said to be mortgages of land and fixed deposits in banks. Such investments as life interests and reversions are naturally almost unknown in the Colonies.1

In the New Zealand Insurance Department the assets at the end of 1896 were invested as follows:

<sup>&</sup>lt;sup>1</sup> One office in New South Wales, and two in Victoria, appear to include among their assets securities of this nature, but the total amount at the end of 1897 was under £100,000.

| Security                      |    |  | Percentage |
|-------------------------------|----|--|------------|
| Government Securities .       |    |  | 32.9       |
| Mortgage on Freehold Property | ٠. |  | 30.7       |
| Loans on Policies             |    |  | 17.6       |
| Local Bodies' Debentures .    |    |  | 6.3        |
| Landed and House Property     |    |  | 5.5        |
| Cash on Current Account .     |    |  | 4.1        |
| Miscellaneous Assets          |    |  | 2.9        |
|                               |    |  |            |
|                               |    |  | 100        |

As regards the second item in this table, it may be remarked that by the constitution of the Department the investment of all funds in securities of this nature is controlled by a Board restricted to lending Government officials, who are restricted to lending not more than three-fifths of the value of the property. The constitution also provides that one-quarter of the total funds must be invested in Government securities and loans to local authorities approved by the Governor in Council. It will be seen on reference to the Appendix to this Paper that the total assets of the Department amount to more than  $2\frac{1}{2}$  millions sterling.

In South Africa the assets of the two native offices may be classified as follows:

|                              |       |      |   |                   | Percentages                         |       |
|------------------------------|-------|------|---|-------------------|-------------------------------------|-------|
| Security                     |       |      |   | ln<br>Cape Colony | Elsewhere<br>than in<br>Cape Colony | Total |
| Government and Municipal Se  | eurit | ties |   | 5·5               | 6                                   | 11.5  |
| Mortgages                    |       |      |   | 39                | 20.1                                | 59.1  |
| Loans on Policies            |       |      |   | 7:9               | 3.3                                 | 11.2  |
| Transvaal Debentures .       |       |      |   |                   | 2.6                                 | 2.6   |
| House Property               |       |      |   | 2.3               | 3.8                                 | 6.1   |
| Cash in Hand and at Bankers' |       |      |   | 3                 | 2                                   | õ     |
| Other Items                  | •     |      | • | 2.7               | 1.8                                 | 4.2   |
|                              |       |      |   |                   |                                     |       |
|                              |       |      |   | 60.4              | 39.6                                | 100   |

Here we see that mortgages again bear a very high proportion to the total assets—higher, in fact, than in any other group of Colonies. The non-forfeiture system is also prevalent in the South African Colonies, and hence the same remarks which have been already made with regard to the item "Loans on Policies" also apply in this case.

From the consideration of the nature of the assets of Colonial life offices we naturally pass to a comparison of the rates of

 $5\frac{1}{4}$  per-cent.

It need hardly be said that throughout the Colonies, with the exception apparently of those in South Africa, this rate has declined rapidly within recent years. In Canada the funds of life offices, which in 1880 yielded an average of nearly  $6\frac{3}{5}$  per-cent, produced in 1891 a mean rate of only  $5\frac{1}{5}$  per-cent, and at the present time  $4\frac{1}{2}$  per-cent may be taken as above rather than below the average yield, and this, too, in spite of the fact that not less than 6 per-cent is charged on loans on policies. In Australasia the rate of interest has Average Rates of Interest yielded by Funds. been maintained better than in Canada, probably on account of the higher percentage which policy loans bear to the total assets, the rate charged in these Colonies for such loans being either 6 or 7 per-cent. The rapid nature of the decline in these Colonies may be judged of by the fact that in 1891 the funds of the combined Australian offices yielded an average of £5. 19s. 1d. per-cent, while in 1896 the average yield had fallen to £4. 18s. 10d. per-cent.¹ During the same period the New Zealand Government Department had experienced a fall from £5. 8s. 2d. per-cent to £4. 19s. per-cent. In the course of his Inaugural Address before the Insurance Institute of Victoria in 1897, the President (Mr. Gillison) gave some striking illustrations with regard to this rapid decline, and arrived at the conclusion that, after making an allowance of 1 per-cent per annum for the depreciation which has taken place in Australian securities, the average rate is very little above that of British offices.<sup>2</sup> As already remarked, these securities usually consist of landed property, whereas the assets of British offices include a large proportion of Stock Exchange securities, from which a profit may be realized from time to time sufficient to neutralize to a great extent the fall in the rate of interest. The South African offices have, on the other hand, been fortunate in not only maintaining the average rate of interest at a respectable figure,

In view of the general decline in the average rate of interest

but have even increased it, since while 20 years ago this rate appears to have been  $4\frac{3}{4}$  per-cent, it now stands at about

<sup>1</sup> A table in the Australian Insurance and Banking Record for January 1899, gives the following percentages relating to the average rate of interest:— 1889, 6·29 per-cent; 1894, 5·44 per-cent; 1898, 4·82 per-cent.

<sup>&</sup>lt;sup>2</sup> In a Paper read before the same Institute in October last, Mr. Pullar discusses this subject at some length, though without expressing any definite opinion as to the future.

yielded by the funds of Colonial life offices, it is satisfactory to observe that these offices are fully alive to the importance of meeting this decline by strengthening their reserves. In Canada, the standard rate of 4½ per-cent, fixed by Statute for valuation purposes as long ago as 1877, is still in force. Most of the life offices in the Dominion have, however, wisely reduced their valuation rate to 4 per-cent; and it is to be hoped that the standard rate in Canada will before long be reduced to this lower rate, as was done some years ago in the United States. In Australia, where no standard rate prevails, the offices value their liabilities at 4 or even 3½ per-cent, while in the last valuation of Standards of Valuation, the New Zealand Government Department the rate adopted was also 4 per-cent, special reserves being made in addition, which reduced the rate to 35 per-cent. Of the two native life offices in South Africa, the older, which has hitherto valued at 4 per-cent, has now lowered its rate to  $3\frac{3}{4}$  per-cent with the object of making a further reduction of  $\frac{1}{4}$  per-cent at its next valuation. The vounger, however, on the advice of two wellknown British actuaries who were consulted on the matter, has decided to continue for the present the rate adopted at its first valuation, namely, 4 per-cent, which is more than 1 per-cent below the mean rate vielded by its funds. It may be added that, with a few exceptions in Canada, where the Table of the American Offices is occasionally used, the Institute of Actuaries' Tables of Mortality are universal in the Colonial offices.1

The rate of expenditure in life offices throughout the Colonies is, it need hardly be stated, considerably greater in proportion to the premium income than in the old country. In Canada this rate is nearly 20 per-cent in the most economically-managed office, the average being a little over 25 per-cent. Here the keen competition with expensively-managed American companies, together with the fact that most of the offices are of recent growth, sufficiently accounts for the expenditure appearing abnormally high. In the Australasian Colonies the percentages

¹ At least three Australian offices now employ the H<sup>M(5)</sup> Table, either wholly or partially. One of these offices states:—"The H<sup>M</sup> and H<sup>M(5)</sup> Tables have been specially blended for the use of this society. For the early years of the policy the H<sup>M</sup> Table only is employed, and for its later years the H<sup>M(5)</sup> Table. Between these periods the values have been graduated, and lie between the H<sup>M</sup> and H<sup>M(5)</sup> values. Such middle period embraces, for over three-fourths of the policies, all durations between three and ten years, while for less than one-fourth of the policies, being a portion of the endowment assurances only, it includes all durations between four and eight years. For assurances on joint lives and term assurances the H<sup>M</sup> Table alone has been employed."

vary from 13 to 33, that of the New Zealand Government Department being 23. The average of all the Australasian offices was, in 1897, a little under 20 per-cent. From a table published in the Australasian Insurance and Banking Record for January 1898, where an analysis of the various items of expenditure is given, it appears that the proportion of the expenditure for obtaining new business may be put down at about 76 per-cent of the new premium income. In these Colonies the competition is also very severe, though it is principally among the native offices which absorb the bulk of the Australasian business.

In South Africa, where the same method of obtaining business is employed as in Australasia, the average rate of expenditure is somewhat less, being about 17 per-cent of the premium income.

In connection with the development of native offices, two unfavourable features have sprung up within recent years, namely, the rebate and assessment systems, which, originating in the United States, have found their way into Canada, though not to any great extent, I believe, into other parts of our Colonial The former of these two systems consists of Possessions. allowing intending policyholders a discount on their premiums in order to entice them away from other companies. evil, which in spite of repeated penal legislation, has grown but little unchecked, has long been a source of bitter strife in the United States. A few years ago the matter was taken up by the influential society known as that of the Life Underwriters of the United States, who were unanimous in resolving to do their utmost to secure the extinction of the system. in consequence mainly of the spread of this system in various parts of Canada that in January 1891 there was formed organization called "The Ontario Life Underwriters' Association", which was successful in obtaining an Act by the Legislature of that Province containing provisions for the effective dealing with the subject, among them being one which prohibited any person except a registered agent attempting to secure life assurance business in the Province.

The assessment system, about which so much has been written in this country, appears to have obtained considerable favour in Canada.<sup>2</sup> At the end of 1896 there were 10 companies

<sup>&</sup>lt;sup>1</sup> In one large Victorian office this proportion amounts to nearly 97 per-cent.
<sup>2</sup> Companies transacting business on this system were officially recognized in the Dominion for the first time in 1885, when an Act was passed for their regulation.

of this description transacting business in the Dominion, of which 7 were Canadian with a total sum assured of \$88,305,500, 3 American with a total of \$44,579,899, making altogether the sum of \$132,885,399 assured under this system. 1 Assessment assurance has also been the subject of legislation, both in the Dominion Parliament and also in that of the Province of Ontario. In the various Acts where it is referred to, it is provided that every publication and advertisement used for the purposes of the system shall bear the words "Assessment System" printed or stamped in large type at the head of such publication or advertisement. In the Report of the Insurance Superintendent of Canada for the year The Assessment System in Canada. 1896 there occurs a long correspondence on the subject of one of the American companies, as to which very great dissatisfaction had been occasioned in Ontario by a large increase in the first assessment levied in 1897 as compared with those of 1896. In the course of this correspondence the Superintendent remarks: "What has just taken "place in the case of this company, and is now taking place, "exemplifies in a striking manner what may be expected in "the case of all similar associations. Policy-holders paying "less than the proper premium for temporary insurance are "led to believe that such rates are amply sufficient to carry "the policies throughout life; they are assured by agents and " officers of such associations that no increase will take place. " and, relying on such assurances and representations, continue "members, and others, by similar representations, are induced " to become members, all only to find, when too late, if the " company insured in does not wholly collapse, that they must " either drop out or pay very largely increased premiums, and "that, too, at a time when, by reason of impaired health, or "for other causes, it may be impossible to obtain insurance "elsewhere." In referring to this correspondence in his Report, the Superintendent adds that, in consequence of the dissatisfaction caused by the levy, the percentage of lapses has been very great. It may be mentioned that a few months ago the Economist of Toronto stated that, while no old-line company had ever failed in Canada, no less than 36 assessment societies had come to

<sup>&</sup>lt;sup>1</sup> Two companies, not unknown in this country, one Canadian and the other American, helped to swell this total by the sum of \$96,791,974. The latter company last year announced its intention of opening branches in Australasia.

grief during the short period within which the system has been in active operation in the Dominion.

Although, however, the assessment system does not seem to flourish outside the American Continent, the principles underlying it may be traced in our Indian Empire, where almost every town has its provident fund or mutual benefit society. In alluding to these principles, the Times of India, in the course of an article on a native provident association a few years ago, stated that the attractive prospect held out by these societies draws cheap Insurance large numbers of unthinking persons to adopt this system of cheap assurance in preference to a recognized life assurance company. The particular association referred to in the article was originally founded in 1887 for the benefit of Government employees, but it afterwards extended its operations to the general public, who, for a premium of 8 annas a month, were assured a possible death benefit of Rs. 1,000.

From the consideration of the growth and present position of the Colonial life offices, I will pass on to a brief outline of their work. I may here state that, in order to qualify myself to deal effectively with this portion of my subject, I applied to the managers of all the principal offices in the Colonies for information as to the various plans and schemes generally in vogue. My applications were liberally responded to, and I have before me a large number of leaflets and papers describing more or less fully the special features of each society. I should like to take this opportunity of offering my sincere acknowledgments to these gentlemen for their courtesy and promptness in acceding to my request. On examining these numerous pamphlets, I find that they contain a vast amount of valuable material, but since a full discussion of their leading principles would alone be sufficient to occupy a whole paper, I shall confine my remarks to the consideration of a few of the more prominent schemes which have been either originated or adopted by the Colonial offices.

For the variety and originality of its different "plans of assurance", the Dominion of Canada easily takes the first place; and both in their character and their nomenclature we observe the distinct influence of the United States. Prospectuses, of the kind to which we are accustomed, are as a rule unknown in Canada, their place being taken by a variety of leaflets printed in different coloured inks, and bearing attractive, if somewhat high-sounding, titles. One office, for instance, issues a little

book under the title of "Open and Read: Life Assurance, "like Truth, shows its gems rarer and purer the deeper you "penetrate its principles." It prefaces the description of its various schemes with a kind of homily upon the moral duty of making provision for the future, illustrated by quotations from Shakspeare, Goldsmith, and other poets. The general character, however, of the literature issued by Canadian offices is of a more prosaic nature. Among the various descriptions of policies granted may be mentioned the following: "The Canadian Plans Unconditional Accumulative Limited Payment Life Policy", "The Reserve Dividend Plan", "The Coupon Bond Life Policy ', "The Reduced Premium Guaranteed Policy", "The Investment Annuity Policy", and "The Automatic, Non-Forfeiting, Self-Premium-Paying Policy." Most of these plans, it may be observed, are merely modifications in some form or other of the tontine principle of allotting bonuses—a principle which appears to be exceedingly popular both in the United States and Canada. The "estimate" element naturally plays an important part in all these plans, but it is only fair to state that in the majority of cases the amounts mentioned in the contract are absolutely guaranteed, and that everything in these plans appears to be straightforward and above-board. In what are called "cumulative" policies, at the end of a specified period the following options are generally given to the assured: (1) To draw the surrender-value, which varies from a percentage of the Government Reserve to the whole of it, together with the accumulated profits; (2) To draw these profits alone, and to convert them either into a paid-up policy or into a life annuity if the former be selected, evidence of health is required in some companies, but not in others; (3) To purchase a life annuity with the cash value; and (4) To convert the assurance into a paid-up policy.

The non-forfeiture system, originated in Massachusetts and developed in its present form in Australia, has also recently been adopted by some of the Canadian offices. The policies issued by one of the largest companies contain a provision for automatically keeping them on foot after two annual premiums have been paid, the reserve by the Government standard being used for this purpose, and interest being charged at the rate of 10 per-cent per annum (7 per-cent being for interest proper, and 3 per-cent for expenses). In another leading office where this system is in force, the reserve value is used to purchase a term

assurance, which has the effect of covering the assured for a longer period, though at the end of this period the policy lapses altogether.

There would appear to be transacted in Canada a considerable amount of Industrial Assurance on the lives of children, which, in the Province of Ontario, is regulated by the provisions of "The Insurance Corporations Act, 1892." With the object of securing some of this class of business, some two years ago, one of the Canadian Companies, with a large connection in ordinary assurances, started what it called a Thrift Department. In this department are issued policies, for amounts not exceeding \$1,000, subject to much lower rates of premium than those charged by Industrial Companies, and containing two features not to be found in industrial policies. One is that an annual reversionary bonus of £1 per-cent is guaranteed, and the other is that only annual or half-yearly premiums are accepted. This scheme, which was designed to encourage thrift among the working classes, and not merely to provide burial money on the death of their children, has, I am told, proved a success, and has been copied, as is the fate of the majority of successful schemes, by other companies on the American Continent. Great care appears to be exercised in the selection of the lives, whether those of children or of adults, in this department, the number of questions asked in the applications, and agents' and medical examiners' reports, which are all on the same form, being extremely numerous and searching.

When we turn from the practice of Canadian life offices to that of societies in the various Colonies in Australasia and South Africa, we find ourselves in quite a different atmosphere. these Colonies, the style of the literature issued, and the schemes adopted by the native offices, resemble those of our own country in much the same way as Canadian ideas on the subject of life assurance reveal the influence of the United States. prospectuses of the societies belonging to the Southern Hemisphere, are very similar in appearance to those of British offices, especially as regards the tables of rates, which are invariably appended. Two features of life assurance are, I believe, universal, or nearly so, in these societies, namely, the non-forfeiture and contingent debt systems. The former system was introduced into Australasia in the year 1869, being based, as already mentioned, on the provisions of a law of the State of Massachusetts, passed in the early part of 1861. It differs, however, from the Massachusetts law in this respect, that, whereas, under the latter system, a term premium only is debited to the assured, and hence the forfeiture of the policy is not avoided but only postnoned under the Australacian system the premium

and hence the forfeiture of the policy is not avoided but only postponed, under the Australasian system the premium actually overdue, together with interest at a high rate, is thus debited until the surrender-value is exhausted. It may be added that the policyholder is always kept fully informed of the course which has been taken in keeping his assurance in force.1 Of the advantages and drawbacks attaching to this system, there is a good deal to be said. It is certainly popular in the Colonies where it is adopted, and apparently proves an effective argument in dealing with an applicant for assurance. The prospectuses of the offices, too, contain a number of illustrations in which the application of the system has proved an undoubted benefit to the family of a deceased policyholder. On the other hand, this system seems to be productive of great laxity on the part of the assured, who, knowing that their policies will be kept in force by the office, are apt to take very little trouble in the matter. It is also a very expensive process, as the interest charged is generally 8 per-cent, and hence it is not an incentive to thrift. Lastly, it is one of the main causes of the heavy lapse rate in the Colonies where it exists, and this circumstance alone is perhaps a very strong argument against the system.2

The other system referred to, namely, that of taking underaverage lives at ordinary rates, subject to a contingent debt on the policy until the expiration of the period covered by the expectation of life of the assured, is also one which is open to criticism. The system was discussed, it will be remembered, at this Institute eight years ago, when the general opinion of British actuaries appeared to be decidedly against it, though on practical rather than theoretical grounds. In the course of this discussion,

Mr. F. W. Frankland, formerly Commissioner of the New Zealand Insurance Department, stated that the system had, after a trial, been abolished by the Department, principally on the ground that a very small proportion

<sup>&</sup>lt;sup>1</sup> In 1896 Mr. Teece gave some figures illustrating the results of this system in his office, from which it appears that of the policies to which it was applied 80 per-cent were re-instated and only 5 per-cent were eventually forfeited. A similar statement is given in the last Valuation Report of the South African Mutual, the percentages in this case being 41 and 25 respectively.

<sup>&</sup>lt;sup>2</sup> In a letter to the *Insurance Record*, some years ago, a New South Wales manager stated that he considered the advantages of this system to be very much exaggerated, and that his own office had adopted it only through stress of competition.

of the class of lives which it was intended to benefit took advantage of its operation. The system has also been discussed by the Insurance Institute of New South Wales, some difference of opinion prevailing among that body as to its merits.1 In India the company which transacts the bulk of life assurance business has recently adopted the principle of this system, by charging its assured British rates of premiums, subject to a debt (without interest) on the policy of the difference between the sums assured which could be obtained at the same premium according to Indian and British rates. This debt diminishes yearly in proportion to the actual length of life attained, being finally extinguished when the expectation of life calculated according to British tables is reached. This system is described in the prospectus as "the latest improvement in life assurance",-not perhaps a very correct description of a method, the principle of which was in vogue 38 years ago.

Another feature, which is characteristic of all policies issued in the Australasian Colonies and the Cape, and which is probably of great assistance to offices in obtaining business, is the statutory protection from creditors afforded to an assured and his representatives. The extent of such protection varies in different Colonies, both as to amount and date of inception. The New Zealand Act of 1884 is generally regarded with most satisfaction in this respect, since its provisions seem to meet every possible contingency.

Of schemes adopted by individual offices I may mention the following, which emanate from Australia. One of these is described in the prospectus of the office with which it originated as "a new system of life assurance, exempting from payment of "premiums when the assured is either temporarily or permanently "incapacitated either by accidental bodily injury, or by illness, or "by mental disorder." To cover this additional risk, which ceases at age 60, a slight addition is made to the premium. The other scheme is of a still more novel character, and consists of an arrangement whereby free hospital treatment is given, when necessary, to the lives assured in the society which has adopted it, the theory being "that one member saved is worth a dozen

<sup>&</sup>lt;sup>1</sup> This system was originated in 1861, by the late Mr. M. A. Black, afterwards to become so much identified with life assurance in the Australasian Colonies. A critical paper on it, from the pen of the late Mr. S. Younger, appeared in the J.I.A., vol. x; while in vol. xxxi there is a letter from Mr. D. Carment, which deals with the subject from an Australasian point of view.

gained." The scheme is stated to be working well, and to be of service to the society.

In their methods of distributing surplus, there is not much variety among Colonial offices. In Canada the tontine system is universal, and is based on the "contribution plan" of Mr. Sheppard As regards Australasia, Mr. D. Carment tells us that "most, if not all, of the Australian offices now "employ, in the division of surplus, what may be "termed the 'modified contribution method', by which each " policy is first credited with the surplus arising from interest "earned on the reserve held for it at the previous investigation, "and the remaining surplus is then allotted in proportion to the "loading on the premiums paid during the period in question." It may be added that, in two Australian offices at least, the net premiums adopted in the valuations of whole-term policies are those for the ages next higher than the ages at entry. This method-which is identified with its originator, Dr. Spraguewas, it may be remembered, severely criticized some years ago by a distinguished Colonial actuary in a paper read before this Institute. The effect in the distribution of surplus is, of course, to reduce the amount of loading contributed during the valuation period, and hence to allot smaller bonuses to recently-effected policies. In the South African and Indian offices, the surplus is distributed as a uniform percentage on the sums assured and previous bonuses.

In the Dominion of Canada, as well as in the Colonies of New Zealand, Victoria, and the Cape, there are offices which endeavour to attract into their ranks the lives of total abstainers, by placing them in a class by themselves, styled a "Temperance Section", and holding out to them the bait of higher bonus prospects in consequence of the greater vitality which they are assumed to possess. In the Victorian Office the last Quinquennial Report of the Actuary indicates very favourable results in this section, where the percentage of actual to expected deaths was only 59.2 as against 68.4 in the whole of the Society's business. The Canadian Office also claims to have experienced a very light rate of mortality in its "Temperance Section." In the New Zealand Government Department, however, the experience of this class of lives seems to have been but little different from that of the "General Section." It will be remembered that, in the

course of his paper on this Department read at the Congress, Mr. Richardson gave some interesting statistics with regard to this experience. He there stated that, during the period in which the Temperance Section existed, four divisions of profit had been made. On the first occasion the "Temperance" policyholders received no bonus, on the second they received the same bonus as those in the "General Section", on the third the "Temperance" bonuses were slightly higher than the "General", while on the last occasion the conditions of the third division were reversed.

At the commencement of this paper, reference was made to the fact that four of the Colonial offices had recently made investigations into their mortality experience. I therefore propose, at this point, to make a few comments on the data which have been published with reference to the mortality in Canada, Australia, and South Africa. The principal results of the investigations made by the oldest offices in these three groups of Colonies may be tabulated as follows:

|                                    | Date<br>of          | Years<br>of      |                 | Number           | rs of             |                |
|------------------------------------|---------------------|------------------|-----------------|------------------|-------------------|----------------|
| Name of Office                     |                     | Observ-<br>ation | Entrants        | Existing         | Discon-<br>tinued | Died           |
| Australian Mutual Provident        | 1888                | 40               | 114,471         | 73,632           | 35,096            | 5,743          |
| Canada Life South African Mutual . | $\frac{1893}{1895}$ | $\frac{46}{50}$  | 35,287 $16,695$ | 19,419<br>10,161 | 13,079 $4,764$    | 2,789<br>1,770 |

For the purpose of comparing the mortality experience in these offices, I have extracted the figures representing the expectation of life at quinquennial ages, which are as follows:

| Лge | Canada Life | А. М. Р. | South African<br>Mutual |
|-----|-------------|----------|-------------------------|
| 20  | 46.2        | 47.1     | 42.7                    |
| 25  | 42.3        | 42.9     | 38.6                    |
| 30  | 38.3        | 38.7     | 34.2                    |
| 35  | 34.3        | 34.6     | 30.6                    |
| 40  | 30.3        | 30.6     | 36.8                    |
| 45  | 26.4        | 26.7     | 23.1                    |
| 50  | 22.5        | 23       | 19.6                    |
| 55  | 18.8        | 19.3     | 16.4                    |
| 60  | 15.4        | 15.7     | 13.6                    |
| 65  | 12.2        | 12.4     | 10.9                    |
| 70  | 9.4         | 9.7      | 8.2                     |

In the above table there will be observed a very close resemblance between the experience of the "A.M.P." and that

of the "Canada Life." It must not, however, be concluded that the rate of mortality in each experience is the same, since the former was to a great extent vitiated in its results by the treatment of the surcharged lives, which amounted to no less than 35 per-cent of the total under observation, as against only 7 per-cent in the Institute Experience. These lives were mixed with the healthy lives, the assumption being that at the increased ages the former were equal to the latter. Hence the rate of mortality is brought out lower than would have been the case had only the healthy been included. Another circumstance which would

also tend to reduce the rate of mortality in the A.M.P. Experience is the effect of selection due to the short period in which a large proportion of the lives had been under observation. In the Canada Experience this was not so much the case, the lives being very evenly distributed over the whole of the period observed. A third reason why a fair comparison between the A.M.P. results and those of any other standard table cannot be made, is that female lives, each of which was rated up three years, were also included in the general experience.

In the Experience of the "Canada Life" the following are the general conclusions which were arrived at:

- (1) The quality of assured male lives in Canada is not surpassed by that in the United States, Great Britain, Germany, or Australasia.
- (2) This favourable experience is not confined to the early years of assurance, but is maintained throughout the duration of assurance.

It may be added that in this investigation, which was the first of the kind ever made in the Dominion, all rated-up lives were carefully eliminated.

The Experience of the South African Mutual, which was published only last year, is divided into two sections, one which includes the separate observations on healthy males and females and the rated-up lives of both sexes, and the other which is confined to those male members of the society who had been the "S.A.M." accepted either at the ordinary rate or with an extra for climate or occupation. It is in this latter section that we find the bulk of the Experience as well as its most salient features. The results of the Experience show that in South Africa life assurance is, as a rule, effected at an earlier

stage than in Great Britain or the other Colonies; that the rate of mortality is less than that of the  $H^{\mathrm{M}}$  Table in the earlier years of life and greater after about age 50; and that the reserve values calculated by the South African Mutual Experience are eonsequently higher than those by the  $H^{\mathrm{M}}$  Table.

In addition to the above experiences of Colonial offices, a fourth has been published in the Experience of the Australian Mutual Provident in Queensland during the period 1875 to 1888. This Colony is semi-tropical in its climate, and hence its mortality might be expected to be much higher than that prevailing in temperate climates. It is remarkable, however, that in this Experience the rate of mortality, though higher than that of the A.M.P. Experience already referred to, is only 77 per-cent of the H<sup>M</sup> Table. The small number of observations, together with the short period during which the experience extended, must necessarily be taken into consideration in making a comparison between the two tables.

The subject of legislation in the various Colonies is a very wide one, and hence I shall content myself merely with drawing attention to some of its most important features with respect to the practice of life assurance. For a more detailed discussion of the numerous Acts which have been passed by the Colonial Legislatures, I cannot do better than refer to the valuable paper on "British Life Assurance Companies and Colonial Legislation", read before the Imperial Institute on 12 December 1895, by Mr. H. R. Harding. Some of the principal Acts, Life Assurance Legislation. too, have been published in the Journal of the Institute of Actuaries, and as, with the exception of those of Canada, which are modelled principally upon the United States laws, they all bear a strong resemblance to each other, their main principles are doubtless already familiar to members of the Institute. In the Australasian Acts these principles have been very ably summarized by Mr. Teece in his paper read at the Congress last year.2 A few words on the general tendency of legislation as regards life assurance in the Colonies must therefore suffice for the needs of this paper.

<sup>&</sup>lt;sup>1</sup> In the "Canada Life" and "South African Mutual" Experiences, policy years were adopted, while that of the "A. M. P." follows the Institute method of calendar years.

<sup>&</sup>lt;sup>2</sup> See also a paper on this subject by Messrs. Day and Hollingworth, read before the Insurance Institute of New South Wales in July 1895.

In Canada, the keynote of such legislation may be described as being of a twofold character, namely, (1) The protection of the native offices from foreign competition, and (2) The guarantee of their solvency by means of the system of annual licenses and Government supervision. I have already alluded to the reports of the Superintendent of Insurance of the Dominion, issued year by year, in which may be found the fullest details as to insurance companies of all kinds, and In Canada. which are compiled and published with a rapidity that might with advantage be copied in the old country. The Act of 1894 requires a preliminary abstract of the year's business to be sent in to the Superintendent within a month, and complete returns within two months, of the close of the year. One office, indeed, in its report for 1897, prides itself on having for the past seven years sent in both kinds of returns to the Government on the first day of the new year! Insurance companies, it may be added, have to submit to the control, not merely of the Dominion Government, but also of the authorities of the Province in which their head office may be situated—a state of things that has been the cause of frequent disputes and litigation. "On the whole", says Mr. F. Sanderson, "the life insurance companies prefer "to place themselves under the jurisdiction of the Dominion "Insurance Laws, and only give their allegiance to the regula-"tions of the Provinces when it is obviously useless to oppose " them." 2

In Australasia and South Africa, successive legislation has followed, in the main, the lines of the British Act of 1870. In Queensland, however, under the Act of 1879, which governs life offices transacting business in that Colony, the full details required in the other Australasian Colonies are not provided for; while in New South Wales, notwithstanding the fact that Bills for the regulation of life assurance have been promised by successive governments, there is still an absence of legislation on the subject.<sup>3</sup> It is the practice, however, of life offices in both these Colonies, to issue, with their Valuation Reports, the fullest information as

<sup>&</sup>lt;sup>1</sup> Under Section 6A of this Act, it is provided that "a license shall not be granted to a company to carry on the business of life insurance in combination with any other branch of insurance."

<sup>&</sup>lt;sup>2</sup> Paper on "Life Assurance in Canada", read before the Actuarial Society of Edinburgh, 11 January 1894.

<sup>&</sup>lt;sup>3</sup> The Insurance Institute of New South Wales has repeatedly endeavoured to induce the Government to legislate on the subject.

to their business. The model Act of these Colonies, is generally regarded as that passed by the South Australian Legislature in 1882 1 and amended in 1885 and With respect to statutory forms of assignment and mortgage of policies, however, "The Life Assurance Policies Act, 1884", of New Zealand, appears to be considered as most complete and workable, containing, as it does, no less than 31 clauses and 12 schedules dealing with various phases of the subject. effect of all these various enactments has not unnaturally produced a great deal of confusion and harassment to the life offices, and has caused a number of legal points to be raised as to the validity of transactions in the different Colonies. This especially is the ease with respect to assignments, each Colony having its own forms, and refusing to recognize those adopted by its neighbours. It is, therefore, but little to be wondered at, that the managers of life offices in Australasia look forward to the much-talked about Federation Act as something approaching to the millennium.<sup>2</sup>

In South Africa, Life Assurance Companies are regulated in the Cape Colony by the provisions of the Acts of 1887 and 1891, and in Natal by the Act of 1885. The first and last of these Acts relate exclusively to the grant of licenses. That In South of 1891 is, however, a more elaborate affair, being similar in form to those of Australasia. No deposit is required by life offices, but annual and valuation returns must

be sent in—the latter to be made "by a duly qualified actuary." The Government Actuary makes an annual report in accordance with the terms of this Act, which, however, is published somewhat tardily, since that for 1895 bears date 26 March 1897.

In all the Colonies, as well as in India, provision has been made by the Legislatures for the grant of policies for the benefit of wives and children. In this respect, Canada has been ahead of the old country, since, five years before the passing of our first Married Women's Property Act, a statute entitled "The Wives and Children's Act", was passed by the Legislature of the Province In Australasia, the germ of the principle involved of Ontario.

in such a provision, may be found at a still earlier period in the Act of Incorporation of the Australian Mutual Provident Society, which gave power to married women to effect policies in that society in

<sup>2</sup> A draft of a proposed Federation Bill is given in the Australian Hand-Book, 1897.

The Protection of Married Women. For the provisions of this Act, see J.I.A., xxvi, 24. In a Paper by

Mr. C. D. Higham (J.I.A., xxvi, 325) reference is made to this and to other Colonial Acts in connection with assignments.

their own names. This power was made general in New South Wales by the Life Insurance Encouragement Act of 1862. "Under these Acts, however", says Mr. Teece, "it is only " stipulated that such policy shall not be subject to the debts of "the husband, and may be disposed of by the member by will." Subsequent legislation, in this and the other Australasian Colonies. has provided ample powers in connection with policies effected either by married women, or by husbands for the benefit of their wives and children. The last Colony to legislate on the subject was, I believe, the Canadian Province of New Brunswick, which in 1895 passed "An Act to Secure to Wives and Children the Benefit of Life Insurance." This Act is somewhat voluminous, containing, as it does, 26 sections, besides several sub-sections. It deals not merely with the protection of policies of this kind from creditors, but also with the apportionment of the insurance money among the beneficiaries, the appointment of trustees and guardians, and the grant of paid-up policies and loans to pay premiums. An unmarried man, too, can effect an assurance under this Act for the benefit of a future wife and children. The provisions of the English Acts of 1870 and 1882, relating to the protection of policies of this nature, have apparently been in the main followed in the drafting of the various Colonial Acts passed since these dates.

The consideration of the tendency of legislation in the Colonies, would not be complete without some reference to the controversial subject of State supervision. On this subject the opinions of leading members of the Institute of Actuaries have been so recently expressed in the discussions that took place in this hall on Mr. Sheppard Homans' paper two years ago, and during the meeting of the International Congress last year, that any repetition of them would be superfluous. I should like, however, to quote the following conclusions arrived at by Mr. T. E. Young, in his address on "The Limits of State Control, and the Business of Insurance",1 which, it seems to me, sum up very forcibly the views of actuaries in this country. He says :- " In a business like ours, we simply " require that legitimate and complete freedom—the essence of "State existence-should be accorded to our capacity and "enterprise, and that, as far as is feasible, our proceedings and " results should be published to the world and to the criticism in " technical questions of competent experts."

<sup>&</sup>lt;sup>1</sup> Delivered before the Birmingham Insurance Institute, 26 October 1894.

In the course of our review of Colonial Life Assurance, we have seen that State supervision is supreme throughout the Dominion of Canada and in New Zealand; that it exists, in a modified form, in most of the Colonies on the Australian Continent, and in South Africa; and that in New South Wales it is totally unknown. Hence it may well be imagined that, with respect to its advantages and disadvantages, the views of Colonial actuaries must vary considerably, according to the particular section of the British Empire to which they belong. Canadians, for example, sturdily uphold the principle of State control, in spite of its necessary accompaniments of obligatory and irksome requirements, such as annual licenses both for companies and their agents, fixed standards of valuation, the severe investigation by Government officials into all the details of their business, and the penalties imposed upon failure to comply with the provisions of the various Dominion and Provincial Acts. They point with satisfaction to the indisputable facts that a Government license includes all the advantages of Government security, and that they are afforded ample protection Views of leading from undue foreign competition. In Australia and ance Managers. at the Cape,1 on the other hand, the views of leading insurance men on this subject are practically in accordance with those expressed by Mr. Young. The Victorian view was well expressed by the Australasian Insurance and Banking Record, in an obituary notice of the late Mr. T. J. Martin, thus: "He was a "consistent opponent of restrictive legislation, and did much, in " concert with others, to prevent the introduction in the various "Colonies of ill-advised life assurance laws: he believed that "freedom and publicity were better safeguards for the insuring "public than grandmotherly legislation, and looked upon the "English Act as the best piece of assurance legislation extant." In New South Wales equally emphatic language has been used on the subject by managers of life offices in that Colony. Mr. Teece, for instance, concludes his paper, referred to above, in the following words:-"As to the general principle, I think "there will be no difference of opinion as to the propriety of "following the English rather than the American model. "have seen how the latter, notwithstanding its apparently "theoretical perfection, has ignominiously failed in practice."

<sup>&</sup>lt;sup>1</sup> In his Annual Report for 1893, the Government Actuary at the Cape refers to this system, which he condemns chiefly on the ground that its costliness was not justified by the benefits it produced.

"And although I confess I have a strong disposition to favour "the establishment of some standard of valuation, I am free to "admit that I fear it would not be found practicable; and we " must remember that we are legislating for men as they are, and "not as they ought to be. I am, therefore, driven against my "inclination to favour the principle initiated in England, and "extended in these Colonies (especially in South Australia), of "compelling the companies to throw open their doors to the " scrutiny of the public gaze, and to leave the rest to the criticism "of a capable, a fearless, and an honest press." Another prominent life office manager in this Colony, Mr. J. C. Remington, remarks:-"State supervision, such as prevails in some parts of "the world, offers no such safeguards against mismanagement as " are ensured by such a full and explicit publication of a life " office's affairs as shall expose them to the searching light of "expert criticism. We have no grievance against laws which " (absurd as it may appear) class our association as a 'foreign " office' in other Colonies, and compel us to publish particulars "of our local assets and liabilities. The mother Colony of "Australasia apparently regards such precautions as unnecessary, "and so far, fortunately, the consequent disasters have been few "and insignificant in amount; but the experience of other "countries should warn our statesmen that their policy of " laissez faire lulls the public into a fancied security from which " nothing short of a catastrophe will awake it."

The above reference to the beneficial influence of the Press upon the business of life assurance naturally suggests a few observations upon this subject. In Canada, the leading insurance paper is The Bulletin, which was started at Toronto in December 1893, by the editor of *The Budget*, which, having been established in 1880, became merged into the former paper. Several financial journals, too, such as the Monetary Times, the Economist, and Money and Risks, which are all published at Toronto, devote considerable space to the discussion of matters connected with life assurance. It is, however, in Australasia that the Insurance Press exercises the highest influence upon companies, on account of the great independence which it exhibits in criticising their In these Colonies, the principal and most widelycirculated journal is undoubtedly The Australasian Insurance and Banking Record, which has been flourishing for 22 years, and can hold its own with any paper of a similar character either in this country or abroad. Other works of a more statistical character published in the Australasian Colonies, and which deal with the subject of life assurance, are Wealth and Progress of New South Wales and The Victorian Year Book. At the Cape there was issued last year for the first time an interesting publication called the South African Red Book, which gives a great deal of information as to insurance of all kinds, as well as a number of portraits of leading officials, and other illustrations. In addition to these publications, most of the companies in Canada and Australasia issue monthly or quarterly periodicals of their own. Among the best known of these may be mentioned, Life Echoes of the Canada Life, Sunshine of the Sun Life of Canada, Our Advocate of the Temperance and General of Canada, The Mutual Provident Messenger of the Australian Mutual Provident, The Mutual Life Chronicle of the Mutual Life of Australasia, and The Government Insurance Recorder issued by the New Zealand Insurance Department.

Of a character not less beneficial than that of the Insurance Press is the influence exercised upon Colonial life offices by the advance of actuarial science, in the promotion of which our own Institute has played no small part. The work of the Institute in this direction has been effected partly by means of its Mortality Tables and its Journal, and partly through the guidance given by its leading members in the endeavours of these offices to conduct their business on sound actuarial principles. As regards the direct influence of the Institute upon life assurance in Greater Britain, it is sufficient to refer to the high standards of valuation based upon its Experience, which have been universally adopted not only by the companies but also by the various Colonial Legislatures in the Aets passed for the regulation of

these companies. Following the practice, too, of the strongest offices in this country, those in the Colonies are year by year making their valuations on a more stringent basis, the HM(5) Table superseding the HM in the later years of assurance, and a lower rate of interest being adopted. indirect influence of the Institute has been strikingly illustrated in more than one way. A number of the Colonial offices have for some years past consulted its leading Fellows as to the methods and principles on which their valuations or other investigations should be conducted, and the reports of the actuaries thus consulted may be placed amongst the most valuable contributions to actuarial literature that we possess. On

the other hand, several members of the profession in the Colonies have added to our stock of knowledge by writing treatises and papers on subjects relating to the theory and practice of life assurance, and the names of Pell, Templeton, Teece, Carment, Macaulay, and others are scarcely less familiar to the members of the Institute of Actuaries than those of our own leaders.

Nor are these the only channels by which the influence of this Institute in placing Colonial Life Assurance upon a sound basis has been exercised. Two others may be added, namely, the formation of Insurance Institutes in the Colonies on similar lines to those of the parent body, and the establishment of centres where the examinations of that body can be held simultaneously with those in Great Britain, Australian Continent there exist three Insurance Institutes, those of New South Wales, Victoria, and South Australia, while in Canada the Actuarial Society of America belongs as much to the Dominion as to the United States. In a recent Presidential Address delivered before the first named of these Institutes, it was stated that the idea of an association of insurance men was thought of and informally discussed both in Victoria and New South Wales several years before any definite action was taken. The first of these Institutes to be formed was that of Victoria, which was opened on 20 August 1884,1 and followed at an interval of a few months by that of New South Wales, and five years later by the Institute of South Australia. The members of these Institutes represent various branches of insurance work, the avowed object of their existence being to promote good feeling and friendly intercourse as well as sound practice, and to disseminate useful professional knowledge by the reading of papers and other means. By far the most successful of these Institutes is that of New South Wales.2 the members of which have contributed to its proceedings a number of useful papers on practical and theoretical subjects connected with life assurance. This Institute, too, has occasionally stimulated the efforts of its younger members in this direction by

<sup>&</sup>lt;sup>1</sup> At the close of the 1897 session, the membership of this Institute, which had declined from 187 in 1894 to 138 in 1896, had risen to 150.

<sup>&</sup>lt;sup>2</sup> At the close of the last session, the number of members on the roll was 170, in addition to 13 honorary members. It seems, however, to be a source of complaint that the majority of the members exhibit but little interest in the proceedings of the Institute, either by attending or taking part in the discussions.

the offer of prizes for the best essays on subjects connected with the three principal branches of insurance. proceedings are duly published every month in a periodical called The Sydney Record. Two papers read before this Institute, as well as a Presidential Address delivered to the recently-formed Actuarial Society of New South Wales, have been considered worthy of being reprinted in the pages of the Journal of the Institute of Actuaries. The other two Institutes also publish their proceedings, which contain some interesting papers on the subject of Australian life assurance. It is eustomary in these Institutes for the paper to be read at one meeting and the discussion to take place at the next, by which time members have had ample opportunity of forming criticisms on the subject before them. The monthly gatherings are of a more social character than is the case in this country, and frequently close with music and songs of a sentimental and humorous nature.

The Actuarial Society of America was founded on 25 April 1889, mainly through the efforts of Mr. D. P. Fackler, of New York—a gentleman now included amongst our honorary members. Owing to the vast distances between the various centres of actuarial life, it is impossible to hold monthly meetings. The Society, therefore, assembles twice a year, in April and October, each session lasting two days. Its first meeting in Canada was held at Toronto in the fall of 1891. Accounts of the progress of this society, the membership of which contains the names of many of the leading British Actuaries, have already appeared in the Journal, from the pens of Mr. T. B. Macaulay and Mr. G. H. Ryan.<sup>2</sup>

The examinations of the Institute of Actuaries were first held in the various Colonial centres in April 1892. Their success may be best judged, not only from the satisfactory results that they have produced, but also from the highly appreciative manner with which their benefits have been recognized in the Colonies.

They have proved a great incentive to students in Canada and Australasia, to keep abreast of the times in the different branches of actuarial science, classes having been formed under capable tutors to assist them in their studies. At the examinations last year some 40 candidates presented themselves at the centres in the above Colonies, of

<sup>&</sup>lt;sup>1</sup> J.I.A., xxv, 250; xxx, 219; and xxxiv, 346.

<sup>&</sup>lt;sup>2</sup> J.I.A., xxvii, 544, and xxxii, 43.

whom 22 or more than 50 per-cent were successful. Up to the present time, eight gentlemen from these Colonies have thus attained the distinction of becoming F.I.A.'s by examination, four of whom reside in Canada, three in New South Wales, and one in New Zealand.¹ It may indeed be asserted that by this step the Council of the Institute of Actuaries have forged one more link in the chain which binds the members of our profession in these distant parts of our Empire to those in the mother-country.

I have now concluded this brief review of the work and development of the native offices in those parts of our Empire, to which Sir Charles Dilke and Sir John Seeley have given the name of "Greater Britain." The subject is one that should possess many features of interest for the student of actuarial science, yet, on looking through the volumes of the Journal of the Institute for the last 20 years, one cannot help being struck with the absence of any paper which has been read at our sessional meetings, bearing directly upon such a subject. This, I cannot but think, is scarcely as it should be, especially since it contains ample material, not only for one, but for several instructive papers. I therefore venture to hope that these notes, in helping to draw attention to our Colonial possessions, may also be the means of stimulating others to make investigations into such a wide field of enquiry as they present to our view.

The Anstrabasian Insurance and Banking Record for October last contains the following paragraph:—The letters 'F.I.A.', so well known in the insurance world, seem to have a fascination for other professions. Members of the various Colonial Institutes of Accountants appear to prefer to drop their final Colonial distinguishing letter, and to advertise themselves as 'F.I.A.' Similarly, an anctioneer and surveyor, with some detached villas to lette advertised himself as an 'F.I.A.' Apparently, the Institute of Actuaries' Charter does not protect its members outside the United Kingdom, but the misuse of its titles by non-members is in questionable taste."

## APPENDIX.

# List of Native Offices transacting Business in Canada, Australasia, and South Africa, in 1897.

## CANADIAN OFFICES.

(Extracted from the Animal Report of the Insurance Superintendent of the Dominion.)

| Name o        |       |     | Office |  | Date when<br>Established | Premium<br>Income Funds |            |
|---------------|-------|-----|--------|--|--------------------------|-------------------------|------------|
|               |       |     |        |  |                          | \$                      | \$         |
| Canada .      |       |     |        |  | 1847                     | 2,025,716               | 17,420,526 |
| Ontario Mutu  | al    |     |        |  | 1870                     | 599,339                 | 3,392,697  |
| Sun of Canad  | a     |     |        |  | 1871                     | 1,529,298               | 6,388,145  |
| Confederation |       |     |        |  | 1871                     | 870,615                 | 5,779,211  |
| London .      |       |     |        |  | 1874                     | 177,004                 | 591,591    |
| North Americ  | an    |     |        |  | 1881                     | 532,962                 | 2,515,833  |
| Federal .     |       |     |        |  | 1882                     | 312,399                 | 607,713    |
| Temperance a  | nd (  | ten | ral    |  | 1886                     | 151,319                 | 507,356    |
| Manufacturer  |       |     |        |  | 1887                     | 355,149                 | 1,200,952  |
| Dominion .    |       |     |        |  | 1889                     | 59,339                  | 245,691    |
| Great West .  |       | ٠   |        |  | 1892                     | 149,833                 | 312,003    |
| ľ             | 'otal | l   |        |  |                          | 6,762,973               | 38,961,718 |

#### AUSTRALASIAN OFFICES.

(Extracted from the Australasian Insurance and Banking Record, January 1898.)

| Name of Office                    | Date when<br>Established | Premium<br>Income | Funds      |
|-----------------------------------|--------------------------|-------------------|------------|
|                                   |                          | £                 | £          |
| NEW SOUTH WALES:                  |                          |                   |            |
| Australian Mutual Provident .     | 1849                     | $1,\!298,\!246$   | 13,728,540 |
| Mutual of Australasia             | 1869                     | 141,501           | 1,135,760  |
| City Mutual                       | 1879                     | 31,359            | 147,556    |
| Citizens' (Ordinary Business) .   | 1887                     | 50,913            | 110,511    |
| VICTORIA:                         |                          |                   |            |
| Vietoria                          | 1859                     | 5,457             | 230,592    |
| Australian Alliance               | 1862                     | 19,729            | 284,786    |
| *National Mutual of Australasia   | 1869                     | 409,525           | 3,604,780  |
| Australian Widows' Fund .         | 1871                     | 149,652           | 1,247,188  |
| Colonial Mutual                   | 1874                     | 307,698           | 2,040,972  |
| Australian Temperance and General | 1876                     | 31,239            | 198,708    |
| SOUTH AUSTRALIA:                  |                          |                   |            |
| Adelaide                          | 1866                     | 928               | 38,143     |
| NEW ZEALAND:                      |                          |                   |            |
| Government Department .           | 1869                     | 250,776           | 2,591,342  |
| 1                                 |                          |                   |            |
| Total                             |                          | 2,700,023         | 25,358,878 |

<sup>\*</sup> The amounts set against this office include those of the Mutual Assurance Society of Victoria, the business of which it took over at the close of 1896.

## SOUTH AFRICAN OFFICES.

| Name of Office                     |  | Date when<br>Established | Premium<br>Income | Funds                    |
|------------------------------------|--|--------------------------|-------------------|--------------------------|
| South African Mutual .<br>Southern |  | 1845<br>1891             | £ 217,285 47,286  | £<br>1,947,701<br>55,770 |
| Total .                            |  |                          | 264,571           | 2,003,471                |

#### Discussion.

The President (Mr. H. W. Manly) said the author referred to the high rate of lapse which occurred amongst the new policies in the colonies, and attributed it to the facility with which loans were granted on them. That seemed to him at first to be somewhat paradoxical, because in this country loans were used very often to keep policies in force; but he thought the explanation seemed to be given further on in the paper, where the author stated that the system of applying the surrender values to keeping the policies in force—the amount of premiums and interest at 10 per-cent. being charged as a loan-was productive of great laxity on the part of the assured, who, knowing that their policies would be kept in force by the office, were apt to take very little trouble in the matter. That did not, however, account for the very high lapse rate in the years 0 and 1. In Canada and in Australia it amounted to 17 and 25 per-cent respectively. He should imagine that it arose rather from the difficulty of up-country squatters being able to send their premiums to the head office. The original assurance seemed to have been made easy for them by expeditions setting out with the agent and the doctor, but the means of forwarding their premiums when they became due were not made quite so easy, it not being so much to the interest of the agent to get the renewal premiums as to get the new assurances. There was one other subject, a rather vexed subject, on which, perhaps, he might make a few remarks-the subject of temperance sections. The author had given the explanation of the New Zealand Government Department with regard to that. He (the President) had been engaged as consulting actuary in the valuation of a company which transacted a large amount of that class of business, which they kept as a separate section, and the profits were distributed with reference to the differences in mortality in the two sections; but the result all depended on how those sections were In the two principal companies in England, the members of the temperance section had to make declarations periodically, in one every time that the premium was paid, and in the other every five years, that they continued total abstainers. If the policyholder made that declaration he remained in the section, but if he did not

sign the declaration he was transferred to the general section. In that way the general section was made the dumping-ground for all the lives that did not keep up to the standard, the consequence being that that section had more than its share of the inferior lives. He could very well imagine that if the section was kept entirely to those who entered as temperance lives, and no means whatever were taken to ascertain whether they continued in their good ways, that the mortality in the two sections would be very similar.

Mr. James Chisholm thought they could not fail to be struck by the great success which had attended the development of life assurance in these young and strong communities over the sea. colonies were still in their youth; he might have said in their infancy, as years were measured in the life of a nation—the great expansion in the colonies had certainly taken place within the last fifty years, and during the whole of that time it had been accompanied by the growth and development of native colonial offices, which were now amongst the most active and most successful that were to be found anywhere. The oldest offices mentioned in the paper, whether in Canada, in Australia, or at the Cape, went back in date over fifty years. It seemed to him that the colonies were very much in the position of a young man commencing business. He would have to go to his bankers or to his friends and get some assistance in the way of capital to start with. He was not thought any the worse of for that. To a certain extent he was mortgaging his future, and depending on what his own industry would bring forth, but he was thought all the more of if it was known that he was saving at the same time, and was able to produce a life policy which he took out at a young age and consistently kept up. It seemed to him greatly to the credit of a young community, and it was also fraught with the best promise for the future, that the principle of thrift and prudence should have accompanied the growth of our colonies almost from their earliest beginnings. He thought that it would be interesting to compare the amounts of insurance in force with the Government Debt both at home and in the colonies. The one was, to a certain extent, a set-off against the other. The community was bound to be the richer eventually by the falling-in of all the policies that were effected, or rather of those among them that were kept in force to the end, which was not quite, unfortunately, the same thing, for whatever enriched the individual or brought back the product of saving would reflect itself in the prosperity of the community. And in making that comparison it must be borne in mind that the colonies had solid assets in the shape of railways, harbours, &c., against a very large part of their debts, whereas at home we had nothing but the memory of a great struggle, which secured freedom for expansion and development, but had left practically nothing in the shape of public improvements. The National Debt of Great Britain amounted to about £650,000,000, and the total estimated amount of insurance in force was about £550,000,000, or 85 per-cent of the debt. He had left out the amount of the industrial insurances. If included, they would amount to £150,000,000 additional, and bring the total insurances in force up to £700,000,000. But perhaps it was better to exclude them, because there was no doubt that, in Great Britain,

life assurance was conducted in a cosmopolitan manner, and the total insurances in force were not gathered from the British islands alone. Turning to the colonies, the author in his paper said, that in Canada the total life assurances were, on the average, £12 per head of the population, in Australasia £21 per head, and in South Africa £25 per head. He had made a table showing the comparative figures worked out on that basis. In Canada there was a population of 5.250.000, a total debt of £53.000.000, and total insurances of £67,000,000—that was 126.4 per-cent of assurances in proportion to the debt. In Australia, in a population of 4.350,000, there was a debt of £223.000.000, which sounded very alarming compared with the Canadian debt, and the total assurances amounted to £92,000,000: that was a proportion of 41 per-cent. In South Africa there was a population of 424,000 whites in Cape Colony and Natal, a very small number in comparison with the great area of the country, but it indicated perhaps the extent to which South Africa might grow in the future. The total debt was £35,000,000, and the assurances in force £10,600,000, a proportion of 30 per-cent. The proportion of assurances in force was small comparatively in Australia and South Africa. In Canada it was actually more than the total debt. But in considering those figures it was necessary to consider what had been done with these debts, and the extent to which the various colonies respectively had been developed, but the figures were nevertheless interesting as showing the degree in which saving and accumulation had gone hand in hand with expenditure mainly for development of the country. India was also a country in which a satisfactory beginning had been made since the founding of the company mentioned in the paper, and he thought the founder of that company. Mr. McLauchlan Slater, deserved the greatest credit for accomplishing a very difficult work. He had succeeded in introducing life assurance amongst the native population, and undoubtedly that was a very valuable thing to do. He had not left himself much time to speak of the other matters mentioned in the paper, but he might very shortly give his impression of South Africa, which he had the pleasure of visiting a few years ago. The population of South Africa might be divided into three parts. First, there was the Boer populationthe earliest inhabitants after the Aborigines; then there were the immigrants, of whom there were two classes, the ne'er-do-wells, whom their friends and relations were glad to see the last of, and there were the teetotallers and temperance people. There was not only a large number of teetotallers in South Africa, but the South African Governments generally were very earnest in promoting temperance as far as this could be done by legislation. The Boers in particular were a very temperate race. The Boer farmer was an industrious man, who turned valleys where a furrow had never been driven into fertile and cultivated land. They were a simple, thrifty, hardworking and industrious people, doing an immense amount of work in developing the soil. The importance of Johannesburg was that it brought a market to the farmers. It was in the towns where the immigrants mostly collected. In 1820, settlers from Europe peopled the eastern province, but the immigrants now congregated in the towns, and it was necessary to discriminate very closely among the

town populations when the question of life assurance came to be considered. One thing struck him very much in going through South Africa, and that was the hardy, robust, healthy aspect of the people. It was a young country, and the people in it were young and vigorous, ready to undertake any amount of work. One curious feature about them seemed to be that they all looked older than they really were. The reason for that, probably, was that there was a greater strain in the rough open-air work they had to do and the earlier age at which they began life, but on the other hand they had a very fine climate to work in, and that was to a great extent a compensation.

Mr. C. D. HIGHAM said it was pleasant to see that the "empiretendency" of the time was spreading into the profession, and perhaps it might be claimed that the Institute had had something to do with the gathering together into one brotherhood those who were pursuing the same avocations in different parts of the world. Certainly the Institute had had its influence, as those members had found who had gone across the seas, because not only did they see the respect in which it was held, but it ensured them a warm welcome which they never could forget. He himself knew very little of Greater Britain, but he had spent a week or two in America, and although the paper was about Greater Britain and would not include the United States, yet that country had so much influence over Canada that one could not help feeling the difficulty of separating the two, especially in life assurance matters. Across the water everything was big; their mighty rivers and vast inland seas all seemed to make bigness a principal idea, and therefore the bigness of insurance companies was thought too much of, because there were smaller societies—some of them with odd names—doing a very useful work. One noticed over there how much greater interest people took in life assurance than in this country. Everybody knew something about it there, and something of the different plans of the different offices, and there was generally a greater activity in such matters. But it was not only that, and the enterprise of the agent, or solicitor, as he was called, which accounted for the pushing of business, but partly it would seem to be from the self-reliant habits of the people themselves. A man there did not seem to leave it to a father to provide for him, so far as he could see. He was prepared to make his own fortune and provide for his own family. That led, as he fancied, to two different classes: those who were going to "make their pile," as it was called, and who wanted life assurance simply as a temporary protection, and these took a whole-life assurance, and let it go in the most casual way when it had served its purpose, or they got into the assessment companies. And there was the other class, which looked to life assurance not only for the life assurance part, but for the saving too, and they took endowment assurances, but with all their friends dealing in stocks and commodities they wanted an element of speculation in their policies, and found it in tontines. confusing to a stranger that there were practically no prospectuses, in the English sense of the word—but it was the same as to railway guides, though they had what they called "fold-overs." They

published little books about the different plans, as the railways did little pamphlets about the different lines. There was such a hurry that no one had time to read a prospectus. A man only wanted to know the pith of it, and trusted a good deal to the personal explanation of the agent and his own information. He thought the lesson might be learned that prospectuses in this country were sometimes too long. He did not think anyone in America would collect a series of prospectuses of different companies and really study them and decide for himself which office he would take up, as people did in this country. He noticed a desire that nobody but a registered agent should attempt to secure life assurance business. That might suit over there, but it would be a strange thing here, especially with regard to certain offices—one in particular, which got nearly half its business from the unpaid services of members and friends. A few words in the paper seemed to him to want further explanation, for it was said that a certain company had only three policies from Rhodesia, and that they restricted business to the principal towns, and then only at a substantial extra premium. That did not seem to him very clear, and, mentioning another trifle, Mr. Tarn would agree that "universal" could not be qualified by less or more. And when Mr. Tarn spoke of the large number of offices in Ontario, as a result of the large towns in that province, he forgot that Montreal, the biggest commercial town in Canada, was in the province of Quebec. The difference was not so much, he thought, from the presence of the towns as the race differences of the English and French—the French having their habits of thrift in different modes. The author did not say whether there was any general Dominion law as to assignments. In the States, each State had its own law, and the assignment depended on the law of the State in which it was executed, not on the law of the State of the issue of the policy, or of the company's head office. If that was the same in Canada it would be very interesting to know it. With regard to India he could say very little, but he remembered when the author spoke of the difficulty of identifying natives, that General Hannyngton had told them that many natives had a document which they could produce if they were made to do so, which would identify them and give their date of birth. Another thing interesting to know would be whether the policies were still assignable at law in India, and if the assignment were still free of duty, provided it was by endorsement on the policy. He thought figures as to assurance per head of population had to be taken cum Even the total sum assured in this country was only an estimate, and it was not known how much was on assured lives here and how much in other parts of the world. The English especially would have a larger proportion abroad than the colonies. The President had said something about Mr. Tarn making out that loan spelt lapse, so he would merely remind the members that in the first volume of the Journal there was a paper arguing on the other view, that loans prevented lapse, and that through the fact that bad lives were kept on the books, and had their influence in the matter of selection. As to laxity caused by non-forfeiture rules, he agreed with the author. The public wanted these conditions, and the public had to have them,

but he questioned if the public did not get as much harm as good from them, because it debarred them from a good deal of freedom. He did not think in former days there were many hard cases of lapse, as the companies took sufficient care of their own assured. But when one came to the advantage of the company, if it could get 8 per-cent or 10 per-cent for its loans, he thought there might be something to be said on that side. A man by non-forfeiture rules lost the stimulus of having to pay his premium, and those who granted extensions of time knew how often an assured took his full extension year after year, so that he paid every premium one year late, and was no better off at the end than at the beginning. The contingent debt system was stated to be diminishing, except in India, and he was glad of it. It always seemed to him the principle was wrong unless a much heavier extra was charged, for it meant that a man who was not a good life at the beginning and recovered was not to pay, and the charge ought, therefore, to be heaped on those who succumbed before reaching the ordinary "expectation." protection from creditors, perhaps the author could say whether that extended abroad to all forms of policies, because there was a distinct difference between the man who wanted to protect himself from the chance of death only and the one who wanted to do his saving, too, by endowment assurance. There was the question in this country whether endowment assurances might come under the Married Women's Property Act, and perhaps the author could say what the law was abroad. The President had spoken of what was called temperance, and the influence of selection in reducing teetotaller mortality, so he would only say he wished a proper mortality table might be published of these lives. All sorts of things were said, but there was not an authentic table of the actual mortality. They did not want statements that the claims were less than the expectation. In conclusion, he hoped Mr. Tarn would have his reward for his labours in having caused a useful discussion.

Mr. Tatlock (Assistant Actuary of the Mutual Life Assurance Company of New York) thanked the President for the opportunity given to him of expressing his pleasure at being present at a meeting of the Institute of Actuaries, a pleasure which he had frequently anticipated. It was a ease, he was happy to say, in which the pleasure of anticipation had not been greater than that of realization. Like all the members, he had been greatly interested in the paper, and it had given him a home-like feeling to see that the author had not neglected to touch upon those two subjects, which, in America, always furnished so much food for thought and reflection—namely, those of rebate and State supervision. It was interesting also to notice how the Colonial Offices had been foremost—perhaps not at the very front, but certainly in the advance guard—of those companies which had been anxious to vie with each other in offering so-called advantages to the insured. He thought that the paper also deserved special mention in the particular importance the author had attached to colonial legislation with regard to titles of different offices, especially the laws regarding assignments, and the rights of married women, and so forth. He was also very glad to hear the remarks of Mr. Higham. His criticisms upon American manners were certainly

well-timed and well taken in many respects. It gave him pleasure to corroborate what Mr. Higham had said in regard to the excellence of those American offices which were sometimes designated as "the smaller offices." It was true that, perhaps, in America bigness was sometimes taken as a synonym for goodness, but among thinking people he did not think smallness was taken as a synonym for badness. He only hoped that next time Mr. Higham went to American he might have the opportunity of educating him on the subject of American time tables; for he might say he could show him a time table which was as useful in its way as the London "A B C", and another which was as embarrassing and misleading as "Bradshaw." He again thanked the President and members for the opportunity afforded him of being present with them that evening, and as an official of the sister society, the Actuarial Society of America, he begged leave to offer to them the sincere greetings of their brethren across the sea.

On the motion of the President, a hearty vote of thanks was

accorded to Mr. Tarn for his paper.

Mr. A. W. TARY, in reply, thanked the members for their vote. He did not think there was very much to reply to; but Mr. Higham had referred to a quotation he had made from the speech of the chairman of one of the South African offices. He had simply given it as it stood, and could neither add to it nor explain it in any way. He was unable to give any particulars as to the methods of assignment of policies in India, and he might add that he had had very great difficulty in getting information about India, except from Mr. Slater's pamphlet, to which he had referred in his paper. Mr. Higham had also referred to the contingent debt system as "going." He himself thought it always bore a prominent part in the prospectuses of Australian offices, but he had no statistics as to its success or otherwise. With regard to the protection of policies from creditors, as far as his recollection went, he believed that no distinction between different classes of policies had been made in the various Acts to which he referred. Mr. Teece, however, in his paper on "Life Assurance Legislation in Australasia," read before the Congress, had dealt with the subject more fully. He quoted the following paragraph from the prospectus of a New South Wales office: - The interest of every member is protected by law against insolveney to the following extent, namely, to £200 after an endurance of two years, to £500 after an endurance of five years, to £1.000 after an endurance of seven years, to £2.000 after an endurance of ten years. This thoughtful provision by the Legislature has been of incalculable service during recent years of trial."

Some Notes on Sinking Fund Assurances. By J. E. Faulks, B.A., F.I.A., F.S.S., Assistant Actuary of the Law Life Assurance Society.

[Read before the Institute, 27 March 1899.]

I DO not think that any apology is necessary for asking the members of the Institute of Aetuaries to devote a meeting to a consideration of the subject of Sinking Fund Assurances. My paper—if paper it can be legitimately called—contains, I am afraid, little, if anything, that will be new to those actuaries who have had to consider the subject, and doubtless in itself demands a very ample apology for the temerity of its author in submitting it to the criticism of the Institute. I am not, however, without hope that the ensuing discussion may bring out some points of practical interest, and if this should be so, and more particularly if fuller consideration of the subject should lead to an arrangement on the lines indicated in the concluding paragraphs of my remarks, I think I may fairly ask that these results may be considered as to some extent a mitigation of my offence.

Sinking Fund or Fixed Term or Leasehold Redemption Assurances—assurances, that is, securing a capital sum at the expiration of a term of years independently of any life-appear to be of comparatively recent growth. They were, I believe, practically unknown 15 years ago, and even after some offices commenced to publish tables of premiums for such policies the policies effected thereunder were, so far as I have been able to gather, for some years only for comparatively small amounts policies, for instance, effected in order to provide for the return of the capital invested in the purchase of leasehold property or other forms of terminable annuities. More recently, however, the praetice of taking out such policies has been extended to the case of limited companies issuing terminable debentures or debenture stock, and desiring to afford to debenture holders some security (necessarily dependent on payment of the premium) that the redemption money of the debentures will be forthcoming at the expiration of the term of years. These transactions are naturally of eonsiderable magnitude, and it has recently been no infrequent occurrence to read in the prospectuses of such companies of undertakings to effect with one or more offices sinking fund policies, for such purposes as have been referred to, to the amount

of hundreds of thousands of pounds. Transactions of this character appear to come under rather a different category to the comparatively small policies effected in connection with the purchase, for instance, of a leasehold house by a private investor for the sum of £1,000 or £2,000, and furnish very strong reasons why the whole question of Sinking Fund Assurances should be carefully considered—reasons which, although of course always present, were not nearly so weighty while the amounts of the policies effected continued to be comparatively small. I propose, therefore, to refer, as briefly as possible, to some points in connection with such policies.

## Premiums.

The first and most important point to be considered is what premiums should be charged for the benefits granted. being involved in the policy, the premiums to be charged are simply such sums as will, after making due allowance for expenses, accumulate to the sum assured by the operation of compound interest during the currency of the policy, i.e., the net single premium for an assurance of 1 is  $v^n$ , and the net annual premium, payable of course in advance, is  $\frac{1}{s_{n+1}-1}$ . There are, then, two points to be considered in framing a table of office premiums for such policies: (i) the rate of interest to be assumed, (ii) the provision to be made for expenses. To deal first with the latter, as the less important of the two, the expenses to be considered (apart from expenses in connection with the investment of the premiums and the interest thereon, which may be more conveniently considered in connection with the rate of interest to be assumed) are (1) commission, (2) policy stamp, (3) printing, postage, stationery, &c. If the stamp duty on these policies, to which I refer later, is only the ordinary agreement stamp, Nos. (2) and (3) are necessarily small in amount. As regards commission this is, in practice, having regard to the rates of premium charged, necessarily small, and generally takes the form of a commuted allowance in respect of the first year's premium. premium policies, being comparatively rare, are not further referred to.) This allowance may be, for instance, 15 per-cent or 20 per-cent of the first year's premium, modified, no doubt, for very short terms. On the whole it appears that the policy expenses will not be large in amount, and, provided of course that the commission allowance is kept low, it may fairly be considered not inadvisable to charge net rates, leaving the expenses to be provided for out of the interest margin.

When, however, we come to consider the second of our bases for the quotation of premiums, namely, the rate of interest to be assumed, we have a problem of infinitely greater difficulty. In the first place, two methods of calculation obviously present themselves, namely, to assume one rate—a low rate—throughout, or to assume a rate of interest approximating at the outset more nearly to the rate now ruling, but decreasing afterwards at periodical intervals. To accurately forecast the future of the rate of interest is, of course, impossible, and any discussion of the general aspect of the matter would necessarily run to considerable length. The following points, however, may, I think, usefully be borne in mind in considering the future rate of interest for the special purpose of these policies:

- 1. An office is manifestly not justified in framing a table of premiums based throughout upon a rate of interest which there is every reason to believe may be realized in the near future, but as to the realization of which there may be some doubt in connection with the longer terms quoted for in the table-unless, indeed, the rate initially assumed is lower than that which is expected to be realized after all deductions, and the excess accumulations are kept intact in the case of the longer term policies (see post s. r. Reserves). In connection with this point, it is obvious that there is a strong selection against offices in connection with sinking fund policies; that is to say, that, ceteris paribus, an office quoting an unduly low rate for a certain term may, and probably will, receive a great deal more business for that particular term than for others for which its rates are not so favourable, and, it may be added, that the persons responsible for effecting the larger policies already referred to are no doubt fully competent to take advantage of any such anomalies.
- 2. We have not to consider the rate of interest which an office may now be making over its whole fund, but the rate it is making in respect of its new investments—a rate which it is admitted is, in most cases, lower than the rate earned by the funds as a whole. These policies are new policies, and the premiums on them must be

- invested year by year. I shall refer to this point again later on (s. v. Security).
- 3. It must not be overlooked that the formula for the net premium assumes that each premium is invested annually, on the due date, so that the full benefit of a compound interest accumulation is secured. In practice this is not, of course, the case. The office is practically compelled to give some (even if few) days of grace, and when the premium has actually been received some delay must necessarily occur in investing it, even where the amount is, as it were, worth investing. For it cannot be doubted that the difficulty of the private investor as to investing promptly small sums of money (to which some offices commiscratingly refer) is one which may be experienced by the offices themselves, if they consider their sinking fund policies strictly apart from their other business.
- 4. The nature of the securities suitable for the investment of the premiums from time to time is worth consideration, if it be assumed for the moment as desirable that such investments should be ear-marked. Having regard to the possibility of surrender, it would be essential that such securities should be easily convertible, and many investments, such as reversions, yielding a somewhat higher rate of interest than the average and eminently suitable in connection with life policies, might therefore, strictly speaking, be considered as ineligible.
- 5. The expenses in connection with investments and, in particular, income tax, should not be overlooked.
- 6. There should be such a margin between the rate assumed and that which, after allowing for all the above facts, may be expected to be realized, as will provide for (i) (if net premiums are charged) the necessary policy expenses, (ii) in any case, the profit for the issuing office.
- 7. It has occurred to me that while in connection with these policies any fall in the rate of interest must necessarily prejudicially affect the interests of the issuing office, who would most certainly be held to their bargain, it is at least doubtful whether they would, as regards a policy already issued, receive any corresponding benefit in the event of the rate, owing to war or other causes, undergoing a marked and, so far as could be predicted,

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permanent rise—where by permanent rise I mean a rise likely to last for the whole, or nearly the whole, of the remaining currency of such a policy. The assured, especially if a company with expert financial advisers, might draw the surrender-value of the policy (thus depriving the issuing office of the opportunity of reinvesting it, or, rather, the reserve-value, at the higher rate of interest presumed to be ruling) and themselves invest that surrender-value, with a further sum annually equivalent to the annual premium under the surrendered policy, at the higher rate, and so secure an advantage to themselves over and above the face-value of the policy at the expiration of the original term.

Bearing the above points in mind, and also that we have to quote for terms up to, say 99 years—far longer, that is, than it is necessary to consider in fixing a life assurance premium—it is obvious that a very cautious estimate of the rate of interest should be made.

At this juncture, the following table of rates of annual premium for such policies, calculated on various bases, may prove of interest:

| Term     | 3 per-cent Net Net throughout throughout                |  | decade to<br>2 per-cent                                 | 3 per-cent for<br>10 years,<br>decreasing<br>\$ per-cent each<br>decade to<br>2 per-cent | 3 per-cent for<br>20 years,<br>decreasing<br>½ per-cent each<br>period of<br>20 years to<br>2 per-cent |  |
|----------|---|--|---|--|--|--|
| (1)      | 2)  | (3)  | (4)   | (5)  | (6)  |  |
| Years    |   | Annua  | al Premiums pe  | er-cent  |  |  |
| 10       | € s. d.<br>8 9 5  | £ s. d.<br>8 14 2  | £ s. d.<br>8 9 5  | £ s. d. 8 9 5  | £ s. d. 8 9 5  |  |
| 10<br>20 | $\begin{bmatrix} 8 & 9 & 5 \\ 3 & 12 & 3 \end{bmatrix}$ | $\begin{bmatrix} 8 & 14 & 2 \\ 3 & 16 & 5 \end{bmatrix}$ | 3 13 9  | 3 13 0   | $\begin{bmatrix} 8 & 9 & 5 \\ 3 & 12 & 3 \end{bmatrix}$  |  |
| 30       | 2 0 10  | $\begin{array}{cccccccccccccccccccccccccccccccccccc$     | $\begin{bmatrix} 3 & 13 & 3 \\ 2 & 3 & 3 \end{bmatrix}$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$                                    | 2 2 8  |  |
| 40       | 1 5 9   | 1 8 11   | 1 9 0   | 1 7 4  | 1 8 0  |  |
| 50       | 0 17 3  | 1 0 0  | 1 1 0   | 0 19 0   | 1 0 4  |  |
| 60       | 0 11 11   | 0 14 4   | 0 15 8  | 0 13 10  | 0 15 3   |  |
| 70       | 0 8 5   | 0 10 6   | 0 12 0  | 0 10 5   | 0 11 8   |  |
| 80<br>90 | $\begin{array}{cccccccccccccccccccccccccccccccccccc$    | $\begin{bmatrix} 0 & 7 & 10 \\ 0 & 5 & 11 \end{bmatrix}$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $\begin{bmatrix} 0 & 8 & 1 \\ 0 & 6 & 5 \end{bmatrix}$                                   | $\begin{bmatrix} 0 & 9 & 1 \\ 0 & 7 & 2 \end{bmatrix}$   |  |
|          | 0 4 9   | 0 5 11   |   |  |  |  |

Even a cursory study of these figures cannot fail to illustrate some of the points already referred to. For instance, if the rates given in Col. 4 be taken as a standard, obviously an office which quotes net  $2\frac{1}{2}$  per-cent rates as under Col. 3 is quoting, for terms

over 50 years, rates which are too low, and may give rise to considerable loss in the future, unless additional interest accumulations, over and above the  $2\frac{1}{2}$  per-cent assumed, are reserved during the earlier years of the policy.

It is, to some extent, a matter of individual opinion which method of calculation is the proper one to adopt. My own personal view is, that rates based upon a decreasing rate of interest are the more logical and are preferable to those based upon a fixed rate, however low.

#### SECURITY.

According to the provisions of the Life Assurance Companies Act of 1870, and its amending Acts, a company which concurrently with the granting of "policies of assurance or annuities on human life" transacts any other kind of assurance or other business is compelled under penalties to frame its annual returns under the third and fourth schedules of the Act, implying separate statements of accounts for the life business and the other business referred to; and, further, the life assurance fund is, as regards contracts entered into after the passing of the Act. exempted from liability for obligations other than to its life policyholders and annuitants. As the policies we are considering are not policies of assurance upon human life, it manifestly therefore becomes necessary to furnish a separate revenue account in respect of such policies. The provisions of the Act making the life assurance fund security for the life policyholders alone are specially important, although perhaps their effect is not fully apprehended from the point of view of the assured; more particularly where the issuing office is a purely life company. If the company be a proprietary one, the shareholders' capital including uncalled capital, as it does not form part of the life assurance fund, can ne doubt be charged in respect of sinking fund policies, but in the case of a purely mutual life office it would appear that the sole security for such policies is, speaking generally, the fund formed by the accumulation of the premiums upon the policies themselves; it should, however, be noted that possibly any deficiency in the sinking fund account could in some cases be made good out of the profits of the life assurance fund, although not out of the corpus of that fund. In any case, however, it appears to me that the effect of this provision is to give an additional reason, if any be needed, for estimating the premiums on sinking fund policies on the most cautious basis

practically possible—particularly in the case of a mutual office. Separate investments it would appear need not necessarily be made in respect of the sinking fund business; and on this ground it may be a sufficient compliance with statutory requirements to transfer annually to the sinking fund account a proportion, estimated, say, on the mean fund, of the total interest earned by the office. It is to my mind, however, open to question whether this course is really equitable to the life policyholders, as there would seem to be good reason, on the grounds already referred to, to believe that the rate of interest yielded by the Sinking Fund Assurance Fund, if it were constituted by means of separate investments specially ear-marked to that fund, should be lower than that yielded by the life assurance fund. Possibly, however, the charge on the profits of the life assurance fund already mentioned may be considered as authorising the course in question. In any case, I need not point out, it is necessary to carefully consider the provisions of the deed of settlement or other instruments constituting a particular company as their provisions, although they will not, of course, override those of general Acts, may impose additional restrictions on the company.

## RESERVES.

Sinking Fund Assurances are, of course, not to be undertaken without the prospect of their yielding at any rate some small profit to the issuing office, and it will therefore be necessary to make a periodical valuation of such assurances in order to ascertain what profit may legitimately be carried over to the proper quarter. The value of a sinking fund policy is manifestly the premiums received (less any loading for expenses) accumulated at interest, and if the rate of interest assumed in the calculation of the premiums be employed also in the calculation of the reserves, we should, of course, have a policy-value constantly increasing until it reaches its limit, namely, the sum assured. If, however, a uniform rate of  $2\frac{1}{2}$  per-cent or 3 per-cent has been assumed in the calculation of the premiums, this rate is not necessarily a sufficiently high one to adopt throughout in the computation of the reserves. For in adopting a 3 per-cent rate for a term of 80 years, the idea is doubtless that while the interest realized may and probably will fall below 3 per-cent before the term has expired, yet more than 3 per-cent will be realized during the earlier years, so that, to speak somewhat

unscientifically, the average rate will not be less than 3 per-cent. And, of course, matters may turn out in accordance with this view; but if the reserves during the earlier years have been formed by accumulating the premiums at 3 per-cent as against the  $3\frac{1}{2}$  per-cent or  $3\frac{3}{4}$  per-cent earned, and the resulting profit has been divided, the experience of the later years will prove a constant drain on the resources of the office. The premiums, therefore, should, at any rate as a counsel of perfection, be accumulated for reserve purposes at a rate only slightly less than the rate actually realized during the valuation period, this slight margin being allowed to provide for the expenses in connection with the policies and possibly some small profit to be treated as immediately accruing. This course will not in any way diminish the total profit to be eventually realized by the office, but will in fact increase it, owing to the reserves being higher and consequently earning more interest. The division of the profit is merely deferred until the policy matures, although it may be said that in the case of policies of such a term as 80 years the division of that profit may safely be left to successors of the present generation of actuaries.

The following figures, which explain themselves, will serve as an illustration of the point in the case of a 50-year policy at £10 annual premium assuring, at  $2\frac{1}{2}$  per-cent net rates, £1,000, taking the rate of interest actually earned by the office at  $3\frac{1}{4}$  per-cent (after deduction of all expenses) for 10 years, decreasing thereafter by  $\frac{1}{4}$  per-cent per annum each period of 10 years.

| Years                      | Reserves on basis of<br>Premiums accumulated at<br>2½ per-cent | Actual sum in land at end of decade, being Reserve at beginning thereof plus Premiums paid accumulated at Rate assumed to be earned. | Profit divided if Reserves<br>be as in Column 11 | Accumulation of actual<br>Premiums paid at rate<br>of Interest assumed to<br>be carned | Profit divided if Reserves<br>he as in Column V |
|----------------------------|--|--|--|--|---|
| 10                         | 115  | 120  | 5  | 120  |   |
| 10<br>20<br>30<br>40<br>50 | $\frac{262}{450}$  | 120<br>273<br>460  | 11   | 120<br>279<br>482<br>732<br>1,028  |   |
| 30                         | 450  | 460  | 10   | 482  | ***   |
| 10                         | 691  | 691  |  | 732  |   |
| =0                         | 1,000  | 976  | -24  | 1.028  | 28  |

Here it will be seen that as an annual premium of £10 is sufficient at the assumed decreasing rate of interest to assure a

policy of £1,028, the  $2\frac{1}{2}$  per-cent nct rate will result as a whole in a profit to the issuing office, but if the reserves be made at 21 per-cent only, while a profit is shown at the end of each of the first three decades, if this profit be divided a loss will have to be met during the fifth decade, at the end of which the sum assured is payable. If, however, the reserves are formed by accumulating the premiums at the assumed decreasing rate, while the profit to be divided at the end of each of the first four decades is supplied only by any margin between the rate of interest actually earned and the assumed decreasing rate, yet, at the end of the fifth decade, when the policy matures, the company will be able to take credit for a larger amount of profit; or, in other words, the effect of reserving the premiums accumulated at a rate approximating to that earned, and higher than the "average rate" assumed in the calculation of the premiums, will be to keep in hand an extra sum available to counteract, at least in part, any fall in the rate of interest carned greater than that assumed to be probable, while if such fall do not in fact occur, this margin will appear as profit on the policy maturing.

### SURRENDER-VALUES.

The surrender-values to be allowed in respect of Sinking Fund Policies, as no risk of death has been incurred during the currency of the policy, will be high, and may not unfairly be fixed at the premiums paid, less expenses, accumulated at a rate of interest somewhat less than the net rate realized. It is generally necessary to fix a scale of surrender-values in advance and guarantee it, and a rule frequently adopted, which would appear to be on the whole equitable, is to allow the premiums paid, less the first, accumulated at a rate of interest not exceeding 2 percent. It would not seem wise to offer surrender-values approaching very nearly to the actual amount of the accumulated premiums, that is, to allow, in the accumulation of the premiums for the purposes of surrender-value, interest at the same rate or rates as has been assumed in the calculation of the premiums themselves. This would enable the assured to surrender the policy with so small a loss to himself that it might be worth his while to do so, in the event of a rise in the general rate of interest occurring, which was substantial, and, so far as could be seen, likely to be maintained for a considerable period. This is point to which I have already referred.

## FORM OF POLICY.

The form of a sinking fund policy presents no very special feature, but it may be well to bear in mind the following points:

- 1. It is probably safer and better to make it clear in the body of the policy that the claim for the sum assured applies only to the funds of the issuing office other than its life assurance fund.
- 2. It is desirable to provide that, under no circumstances shall the claim or any surrender-value carry interest as against the issuing office, on account of any delay in claiming it, or in proving the claimant's title.
- 3. It is questionable whether the 30 days' grace ordinarily allowed in respect of life assurance premiums should be extended to the premiums under these policies, and whether it is not rather better to allow only seven, or at most 14 days' grace.
- 4. It is eminently desirable to provide against the possibility of the issuing office being called upon to pay a surrender-value at a day or two's notice. This statement applies, of course, specially to the case of the very large policies already referred to, no portion of which is generally (or at least frequently) re-assured by the issuing office. The office, as there is no chance of an early claim, is, from one point of view, justified in retaining any amount under such a policy, assuming that its rates are sufficient to provide against any humanly possible fall in the rate of interest, and as the date of payment of the claim is known beforehand it can easily make the necessary practical arrangements to have the sum assured ready on that day. But the case is different as regards a surrender, the date of which is unknown, and it would cause considerable inconvenience to an office to be required to provide a large sum of eash at a few days' notice. It is suggested, therefore, that notice (say three months) should be required of any intention to claim a surrender-value of the policy, and the same provision would, of course, apply if there is any undertaking to grant a loan on the policy to an extent covered by its surrender-value.

## STAMPING OF POLICIES.

It would appear that Sinking Fund Policies at annual premiums need be stamped only with the agreement stamp (6d. if under hand only, 10s. if under seal).

It has been stated that single premium policies under hand are sufficiently stamped with 1d.

Other views have, I believe, been taken as to the stamping of these policies (which will, of course, differ to some extent according to the actual policy form), and it would be interesting to have the question settled definitely.

## CONCLUSION.—A SUGGESTION.

In bringing to a close these somewhat disconnected remarks, I venture to make a suggestion for the consideration of members of the Institute—a suggestion which forms perhaps the principal reason for my having submitted the paper to you.

I have, so far, carefully avoided any reference to the rates and practice of individual companies, nor is it my intention to refer to them now, except for a moment in a perfectly general way. Although there are, it would seem, very many fewer points as to which difference of opinion may arise in fixing the premiums under these policies than in fixing those for life policies, it is probably well known to everyone here that the rates quoted by companies transacting the business differ widely, in spite of some modifications that have recently been made. Thus, to take two offices only, I find that for a term of 10 years, their rates are identical, while for a term of 50 years, they differ by nearly 20 per-cent of the lower, and for a term of 90 years, by over 60 percent of the lower. Such results as these are, perhaps, somewhat similar to Dr. Sprague's illustrations of figures quoted in different quarters in respect of the same reversionary transaction. They certainly show very wide differences of opinion as to the future rate of interest—a point which is, of course, of far greater importance in other departments of life assurance business. No doubt certain companies may, for legitimate business reasons, be more anxious to attract policies of this description than others, but making full allowance for this, the differences mentioned still seem wider than an outsider would have thought probable. What I desire to suggest for your consideration is, whether it would not be possible for all the offices transacting the business to join together

and agree upon a common basis for the calculation of premiumsupon a tariff, in fact-necessarily one framed upon a cautious The issue of the large policies I have more than once mentioned must, I think, cause some uneasiness in the minds of those who have quoted premium rates lower than the majority of other offices, especially where the whole risk is retained by the There would, of course, be many points to be considered in attempting to arrive at such an agreement—e.g., the difference in the security afforded by the offices-but the difficulties need hardly prove insuperable. If a common tariff were settled (as has, I believe, been more than once suggested for annuity business), the discrepancies between the rates of various offices, now rather inexplicable to an outsider, would disappear, and with them the selection against the offices, which, it may safely be predicted, the assured will display in placing the policies, ceteris paribus, with that office quoting the lowest rate for that particular term of years,

If such a tariff were agreed upon, too, some of the reasons for an office keeping the whole of one of these very large policies would be much weakened, probably to the advantage of the offices. All amounts over and above a certain fixed sum, which might fairly be fixed at eonsiderably higher than the office's ordinary assurance limit, might be given off by way of re-assurance without loss, and if it be argued that the original office would thus be giving up a large portion of the profit it expects to make over the transaction, the answer would be twofold: (i) that in theperhaps improbable, but possible—event of a fall in the rate of interest beyond that contemplated by the premium tables, it would be keeping any loss within manageable limits; (ii) that it would receive, as an integral part of the tariff arrangement, similar re-assurances from other offices upon which it might expect to realize a profit. In this way, too, it would be possible to avoid the unpleasant process, from a business point of view, of having to pay a very large claim, £50,000 or £100,000, in one year; a process which, although the premiums may have worked out as amply sufficient, and the transaction resulted in a profit, is not desirable, as it may imply a fall in the funds, with its accompanying disadvantages. If the large policies be spread by way of re-assurance over several offices, the risk of a disproportionately large sum being claimed in any one year would be very much lessened. Even if the tariff were confined at first to those offlees whose connections give them a fair prospect of obtaining

the large policies from time to time, and if each office re-assured only sums over £20,000 or £25,000, I think the plan would be on the whole not less advantageous, and certainly safer, than the present practice of retaining almost unlimited amounts under these policies, even if the rates are calculated on a basis that seems to present every prospect of being realized. And surely the spirit of competition which would prevent, if nothing else did, such an arrangement in connection with life policies need not form a disturbing element here—for many will think that the large sinking fund policy, as at present quoted for, and as at present managed, is not altogether a desirable policy for a life office to issue.

## Discussion.

The President (Mr. H. W. Manly) cordially welcomed the paper as one which was calculated to raise an important discussion upon a subject which was almost the burning question of the time, namely, the future rate of interest. The author had mentioned almost every point that could be raised with regard to this form of insurance, if it could be called insurance. It was nothing more than disguised banking. Small sums of money were taken and accumulated for a great number of years at rates which certainly no bank would offer unless it was transacting some other business than banking. There was one additional point, however, which might be considered in the discussion, and that was whether it was fair and equitable that insurance companies should allow a higher rate of interest for this form of policy than they were prepared to allow for non-profit life assurance policies, for, if the rates charged for non-profit policies were examined, it would be found that, excluding the question of expenses (which was generally excluded in calculating these sinking fund premiums), the premiums were little higher than the  $2\frac{1}{2}$  per-cent The author used the word "logical" in one of his remarks. He said: "My own personal view is, that rates based upon a decreasing rate of interest are the more logical and are preferable to those based upon a fixed rate, however low." As a rule, he (Mr. Manly) did not like the word "logical", but he thought if the suggestion were to be earried to its logical conclusion, then, it would be necessary to alter the rates every year. If they started on the basis that they were going to receive 3 per-cent for the next ten years, and  $2\frac{3}{4}$  percent for the following ten years, the next year they ought to make their rates on the assumption that they were going to have 3 per-cent for the next nine years, and  $2\frac{3}{4}$  per-cent for the following ten years, and so on; and therefore, to be extremely logical, they ought to alter the rates every year.

Mr. G. S. Crisford said that sinking fund policies were useful for various purposes, and they appeared to meet a requirement of the public. For instance, they were very useful indeed in connection with leaseholds. They were also useful in connection with securities which were bought upon the Stock Exchange at a premium. it convenient to deal with those premiums as a certain amount which had to be written off at a certain date. Especially in the case of terminable debentures, where the date at which they were to be redeemed was known, a sinking fund policy would provide the premium when the amount was paid off at par. With reference to larger policies for the purpose of securing the redemptions of debenture issues, these opened a very important question indeed, and he thought it ought to be seriously considered by the profession what would be the effect upon the business, if policies of this kind were introduced on anything like a large scale. With reference to the calculation of the premiums to be charged, he did not think it desirable, or possible, to set up any standard rate of interest which was going to last for sixty, seventy, eighty, or ninety years, and to assume that for the first period it would be 3 per-cent, or  $2\frac{1}{2}$  per-cent or 3½ per-cent, as the case might be, and then that it would gradually reduce by a certain percentage in certain decades or decennial periods. The fact should not be lost sight of that the same sort of argument would apply to the calculation of life assurance premiums. policies might have a duration of a few years or of a great number of years, he thought, as a general principle, in considering the rate of interest, they must rather go upon the broad principle of an average rate, and rectify it if necessary, as experience pointed out, in the matter of reserves at valuation periods. It was impossible to forecast the future. He believed as long ago as 1792 Three per-cent Consols stood at 97, but everyone knew how very much they had varied from that time up to the present, and it was not possible to tell what changes might take place in the financial affairs of the world, which might either reduce or increase the rate of interest. With reference to the rates of premium charged by different offices, he noticed there was as much difference in a twenty years' policy as half-a-guinea per-cent, and in twenty-five years 5s. per-cent; in fifty years 3s. percent; in seventy years 2s. per-cent; and in ninety years as much as 1s. per-cent. Taken as percentages on the premium, they would represent a very striking difference in the amount of loading. If the lowest rate was justifiable then the higher rate had a very heavy loading upon it. As to the reserves, the author had suggested that the method of forming the reserve should be the accumulation of the premium at the rate of interest a little more than the rate of interest assumed, and a little less than the rate of interest realized. He (the speaker) did not think that that was the best method of calculating reserves. If the rates of interest in the future were fixed quantities and known beforehand, then that method was correct for adoption, but the usual practice was to take the present value of the liability on the one side, and the value of the future premiums on the other, and he thought that at any particular period for valuing policies in which future contingencies had to be considered, that was the right method. The effect, of course, was, that if such premiums were charged, for instance, as were shown in column (4) of the author's table on page 566, and if the reserves were made at the valuation on the general methods adopted of putting the liabilities on the one side, and the value of the

future premiums on the other, instead of reserving a sufficient sum at the earlier years, when it is important that the reserves should be built up, it would be found that a very small sum would be reserved, and that the future years would be left to make up for the deficiency of the earlier years. He did not think that in business of this kind there was any necessity for a tariff association. It was a perfectly clear transaction, and there was not likely to be any deception of the actuaries and managers of the companies as to the responsibilities which they were undertaking; and if offices were willing to assume the risk at a rate which was not commensurate with the responsibility, he thought they must take the consequences. If, however, there was to be anything like a combination in which company promoters were going to use insurance companies for the sake of getting the repayment of their capital upon any basis which was unsound, then it was desirable to combine together to meet it.

Mr. L. F. Hovil said that there were very few actuarial points in the paper, and those there were arose in connection with the calculation of the premiums, and the calculation of the reserves for the policies when granted. The question of expenses was not a very serious one, and the point of importance was the rate of interest. Two methods for calculating the premium were given, the first method assuming a fixed rate of interest and the second assuming a decreasing rate of interest. Of the two, he considered the decreasing rate of interest was the one which was required, and he was very disappointed to hear Mr. Crisford thought so little of it. He did not think that ordinary life premiums could be compared with the sinking fund premiums. Of course, in the case of the life premiums the ordinary life policy was a contract which might be in existence a very short time compared with the average duration of a sinking fund insurance. As regards life policies, too, there were opportunities at the periodical valuations for providing for the decreasing rate of interest by adding to the reserves out of the policyholders' bonus, but there were none of those opportunities with the sinking fund policies. One of the best arguments for the decreasing rate Mr. Faulks supplied himself in the method he advised for calculating the reserves, a method that would immediately be suggested if the premiums were based in the first instance on the decreasing rate of interest. Single premium policies were very rarely granted, but in connection with one he had heard of there was exhibited a danger that was apt to be incurred in granting that form of policy. It was a single premium policy in connection with one of the debenture issues, and before the policy was actually granted the office who proposed to grant it had had placed before it a draft prospectus of the debenture issue, in which they were startled to find a bald statement to the effect that they absolutely guaranteed the repayment of the debentures. course that was literally true, he supposed, because they had been paid a single premium, and were going to pay a certain sum at the end of a definite term; but it would have done the office some harm if that had been made public. The author mentioned several points to be considered in the calculation of the premiums, and in one case referred to the question of selection. He mentioned a large policy that had just been effected, but that was rather a case against him,

because it was at a full rate, in fact a rate well above the average rate charged for the term in question. The second part of the paper headed "Security" would be almost better placed before that headed "Premiums", because there were points to be considered in connection with security that had to be taken into account in calculating the The author pointed out that sinking fund assurances were not policies of assurance or annuities on human life, and that therefore they were excluded from the returns to be made as regards that class of business. It also followed, of course, that they need not be included in the valuation returns. The author referred to the case of the purely mutual office where the sinking fund policyholder might have some charge on the profits of the life assurance fund. He could not understand under what circumstances that could be so, and he thought the author might explain what case he had in The Act seemed to be so explicit that it would be thought that once the life premiums had been carried to their separate fund, under no circumstances could they be under any liability to the sinking fund policyholders any more than to the fire policyholders. The President had referred to sinking fund assurance as banking He came across a very good definition by the late Mr. Newbatt, in a paper read before one of the Insurance Institutes in Mr. Newbatt said they (sinking fund insurances) were wholly deficient in the life assurance element, and unless needed by offices for their own protection in connection with loans or leasehold properties, savoured too much of money-dealing pure and simple. (Mr. Hovil) thought that conveyed exactly what the President With regard to the stamp, he had it in himself had said just now. his mind that a stamp for a single premium policy was the mortgage stamp half-a-crown per cent, but he was not sure whether that was so or not. With regard to the suggestion of combination, he was afraid the author must have had some misgivings when he put it forward—there had been so many suggested plans for combination, pleas for co-operation, and pleas for uniformity, and they had all met Many offices would, nevertheless, welcome with little success. combination in this class of insurance as he did not think life offices wanted to compete for the business, and if there was a combination it would no doubt mean a revision of rates and an increase in the very small margin of profit they could possibly have under the average rates now charged.

Mr. S. G. Warner thought that the most interesting question was that of the method of calculating premiums, and, while he did not know that he could use very cogent arguments in its defence, he agreed with the author as to the advisability of using a sectional method and a decreasing scale of rates of interest. Strictly speaking, of course, that method was unanswerably open to the objection that it would involve a revision of rates every year. Still, it seemed to him that what was wanted was to get as near to the facts as possible, and that was peculiarly desirable with a class of business which stood entirely on its own footing, and depended purely and simply upon the rate of interest. It was known what banking business was and how entirely it depended upon the rate of interest, and how, as a natural consequence, its rates were fluctuating

from day to day. In a sense, sinking fund business, if it was to be extensively done, must partake of the same character, and the calculations must be adjusted to the facts as they developed them-He quite thought, if the business was to be largely done, and large amounts involved in it, the rates would have to be revised with tolerable frequency. It seemed to him that there was a tendency in Mr. Crisford's remarks rather to confuse the issue by importing ideas connected with ordinary life assurance into the consideration of sinking fund assurances. He failed to see any point in the remarks with regard to reserves, that it was necessary to consider and compare future liabilities and future assets in making a It did not seem to him that that was the way to view that kind of contract. The whole thing lay within the four corners of the accumulation of premiums, and the liabilities had to be met by a pure and simple accumulation of premiums at compound interest. Surrender values had to be provided in the same way, and, therefore, the natural and simple method, the least likely to tend to confusion of ideas on the subject was, when dealing with that particular class of business, a class which stood entirely alone, to simply take a suitable rate, with regard to which, of course, there might be a difference of opinion, but to take what was thought a suitable rate, and then accumulate the premiums at that rate, and let those accumulations be the reserve for that particular class of business. That was a method which was exceedingly simple, and open to no possible misunderstanding, and open, as far as he could see, to no possible objection. He was in emphatic agreement with the author of the paper in his suggestion regarding a tariff. His reason for advocating something of the nature of a tariff was that the foothold was too slender to admit of competition. Here again, he thought, the difference between that form of insurance and life business forced itself upon their notice. They were dealing with financial facts within a very narrow compass, and it was undoubtedly of vital interest and importance to fit the calculations to those facts. If that was not done there would be a very heavy loss to meet, and there was no corresponding gain that he could see in reckless competition for business of that kind. Reckless competition doubtless did not exist, but if, as had just been said, the offices had no desire to compete for that kind of business, it seemed to him that the obvious course, and the most advisable one, would be to form such an association as had been suggested. There were many elements entering into life assurance business which tended to make the issues far less certain, far less confidently predicable than was the case with sinking fund business. It was known very well where the chief foothold for life assurance competition came in—it was in the question of future If it was desirable to compete for sinking fund business the matter should be looked at and approached in an entirely different way. Premiums should be charged calculated to provide the sums assured without any interest entering into the case at all, and then the offices might compete as much as they liked, making the policies profit-sharing, and distributing by way of profit the interest they earned. Within the four corners of a definite contract, and with no question of profit, it seemed to him that the foothold was too slender

for competition, especially competition for large policies spread over large periods, which was very dangerous, and, with the best intentions, a mistake might be made, which might, long after all those at present engaged in the work were out of the insurance world

altogether, involve the offices in serious difficulty.

Mr. G. Todd said that sinking fund business was not a class which many insurance managers would care to cultivate. in fact, like another class of business, the grant of annuities, forced upon them by competition. Many, whose business was dependent upon the goodwill of a large agency connection, could not afford to allow their agents and other connections to go to offices other than their own where such policies could be readily effected. connection he was referring more to what he considered the legitimate object for which such policies were desired, namely, for protecting leaseholds and other wasting securities, and in many cases for providing endowments for children. While he agreed with all Mr. Faulks' principles, he joined issue with him as to the general tone of his paper. He thought him over-cautious in the view he took of the business. If everything that was said in the paper were taken at its literal value, he doubted whether policies of that kind would ever be granted at all. It seemed to him that the premiums which they would be forced to quote would be practically prohibitive, and generally it looked almost as if the author considered that they were going to issue nothing but 99 years' policies. As far as his experience went, the average term of those policies was, at the outside, between 30 and 35 years. Surely, what had to be dealt with in the way of assurance business were averages. It was not necessary to consider any individual policy in the class because it happened to be granted for 99 years, or 70 years, or any other term, and deal with that separately from the rest. In that connection, of course, the principal point in the paper was as to the rate of interest, and, as Mr. Faulks had very justly said, it was absolutely impossible for any one of them, or any combination of them, to tell what that was going to be in the next 15 or 20 years. It seemed to him, however, looking at it as a question of averages, that the rate upon which it was considered safe to calculate ordinary non-profit premiums should be also a fairly safe rate on which to grant sinking fund or fixed-term policies. If ordinary non-profit policies were granted on, say, a 3 per-cent pure rate, he saw no reason why sinking fund policies should not be granted on similar terms. He thought there was no doubt that the author of the paper was perfectly correct in his statement that the granting of policies of that class would necessitate having a separate fund, and making returns under the 3rd and 4th Schedules instead of the 1st and 2nd. Yet he (Mr. Todd) saw no reason why that class of business should not be legitimately included within the ordinary powers of a life office. When the 1870 Act was passed such policies were unheard of, as were also various other policies which he considered not strictly life assurance policies, such as policies against issue, against the recovering of reason, name and arms risks, and various contingencies of that kind. In some of them there were, no doubt, life contingencies involved, but it was rather straining the point to consider them policies of assurance upon lives. The Board of Trade were supposed to regulate the

proceedings of the offices in reference to the returns, but they did not appear to have noticed these points. There appeared to be in the fixed term policies no risks which were not already inherent in There was no life ordinary life policies—rather less in fact. contingency, and that, especially in non-profit policies, was generally rather above the usual rate of risk. Possibly when that long-talkedof and much-to-be-desired revision of the 1870 Act was brought about, these points might obtain consideration, but he had not the slightest doubt in his own mind that the object for which the third and fourth schedules were instituted was to separate off business of the nature of fire and marine classes, and not business of a nature so closely cognate to that of life assurance. However, looking at it from that point of view, it was no doubt desirable that in some way the amount retained at risk in respect of the policies should be limited—he would not say in respect of individual policies, but certainly in respect to years of maturity—and that it would not be advisable for any office to retain policies to a very large amount maturing in any one particular year. With regard to these insurances, there was not the slightest doubt that when they were first undertaken the objects were perfectly legitimate, that of covering leasehold securities, and in many cases they were a real necessity. Trustees and others who took up under wills or otherwise leasehold securities found themselves very often in an awkward position. There were also many cases in which trustees and people similarly situated got opportunities of purchasing leasehold properties, and they had not the power to do so unless the properties were covered by policies. There had been two or three cases in which the Courts had definitely allowed a security outside the statutory term to be purchased by trustees covered by a policy of that nature. reference to the other class, the larger policies in connection with the covering of debenture and other stocks, he fully agreed with the speakers that the utmost caution was necessary in dealing with them. In the course of his official duties he had had a great many propositions of that kind made to him, but in almost every instance he had been unable to come to terms with the proposers. Either there was a requirement that the name of the insuring office should be made public in such a way that it was unwise to enter into the transaction, or special terms had been asked for as to surrender values and various things of that kind. It seemed to him that that was not legitimate growth. The companies requiring these policies to cover debentures were as a rule companies expected to pay considerable rates of interest upon their junior securities, and the correct thing to do was, either by means of drawings or by means of a sinking fund, to provide the policies or an equivalent fund out of their own resources, and by the employment of their ordinary business operations. It seemed rather giving something away to come and take a policy at  $2\frac{1}{2}$  per-cent, or any such rate, when they were making 6 or 7 per-cent on their own securities. He could not help thinking that in nine cases out of ten the object in taking out policies of that kind was in order that the names of the insuring offices might be used in the prospectuses in a way more or less misleading. Should the assured company come to grief, and the

premiums not have been kept up, the insuring offices would not make many friends amongst debenture or other stockholders who were relying on them, rightly or wrongly, for the return of their eapital. With reference to the author's final paragraph, he confessed he could not agree with the last speaker in his hopeful view of the result of their carrying out such a suggestion. certain that sooner or later the Charter of the Institute would be enlarged, and actuaries would have an acknowledged legal status, and then it would be incumbent upon the Institute to take up the more practical side of insurance work, and regulate generally the proceedings of all who would then be recognized as actuaries. Then the Institute would be entitled to say who was, and who was not, an actuary, and have a right to some control over the individual actions of the members. Until then he doubted whether any combination of offices joining together would produce a result adequate to the trouble and time that would be taken up in considering the matter. The suggestion, of course, really amounted to having a tariff committee. A similar committee had been proposed for annuity business, and if it came to anything, he suggested that that tariff committee should deal also with the whole of the non-profit rates of the whole of the offices. It seemed to him there was as much reason for taking in the ordinary non-profit rates, as for the leasehold policies or annuities. But apart from the question as to whether such a committee would be successful or not, he deprecated the suggestion from another point of view. He thought the success of the insurance institutions of this country had been the result of full and free competition, and he saw no reason for taking any in lividual class of policy, or classes of policies, and putting a stop to that competition. It must, he thought, be left to the individual offices to consider whether in carrying on that competition they were doing so within safe lines.

Mr. W. P. Pulley referred to the investment of the premiums received under the sinking fund insurances. The author stated that, having regard to the possibility of surrender, it would not be suitable to invest in reversions or such like securities which yielded a better rate of interest; but he (Mr. Pulley) thought the surrenders would only form a small proportion of the premiums which would have to be invested. Reversions, he thought, would be very suitable investments, because they would have long terms to run similar to the insurances themselves. The author spoke of it being unfair to the life policyholders in a mutual office for the sinking fund policyholders to take their proportion of the total interest receipts, as the investments in which they would be likely to be put, would yield a lower rate of interest. There, again, he ventured to think that, as those insurances carried no profits, and, as the author stated further on in his paper that the insurances would not be undertaken without the prospect of their yielding at any rate some profit, in a mutual office the policyholders would share in those profits—in fact, have them all. Therefore that objection would hardly hold. With regard to the rate of interest at which the premiums should be calculated, he thought that a decreasing rate should be used; but, in view of the remarks of the President as to the rates having to be revised at short

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intervals, he thought that possibly one rate of interest might be adopted for the next 20 to 40 years, and a somewhat lower rate of interest for the remainder of the term. That would work out well in practice, as the premium for an insurance for a long term being small, the lower rate of interest would produce an addition of only 10 to 20 per-cent, which would be a very small addition to the premium for

Mr. G. Marks said there was one point he should like to notice. He should not have mentioned it except that Mr. Todd, who was associated with a mutual office, agreed with the author that mutual societies were, equally with proprietary offices, under the Act of 1870, obliged to render a separate revenue account in respect of sinking fund policies. He should like to put it on record that he did not agree with that conclusion, although he was open to conviction on the point. The statement depended on the true interpretation of a very badly worded section of the Act, but he should not like it to go from that room, without protest, that mutual offices considered themselves bound by Mr. Faulks' interpretation of the section.

Mr. Todd did not think he particularized mutual offices. He was referring to a general principle. Probably from his connection with a mutual office, Mr. Marks might have been led to believe that

he was talking of mutual offices, but that was not so.

The President, before asking Mr. Faulks to reply, proposed a

vote of thanks to him for his eminently useful paper.

Mr. J. E. FAULKS, in reply, thanked the members for the cordial reception given to his paper and for the many valuable ideas elicited by the discussion. The chief point in the paper was the suggestion of a tariff. The quotation given by Mr. Hovil from Mr. Newbatt recalled to his mind another remark made by Mr. Newbatt in that room, an adaptation of an old saying, namely: That in life assurance there should be—

In essentials, liberty. In non-essentials, unity. In all things, charity.

His (Mr. Faulks's) contention was that sinking fund policies were "non-essentials", and that unity was the proper thing there, and not liberty. The idea of a tariff committee appeared to frighten some of the speakers, but it seemed to him that very few meetings of the committee would be required. It was merely a question of settling rates and reconsidering them periodically. Although life insurance offices had attained their present position by competition, rather than by combination, still there was a fire offices committee and a fire tariff, and no one would contend that fire insurance was less successful than With regard to what the President had said as to the decreasing rate of interest, there was a good deal in the objection to the word "logical." No doubt, strictly speaking, it was necessary to do two things—(1) not to take such long intervals as ten years before allowing for a decrease in the rate of interest, and (2) to reconsider the rate of interest very frequently. Mr. Warner had mentioned the second point. With regard to the other, it had originally been his intention to insert a table based on a rate of interest decreasing by 1s. per-eent per annum. He reminded the members that Mr. R. P. Hardy

had made use of a similar idea for a particular purpose in connection with life assurance in a recent note published in the Journal. It afterwards seemed to him, however, that that would have been attempting an accuracy which could not in reality be attained, and therefore it was better to obtain such accuracy as was possible by taking periods of ten or twenty years. With regard to the method of valuation. Mr. Crisford suggested that the proper method was the prospective method, not the retrospective method—that was, not to accumulate the premiums, but to estimate separately the values of the sums assured and the future premiums as at the date of the valuation. As Mr. Warner pointed out, that idea appeared to arise from a confusion of sinking fund policies with life assurance policies. There was no reason for estimating separately the values of the sums assured and of the premiums, because, as he had mentioned in the paper, there could be no doubt those insurances did not come into the life assurance fund. They should not, therefore, apparently, be included in the 5th and 6th schedules of the valuation returns. Mr. Todd had referred to the question of averages. Perhaps the term "averages" might, in connection with sinking fund policies, lead one a little astray. The term of each individual policy was fixed, and there was no occasion to bring in any idea of average with regard to the term. The only average to be dealt with was the average rate of interest for the duration of the particular policy quoted for. With regard to Mr. Pulley's remarks as to the proportion of surrenders, of course it was quite possible that only a small proportion of the policies would be surrendered, but the worst had to be provided There was only one other point to be dealt with, the point that Mr. Marks had raised as to whether the sections of the Life Assurance Companies Act applied to purely mutual life offices: that was to say, whether in the case of a purely mutual life office it was clear that a separate fund should be created for the sinking fund policies, and that those sinking fund policies could not be charged on the life assurance fund. In the first place, there were two sections in the Act of 1870 which dealt with the matter—sections 4 and 6. Section 4 dealt with keeping the life fund separate, and there was an exception; and section 6 dealt with the question of accounts, and there was no exception. Therefore. even if the sinking fund policies could be legitimately charged against the life assurance fund, in the case of a purely mutual life office—notwithstanding this—its returns must, by section 6, be framed under the third and fourth schedules, and not under the first and second. Secondly, the words of section 4 were: "In respect to all existing companies the exemption of the life assurance fund from liability for other obligations than to its life policyholders shall have reference only to the contracts entered into after the passing of this Act, unless by the constitution of the company such exemption already exists: Provided always that this section shall not apply to any contracts made by any existing company by the terms of whose deed of settlement the whole of the profits of all the business are paid exclusively to the life policyholders, and on the face of which contracts the liability of the assured distinctly appears." In the first place, it would be seen that the exception was limited to mutual

life offices existing at the date of the passing of the Act. second place, the whole of the profits must be paid to the life policyholders, and on the face of the contract the liability of the assured must distinctly appear. He took it that the only possible meaning of this was that it must appear on the face of every life policy that the life assurance fund of the company was liable also for the liabilities of the company under its sinking fund policies. Porter's Laws of Insurance said—" This provision as to a life assurance fund does not apply to companies the whole of whose profits are divided among the policyholders, and whose policies bear on the face of them a distinct declaration of the liability of the policyholders. Such a company is a purely mutual company, where all must contribute, and in the profits of which all share. There was at the passing of the Act only one such not coming within the Friendly Societies Acts." A reference was given to *Hunsard*. Finally, the Life Assurance Companies Acts Amendment Act 1872, section 2, stated: "Whereas by section 4 of the Life Assurance Companies Act 1870, it is enacted . . . and further provisions were made by the same section with respect to the application of the above recited part of the said section to existing companies, and doubts have arisen with respect to the construction of the said provisions, and it is expedient to remove such doubts; be it therefore enacted that the portion of section 4 of the Life Assurance Companies Act 1870, above recited, shall apply to every company established before the passing of that Act, provided that the Life Assurance Companies Act (1870) and this Act shall not diminish the liability of the life assurance fund for any contracts of the company entered into before the passing of the Life Assurance Companies Act 1870." To his mind it was therefore abundantly clear that a mutual life office was in precisely the same position as a proprietary life office on the point raised by Mr. Marks.

## REVIEWS.

## The Declension of Phthisis.\*

THE Oration delivered by Sir Hugh Beevor before the Hunterian Society is of very considerable actuarial as well as medical interest, as it deals with problems occurring in the daily work of insurance, and presents a succinct and able summary of the English National Statistics relating to phthisis.

The first point discussed is the reliability of the official figures, and the conclusion reached is that they may be substantially trusted. We quite agree that there has been a steady and substantial decline in the mortality from phthisis between 1851 and the present time;

<sup>\*</sup> The Declension of Phthisis. An Oration delivered before the Hunterian Society on 8 February 1899, by Sir Hugh Beevor, Bart., M.D., F.R.C.P. Lond. Reprinted from *The Lancet*, 15 April 1899.

but, unlike Sir Hugh Beevor, are of opinion that the decline in pulmonary phthisis is a little over-stated in the official figures, on account of the fact that many deaths that would have formerly been returned as phthisis are now certified as due to "tuberculosis", or "general tuberculosis." It is difficult to estimate the extent to which this cause of error is in operation; and it cannot be rectified by a comparison between the past and present incidence of tubercular diseases other than phthisis, because in respect of one at least of these, viz., tabes mesenterica, there is an increasing tendency to enter under this head deaths which would formerly have been certified more vaguely as due to "wasting" or "marasmus", and the tubercular nature of which is very doubtful.

Chart I. in Sir Hugh Beevor's paper enables a comparison to be made between the death-rate from phthisis in the two sexes at each group of ages in 1861–70 and in 1891–5 respectively. It shows that the greatest saving of life (i.e., total lives, not ratio) has occurred at those age-groups when phthisis is most prevalent. The result is summarized in the following tabular statement:

Fall of Death-rate from Phthisis in 1891-5, from that in 1861-70, the rate of 1861-70 being taken as 100.

| Age in Years    | Males | Females |
|-----------------|-------|---------|
| 0-              | 47    | 44      |
| ō               | 44    | 54      |
| 10-             | 41    | 53      |
| 15-             | 47    | 45      |
| 20-             | 51    | 43      |
| 25-             | 61    | 49      |
| 35-             | 77    | 59      |
| 45-             | 83    | 61      |
| 55-             | 78    | 62      |
| 65-             | 76    | 64      |
| 75 & upwards    | 85    | 78      |
| t a di di marti |       | 1       |

Thus the greatest reduction is under the age of 25 years, and the amount of reduction is almost equally great among all ages below this, which are of least consequence for ordinary insurance purposes.

To determine the relative incidence of mortality of phthisis at different ages another table is necessary, as follows:

Death-rate from Phthisis per 100,000 living in 1891-95.

| Age in Years | Males | Females |  |  |
|--------------|-------|---------|--|--|
| 0-           | 46    | 42      |  |  |
| 5-           | 19    | 26      |  |  |
| 10-          | 26    | 56      |  |  |
| 15-          | 107   | 142     |  |  |
| 20-          | 202   | 174     |  |  |
| 25-          | 254   | 215     |  |  |
| 35-          | 326   | 230     |  |  |
| 45-          | 320   | 174     |  |  |
| 55           | 268   | 129     |  |  |
| 65-          | 157   | 80      |  |  |
| 75 & upwards | 56    | 35      |  |  |

It is plain from this table that the greatest incidence of phthisis mortality in males is at ages 20 to 65, and that even at the age-group 65-75 the death rate from this disease is higher than at any age below 20. Among females, the chief incidence of phthisis mortality is at the ages 15 to 65, and at the age-group 65-75 the death-rate from this disease is higher than at any age below 15.

Putting the two tables together, we arrive at the conclusion that the greatest reduction of phthisis mortality has not been at the ages of greatest mortality from this disease, but in males, at least, at ages under 20, before the full force of mortality from phthisis is

experienced by the community.

Sir Hugh Beevor next enters into a profoundly interesting discussion of the causes of the different age-incidence of phthisis mortality displayed in the last table set forth above. His view is that the growing lung is able to resist infection, and that as women cease to grow a few years earlier than men, this fact accounts for the earlier increase of phthisis mortality in their sex. We are not satisfied with this explanation, although it is ingenious. hypothesis that differences in the manner of life of boys and girls explain the higher mortality from phthisis among girls, appears to us to furnish a more satisfactory explanation of the facts. After the age of five, and between that and 20 years of age, girls live a more sedentary and indoors life than boys, and show a great excess of Under the age of five, boys and girls are phthisis mortality. exposed to very similar conditions and they have a nearly equal phthisis mortality. After the age of 20, the evil occupational influences of men's lives come into operation, and male phthisis mortality now becomes very excessive.

A word en passant on "deaths to deaths ratio", to which much space is devoted in the oration. It would appear that Sir Hugh Beevor regards the curve based on these ratios as a "liability diagram", while that based on the death rates is a "mortality diagram." We cannot admit that the former shows the liability to phthisis. This could only correctly be shown by the ease-rate

of phthisis in terms of the population, for which the data will be unavailable until it is made compulsory for every recognized case of phthisis to be officially notified. Conclusions based on the ratio between deaths from phthisis and deaths from all causes must necessarily be fallacious, inasmuch as this constitutes a ratio between two factors, of which both may be variable, and one of which (total mortality) does actually vary enormously from year to year.

Sir Hugh Beevor next gives a most suggestive discussion of the relative influence of infection, and of hereditary and acquired susceptibility in the causation of phthisis, his conclusion being that the facts "suggest the answer that infection is quite subsidiary to other influences which create susceptibility." The British public eat more and more, and the improved nutrition resulting from this is evidently, in Sir Hugh Beevor's opinion, the great factor in the improvement which has occurred. The improvement of nutrition might, he adds, be expected to influence the ages of youth and adolescence more than later vears; hence the greater fall in the phthisis rate during the first 25 years of life. One would naturally expect, if this hypothesis be correct, that the higher classes would have shared less in the improvement that has occurred, and the stationary mortality from phthisis in five septennial periods of the experience of the Scottish Widows' Fund is quoted in this connection. Exactly the same line of argument would, we may point out, apply if we substituted the words "diminished overcrowding" for "increment of nutrition"; and it is more consistent with what we now know of the methods of spread of phthisis to believe that overcrowding does not simply connote the chronic inhalation of what has been called "air sewage", but also of the specific infective organisms of tuberculosis in the form of dust. In other words, that infection occupies a much more prominent part in the spread of phthisis than Sir Hugh Beevor is prepared to admit.

We next have some interesting figures showing that in London districts there is a coincidence of increase of phthisis with over-crowding, and that this overcrowding produces its effects in a small degree only upon growing lungs, the effects being greatest at ages higher than those at which phthisis mortality attains its usual

maximum.

Coming next to the question of local variations in phthisis mortality, an interesting chart is given showing that in urban populations the maximum is reached at a later age, and is comparatively high in old age. The three urban populations (Cambridge. West Ham, and Dudley) are, we think, very unfortunately selected. The large student population of the first would probably contribute practically nil to the phthisis returns of the town; while Sir Hugh Beevor states that 10 per-cent of the deaths of the West Ham population occur in Metropolitan hospitals and do not appear in its official returns. The inclusion of the external deaths from phthisis might altogether change the apparent age-incidence of this disease. We have not, in the preceding remarks, exhausted the interesting subject with which Sir Hugh Beevor's oration supplies us. Although we have ventured to criticise a few points in a paper which

is necessarily highly technical, we are none the less grateful to the author for his very exhaustive study of this most important subject, and for having condensed into small bulk a large mass of most valuable information.

A. N.

## On Centenarians.\*

MR. Young's book on Centenarians displays an amount of learning and research for which even those who had listened to his memorable Presidential Addresses were scarcely prepared. The references and footnotes on every page, the constant and rapid transition from questions of Medicine and Physical Science to Classical Mythology and Biblieal Criticism, from modern Philosophy to Mediæval Italian Literature, prove a width of reading and appreciation which fairly bewilders the average mind, and brings back to one's memory the reverential wonder with which the schoolmaster in Goldsmith's Deserted Village was regarded by his neighbours. This universality of knowledge, if we may call it so, on the part of the author, has operated somewhat to the disadvantage of his book. treatment of the subject is too elaborate. The book is, in fact, overweighted. It contains so much not strictly germane to the subject that few people will care to give the very large amount of thought and time that are necessary to its proper appreciation. But when this has been said by way of criticism, nothing remains but unqualified praise for the matter of the book and for its arrangement. Mr. Young begins by an account of the object of his enquiry and its result. Twenty-four authentic, indisputable, Centenarians are mentioned. Of these, one is the case referred to by Mr. C. D. Higham, in vol. xxx of the Journal, and one is Mrs. Margaret Ann Neve, described as a native of Guernsey. The remaining 22 cases were obtained from the current investigation of the Institute of Actuaries and the Faculty of Actuaries, and the Government Annuitants' Experience, Four of these 22 were Assured Male Lives, three were Male Annuity Nominees, and fifteen Female Annuity Nominees. Mr. Young states that "no Assured Female had reached 100." He said this, however, with information probably incomplete. Through the kindness of Mr. Ackland we are enabled to give a table of particulars of four Assured Female Lives, two of whom had passed the age of 100, and one more had probably done so, whilst the fourth at death was aged 99 years, 5 months and 26 days. Mr. Young's stringent tests have not yet been applied to the three eases above mentioned, but there is no reason to suspect mistakes, and presumably, therefore, the three should be added to Mr. Young's list of undoubted centenarians, bringing up the number to 27.

<sup>\*</sup> On Centenarians, and the Duration of the Human Race: a Fresh and Authentic Enquiry, with Historical Notes, Criticisms, and Speculations. By T. E. Young, B.A., &c. London: C. & E. Layton, 56, Farringdon Street, E.C.

# Female Assured Lives-Age 100 Years and upwards.

| Initials of | Date<br>of       | Date<br>of    | Асе ат Беатн |      |       | Age if Living on<br>2 December 1898 |      |        | Social    | Single (S),<br>Married (M), |
|-------------|------------------|---------------|--------------|------|-------|-------------------------------------|------|--------|-----------|-----------------------------|
| Name        | Name Birth Death | Years         | Mths.        | Days | Years | Mths.                               | Days | Status | widow (W) |                             |
| М. А.       | 179              | 7 25 May 1897 | ş            |      |       |                                     |      |        | £50 case  |                             |
| J. C. C.    | 2 Dec. 1798      | 3             |              |      |       | 100                                 | U    | 0      | Baroness  |                             |
| s. w.       | 24 Aug. 179      | 9 4 Jan. 1891 | 100          | 1    | 11    |                                     |      |        | Superior  | М                           |
| М. А. Т.    | 21 Oct. 1798     | 17 April 1898 | 99           | 5    | 26    |                                     |      |        | Moderate  | W                           |

<sup>\*</sup> Death in 100th year. See p. 13 of Mr. Young's book.

The fact that as many as 15 Female Annuity Nominees are included in his list, and that their average age exceeded 102 years. leads Mr. Young to some reflections on female vitality which are well worth attention. "It would almost appear," he says, "that, in the "constitution of the female sex, an intenser inherent power of " survivance exists; or, possibly, this enhanced vitality may be due— " and certainly this feature forms an important partial factor—to the "circumstance that Annuitants are self-selected lives. . . . I am "inclined to believe that the application of the antiseptic principles " of Surgery in the innocuous removal of internal tumours, peculiar "and frequent to females, has also definitely contributed to the "prolongation of their life-time on the whole. . . . Moreover, "in Annuity experience, the preponderant purchase of Annuities "upon the lives of female nominees after the age of 50, when the "danger from child-bearing has ceased, attests that, practically, " specially select lives are alone involved." Mr. Young is of opinion. therefore, and most people will agree with him, "that the practice of "Annuity business,-now largely increasing in consequence, mainly, " of the continuous decline in the market rate of interest upon sound "investments,—is in no degree likely to contribute a profit to the "Companies which favour this description of transaction. " contention is enforced, also, by the fact that, in Life Assurance, an "improvement in the general rate of mortality is favourable to the "Companies and partially counteracts the adverse influence of the "reduced return from interest obtainable upon their Funds, while "in Annuity business, both these elements combine in joint action "against the prosperity of the offices."

Mr. Young passes on to a discussion of the difficulties attending an investigation such as he has made: namely, the difficulties of determining the age and identity. With regard to the date of birth, a review of such forms of evidence as an entry in a Family Bible, or the age recorded in the Certificate of Death or on a tombstone, or the personal statements of contemporaries, leads Mr. Young to the conclusion that all are untrustworthy, and that only a Certificate of Birth is, as a rule, absolutely reliable. In all the 24 cases of

Centenarians referred to above, these difficulties have been satisfactorily overcome, and the age and identity established beyond a doubt. It is fortunate for the future enquirer into this subject that we have now a system of compulsory registration, so that in most cases it will be possible to obtain the necessary Certificate of Birth. Chiefly because age and identity are not satisfactorily proved, Mr. Young is able without difficulty to dispose of most of the alleged examples of Centenarians in previous investigations on the subject. The works of Mr. Thoms, M. Flourens, Dr. Farr, and others are discussed, and a careful analysis of their results made. interest attaches to the enquiry made by Dr. G. M. Humphry, of Cambridge, who attempted to discover by statistical evidence what were the most important factors that contributed to longevity. His results are summarized thus: "Sound family history; a well-"made frame of average stature—5 ft. Sin. in males, and 5 ft. 3 in. in "females; a spare habit; continuous good health; a vigorous "digestion; the regular daily action of the bowels; reasonable "muscular activity and its exercise; the capacity of sound sleep; early rising; a good appetite, moderately indulged; little alcohol " or animal food; and a placid temperament." This sounds all right, but as the first essential appears to be that one should have a thoroughly healthy father and mother, and as it is not permitted us to choose our parents, we are afraid that for most of us the problem, "How to become a Centenarian", must still remain unsolved.

Leaving the firm ground of modern investigations, Mr. Young proceeds to make an exhaustive enquiry into the alleged longevity of the Antediluvians. No attempt is made, of course, to reconcile Geology with Tradition on the subject of a Universal Deluge, but a most interesting account is given of the various hypotheses invented by different people to explain the alleged longevity of the Patriarchs. One curious suggestion is that of Hensler, that from the date of Adam to the period of Abraham the year comprised only three of our months, that by Joseph's time it was enlarged to eight months, and subsequently it increased to its present duration of twelve months. This assumption has no foundation whatsoever in fact, but, as Mr. Young points out, there is a striking analogy to it in the length of our terrestrial day, which was originally about three or four hours only, and will eventually be prolonged by tidal retardation to something like 1,400 hours. Without a knowledge of the actual length of our day or of our year, will the investigator of the future have very much faith, we wonder, in the 22 indisputable Centenarians discovered by Mr. Young? Whiston's speculations, too, are interesting as well as amusing, and the more serious hypothesis of Gompertz and the theory recently suggested by Prof. Karl Pearson on the duration of life will repay careful study. Mr. Young finds little difficulty in establishing a verdict of "Not proven" in regard to the hypotheses of the theologians and others. In a series of eloquent notes he shows how they have received their currency partly from the tendency which is strong in us all to exalt the past, partly through the influence of tradition and superstition, and after a full discussion of the subject he concludes "it is not a subject of wonder that the final result of "investigation must be that credibility cannot be attached in any degree to these alleged and exaggerated ages of the Antediluvians. And this conclusion . . . . is amply confirmed and verified by the results of Geology and the Natural Sciences in general, which have rigorously proved that, so far as keen and cultivated scientific enquiry can extend, a definite continuity of experience has been discerned through all the ages of time, both in the domains of physiology and physics. The investigations of Geology . . . clearly indicate that no Universal Deluge then occurred: Astronomy and the various Physical Sciences demonstrate that no abrupt alteration separated the modern from the ancient world, or the existing relations of the Universe from those which previously prevailed; and the researches of Physiology can detect no solution of continuity in the structure of the human frame or the general conditions of terrestrial life and environment which would begitimately assign an immensely prolonged existence to the

" primitive races or individuals of mankind." Two chapters are added by Mr. Young, one indicating the thoughts which have occurred to the author himself as the result of his investigation; the other containing a simple record of various suggestions connected with longevity. It is in the earlier of these chapters that the chief interest of the book lies, and it is perhaps with regard to Mr. Young's conclusions that most difference of opinion will prevail. The question is first discussed: Are Centenarians mere "sports" of Nature, absolutely independent of Law? and the conclusion arrived at is that they do not constitute a mere lusus Naturæ. "They will in due time be rationally ranged "among aberrant, but not arbitrary, examples; their apparent "lawlessness will be comprised within the scale of a uniformity of "rule", and the hope is indicated that, with an increase in the number of cases, the possibility of discovering this rule may be strengthened. Under what conditions is the requisite increase in the number of Centenarians likely to occur? Mr. Young argues that what is necessary is "a Generation of men which displays not an "unusually extended average duration of life on the whole, but one "which contains an increased proportion of aged lives", and in this connection reference is made to the wonderful discoveries of Bacteriology, and their influence on longevity. Passing on then, to "The Age-Relationship of the Human Race, the Earth, and the Sun". Mr. Young finds that "the Earth as a material fund of energy for "Man is thus incessantly reduced in vital sustaining power by the " conjoint process of thermal abstraction with diminished Solar supply; "and these considerations . . . appear reasonably to suggest that "the Solar System, and consequently the Human Race, have attained "their practical maturity or midway stage, at the present period, and " may be expected to exhibit the evidences of decline." This decline. too, is, in Mr. Young's opinion, mental as well as physical. "The " average intellectual condition of the present period . . . exhibits "no sign whatever of ampler development . . . ; the general " mental structure, as a whole, is lapsing more and more completely under the dominion of memory alone." And with regard to Man's physical development, the influence of Civilization has not been wholly for good. "Her lavish profusion of material services towards "the convenience and torpid ease of mankind tends to arrest the " native activity and the power of expanding growth in readiness, refinement of action, and scope of vigorous exercise, which, otherwise, " the more primitive and unsupplemented necessities of life would "cultivate." We confess that, in spite of Mr. Young's eloquence, we find it possible to take a more hopeful view of the position. It appears to us that he has, to a certain extent, given way to the tendency to exalt the past on which he dwelt in an earlier chapter. It is true that the present age has produced no sculpture like that of the Greeks, no paintings like those of Mediæval and Renaissance Italian art, no literature to rank with the writings of Shakespeare. But these facts do not necessarily mean mental decline, or even lack of mental development. For the energy of the race has been diverted, in the present century, towards Science, and particularly Mechanical and Medical Science, rather than towards Literature and Art. We do not propose to discuss the question at length, but would refer simply to the tabular statement of the great inventions and discoveries of the Nineteenth Century compared with those of all preceding ages given by Dr. A. R. Wallace (*The Wonderful Century*, pp. 154-6). Is it not possible that Greek Sculpture, Italian Art, and Elizabethan Literature, may have simply marked the culminating points of different phases of mental development, and that the inventions of the present age may mark a still different phase, and may lead to an even wider mental evolution in the future?

A. L.

# CORRESPONDENCE.

## INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In continuation of the Tables contributed by Dr. Sprague,\* giving certain particulars of Issue Insurances granted by British Life Offices, I annex a table, the figures in which are taken from the Board of Trade Returns for the five years 1891 to 1895 inclusive, except in the case of the *Rock* Office. As that office makes a septennial valuation, I have taken the figures from the Blue-Book of 1896.

Combining these figures with those formerly given, we have the following results:

| Years       | No. of<br>Policies | Net Sums<br>Assured | Net<br>Premiums<br>Received | Average<br>Premium<br>per-cent |  |  |
|-------------|--------------------|---------------------|-----------------------------|--------------------------------|--|--|
|             |                    | £                   | £                           |                                |  |  |
| 1871-1875   | 261                | 875,558             | 62,238                      | 6.75                           |  |  |
| 1876-1880   | 383                | 1,264,166           | 97,495                      | 7.51                           |  |  |
| 1881-1885   | 539                | 1,696,747           | 116,704                     | 6.82                           |  |  |
| 1886 - 1890 | 698                | 2,076,859           | 133,200                     | 6.08                           |  |  |
| 1891-1895   | 987                | 2,836,634           | 139,137                     | 4.75                           |  |  |

<sup>\*</sup> See J.I.A., xx, 151; xxiii, 220; xxvi, 391; and xxix, 548.

|                       | Date of                     | er of<br>sies         | NET AMOUNTS,<br>DEDUCTING<br>RE-ASSURANCES |                           | Gross Amounts   |                           |               |                                |
|-----------------------|-----------------------------|-----------------------|--|---------------------------|-----------------|---------------------------|---------------|--------------------------------|
| Name                  | Valuation                   | Number of<br>Policies | Sums<br>Insured                            | Pre-<br>miums<br>Received | Sums<br>Insured | Pre-<br>miums<br>Received | Reserve       | Average<br>Premium<br>per-cent |
|                       |                             |                       | £  | £                         | £               | £                         | £             |                                |
| Legal and General .   | 31 Dec. 1891                | 92                    | 370,447                                    | 14,586                    | 499,159         | 19,667                    | 19,667        | 3.94                           |
| Eagle                 | 31 Dec. 1892                | 72                    | 304,525                                    | 14,528                    | 413,990         | 20,004                    | 10,049        | 4.83                           |
| Law Union             | 31 Dec. 1894                | 65                    | 212,750                                    | 5,526                     | 373,500         | 10,267                    | 10,267        | 2.75                           |
| Equity and Law .      | 31 Dec. 1894                | 51                    | 155,347                                    | 11,075                    | 216,968         | 15,636                    | 15,636        | 7.21                           |
| Guardian              | 31 Dec. 1894                | 69                    | 152,313                                    | 8,462                     | 248,656         | 13,696                    | 12,350        | 5.21                           |
| Law Life              | 31 Dec. 1894                | $-49^{-1}$            | 145,880                                    | 8,070                     | 209,280         | 9,728                     | 9,728         | 4.65                           |
| North British & Mere. | 31 Dec. 1890                | -35                   | 130,876                                    | 8,555                     | 169,276         | 10,167                    | 7,003         | 6.01                           |
| Scottish Equitable .  | 1 Mar. 1893                 | -52                   | 98,800                                     | 6,903                     | 105,150         | 7,170                     | 6,817         | 6.82                           |
| Rock                  | 31 Dec. 1895                | 19                    | 79,800                                     | 3,868                     | 158,300         | 9,780                     | 9,305         | 6.18                           |
| Standard              | 15 Nov. 1890                | 20                    | 74,584                                     | 4,038                     | 192,584         | 10,780                    | 1,178         | 5.60                           |
| Royal Exchange .      | 31 Dec. 1890                | 28                    | 70,970                                     | 1,805                     | 72,470          | 1,852                     | 1,852         | 2.56                           |
| Commercial Union .    | 31 Dec. 1892                | 27                    | 61,768                                     | 2,245                     | 69,768          | 2,295                     | 2,296         | 3.29                           |
| National              | 31 Dec. 1894                | 40                    | 63,138                                     | 2,347                     | 63,878          | 2,362                     | 2,414         | 3.70                           |
| Eng. and Scot. Law .  | 25 Dec. 1890                | 28                    | 60,214                                     | 4,651                     | 63,714          | 4,756                     | 4,756         | 7.46                           |
| Reliance              | 31 Dec. 1892                | 11                    | 58,173                                     | 3,303                     | 75,173          | 4,374                     | 4,130         | 5.82                           |
| Pelican               | 31 Dec. 1890                | 12                    | 57,000                                     | 2,814                     | 75,000          | 3,003                     | 3,003         | 4.00                           |
| Alliance              | 31 Dec. 1893<br>5 Apr. 1891 | 17                    | 56,941                                     | 2,886                     | 88,944          | 4,230                     | 4,230         | 4.76                           |
| Life Association .    | 5 Apr. 1891<br>31 Mar. 1892 | 32                    | 55,750                                     | 1,644                     | 106,250         | 2,482                     | 2,482         | 2.34                           |
| Edinburgh             | 31 Dec. 1894                | 23<br>22              | 54,966<br>53,890                           | $\frac{2,496}{3,145}$     | 54,966 $83,790$ | 2,496                     | 2,247         | 4.54                           |
| Atlas                 | 31 Dec. 1893                | 10                    |  | 5,145<br>860*             |                 | 5,114                     | 5,084         | 6.10                           |
| Liv. & London & Globe | 31 Jan. 1891                | 15                    | 46,100 $41,765$                            | 1,975                     | 76,100 $44,165$ | $\frac{1,419}{2,047}$     | 867           | 1.86                           |
| Imperial              | 30 June 1891                | 16                    | 38,610                                     | 1,575 $1,515$             | 39,610          | 1,567                     | 2,047 $1,568$ | 4·63<br>3·96                   |
| Scottish Union & Nat. | 31 Dec. 1894                | 9                     | 38,308                                     | 1,581                     | 48,308          | 2,031                     | 2,031         | 4.20                           |
| Caledonian            | 31 Dec. 1892                | 19                    | 33,100                                     | 1,652                     | 37,600          | 1,794                     | 1,470         | 4.77                           |
| Mutual                | 31 Dec. 1891                | 17                    | 31,700                                     | 1,191                     | 46,700          | 1,341                     | 1,357         | 2.87                           |
| Northern              | 31 Dec. 1890                | 7                     | 28,654                                     | 1,685                     | 28,654          | 1,685                     | 1,685         | 5.88                           |
| London Assurance .    | 31 Dec. 1890                | 9                     | 27,551                                     | $\frac{1,030}{2,334}$     | 27,551          | 2,334                     | 2,217         | 8.47                           |
| Clerical, Med. & Gen. | 30 June 1891                | 6                     | 27,154                                     | 1,888                     | 27,154          | 1,888                     | 1,888         | 6.95                           |
| British Empire .      | 31 Dec. 1893                | 13                    | 27,080                                     | 1,160                     | 27,080          | 1,160                     | 1,102         | 4.28                           |
| Royal                 | 31 Dec. 1894                | 9                     | 23,300                                     | 606                       | 25,400          | 672                       | 672           | 2.65                           |
| West of England .     | 31 Dec. 1892                | 25                    | 23,300                                     | 1,004                     | 56,300          | 1,697                     | 1,414         | 3.01                           |
| Economic              | 31 Dec. 1893                | 6                     | 19,682                                     | 343                       | 19,682          | 343                       | 342           | 1.74                           |
| General               | 31 Dec. 1892                | 9                     | 18,784                                     | 1,468                     | 19,784          | 1,568                     | 1,568         | 7.93                           |
| Yorkshire             | 1 Mar. 1895                 | 3                     | 14,000                                     | 746                       | 14,000          | 746                       | 853           | 5.33                           |
| Westminster& General  | 31 Dec. 1891                | 8                     | 11,562                                     | 410                       | 11,562          | 410                       | 368           | 3.55                           |
| Universal             | 31 Dec. 1894                | 5                     | 11,200                                     | 1,000                     | 11,200          | 1,000                     | 1,117         | 8.93                           |
| Scottish Life         | 31 May 1891                 | 7                     | 8,800                                      | 163                       | 26,800          | 343                       | 235           | 1.58                           |
| City of Glasgow .     | 20 Jan. 1894                | 6                     | 6,450                                      | 448                       | 6,450           | 448                       | 148           | 6.95                           |
| Union                 | 31 Dec. 1892                | 3                     | 6,000                                      | 914                       | 6,000           | 914                       | 671           | 1.52                           |
| Scottish Metropolitan | 3I Dec. 1893                | 4                     | 5,550                                      | 161                       | 8,700           | 393                       | 185           | 4.52                           |
| London & Laneashire   | 31 Dec. 1892                | 3                     | 5,377                                      | 630                       | 5,377           | 630                       | 630           | 11.72                          |
| University            | 30 Apr. 1895                | 2                     | 4,124                                      | 604                       | 4,124           | 604                       | 604           | 14.65                          |
| Lancashire            | 31 Dec. 1894                | 1                     | 3,000                                      | 32                        | 3,000           | 32                        | 21            | 1.06                           |
| Provident Clerks      | 31 Dec. 1892                | 3                     | 3,000                                      |                           | 4,000           | 168                       | 168           | 4.20                           |
| United Kent           | 25 Mar. 1892                | 1                     | 2,263                                      | 525                       | 2,263           | 525                       | 577           | 23.20                          |
| Hand-in-Hand          | 31 Dec. 1892                | 1                     | 2,205                                      | 525                       | 2,205           | 525                       | 550           | 23.81                          |
| U.K.Temp.&Gen.Prv.    |                             | 1                     | 2,100                                      |                           | 2,100           | 525                       | 525           | 25.00                          |
| National of Ireland . | 31 Dec. 1892                | 1                     | 2,000                                      |                           | 2,000           | 84                        | 60            | 4.20                           |
| Midland Counties .    | 24 Mar. 1892                | 1                     | 1,000                                      | 42                        | 1,000           | 42                        | 42            | 4.20                           |
| Marine and General .  | 31 Dec. 1894                | 1                     | 1,000                                      | 25                        | 1,000           | 25                        | 25            | 2.50                           |
| Sun Life              | 31 Dec. 1891                | 2                     | 780  | 46                        | 780             | 46                        | 51            | 5.90                           |
| 52 Companies .        |                             | 987                   | 2,836,634                                  | 139,137                   | 3,977,465       | 188,865                   | 161,852       | 4.75                           |
|                       | •                           | •                     | •  | 1                         | <u> </u>        |                           | 1             | 1                              |

<sup>\*</sup> Approximated.

While there has been a very marked increase in the number and amount of insurances during the five years, the premiums received have not increased correspondingly, and the average premium is much lower than it has ever been before.

I am, Sir,

Your obedient servant,

M. M. LEES.

Edinburgh, 2 June 1899.

# THE INSTITUTE OF ACTUARIES.

# Prizes offered, 1899.

THE Council have resolved to offer two sets of Prizes, to be competed for by Members of the Institute of Actuaries. Particulars are given below. It will be seen that, while the Brown Prizes are open to all Members of the Institute except Members of the Council, only those may compete for the Chisholm Prizes who have qualified for the Fellowship since 1 January 1897. This restriction has been introduced in order to encourage the younger members, both at home and in the Colonies, and in the hope that some of them may send in valuable essays.

# SYLLABUS OF BROWN PRIZES, 1899.

The Council of the Institute of Actuaries have resolved to offer Two Prizes, of the value of Forty Guineas and Twenty Guineas respectively, for the best two Essays on the following subject, namely:

"The Actuarial Aspects of recent legislation in the United Kingdom and other Countries on the subject of Compensation to Workmen for Accidents."

The provisions of the various laws should be briefly described, and the compensation benefits secured under them should be analyzed.

Actuarial formulas and methods should be investigated for the calculation of the values of the benefits, with the corresponding premiums; and the nature of the statistics required for the purpose should be discussed, and suggestions should be offered as to the best means of collecting such statistics.

N.B.—In the forthcoming volume of Transactions of the International Actuarial Congress of 1898, will be found various papers containing much information which will assist in the enquiry.

### CONDITIONS OF THE COMPETITION.

- The Essays must be sent to the Honorary Secretaries of the Institute of Actuaries, at Staple Inn Hall, not later than 31 May 1900.
- 2. The Adjudicators shall be the President and Vice-Presidents of the Institute.
- Each Competitor must send in his name, under seal, with a Motto corresponding to one to be prefixed to his Essay. Such Essay and Motto must not be in the handwriting of the Competitor, and should be, where practicable, type-written.
- 4. Successful Essays shall become the property of the Institute.
- Unsuccessful Essays will be returned, on application at the Institute, with the corresponding envelopes unopened.
- Neither Prize will be awarded unless the Adjudicators consider some Essay worthy of the distinction.
- The Competition shall be open to all members of the Institute, except members of the Council.

## SYLLABUS OF MR. JAMES CHISHOLM'S PRIZES, 1899.

THE Council of the Institute of Actuaries have the pleasure to announce that Mr. James Chisholm having kindly left the balance (£25) of the prizes offered by him in 1896, but not awarded by the Adjudicators, at their disposal, they have decided to offer Two Prizes of Fifteen Pounds and Ten Pounds respectively, for the best Two Essays, to be written by Members of the Institute who have qualified for the Fellowship since 1 January 1897, on the following subject:

<sup>&</sup>quot; The rationale of Discounted Bonus Premiums."

Competitors have perfect liberty to treat the subject in any way they like, but it is expected that, *inter alia*, the questions of the present and future rates of Interest, the rate of Expenditure, Lapses, Surrender Values, Valuation Reserves, and Profits, so far as they relate to the subject of the Essay, and the difference between Non-Profit Rates of Premium and Discounted Bonus Premiums, will be discussed.

#### CONDITIONS OF THE COMPETITION.

- 1. Reference must not be made to individual Companies by name.
- The Essays must be sent to the Honorary Secretaries of the Institute of Actnaries, at Staple Inn Hall, not later than 31 March 1900.
- 3. The Adjudicators shall be the President and Vice-Presidents of the Institute.
- 4. Each Competitor must send in his name, under seal, with a Motto corresponding to one to be prefixed to his Essay. Such Motto and Essay must not be in the handwriting of the Competitor, and should be, where practicable, type-written.
- 5. Successful Essays shall become the property of the Institute.
- Unsuccessful Essays will be returned, on application at the Institute, with the corresponding envelopes unopened.
- No Prize will be awarded unless the Adjudicators consider some Essay or Essays worthy of the distinction.
- The Competition shall be open to those members of the Institute who have qualified for the Fellowship since 1 January 1897.

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